Math you need--vectors and vector addition

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To work out how much an area stretches, crunches, turns, or twists over time with GPS data, you need to work with vectors. Fortunately, if you've ever walked, ridden a bike or horse or train or plane or in a car, you have a good start on understanding them. This article spells out their basics—what they are, one way to show them, and how you can combine them.

Understanding vectors

Stand up and walk a few steps in a straight line. A vector for that motion would be an arrow pointing the direction you moved. The arrow's length would show the distance you moved. It could be the real length, but typically you would scale it to fit a map. For instance, if you walked due north five feet, a vector showing your walk would point north and could be five feet long, five inches, or five millimeters long. It could also be some other length at a particular and logical scale. This vector would be a "displacement vector."

All vectors must have both a magnitude and a direction. For example, to scientists and mathematicians, speed isn't a vector but velocity is. Speed is how fast you go—your displacement per time, such as 50 miles per hour. That's a magnitude, a number. But, in the world of science and math, velocity requires both speed and direction. A vehicle's velocity is its speed in a particular direction. The velocity of a truck, for instance, could be 50 mph heading west or 75 km/hr at an azimuth of 139 degrees.

We often show vectors as arrows (rays). The length of the arrow stands for the magnitude, and the direction the arrow points, not surprisingly, is the vector's direction. The vector's head is the pointed end of the arrow, and the vector's origin is the other end, the butt end. (Fig.1.)

Figure 1. A generic vector leading from an origin to an endpoint, where the head is.

We often draw vectors on a map or in a Cartesian coordinate system. The vector's origin could be at the graph's origin (0, 0) but it doesn't have to be. A scale makes the magnitude meaningful, and there is always some frame of reference to make the direction meaningful. You will be drawing vectors on a grid overlaying a map to show the displacement of a GPS station over time, i.e. its velocity. (Fig. 2.)

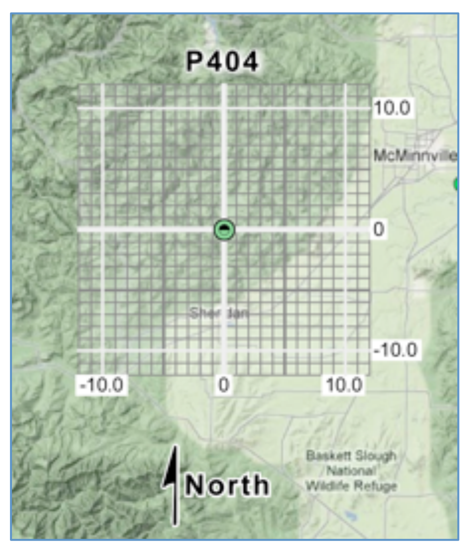

Figure 2. A google map with a grid on which you will draw vectors. Note the north arrow.

Adding vectors

When you walk in a city, you usually stick to sidewalks, so you might walk a block and then turn a corner and walk another block. If the buildings were to disappear, you could instead cut across the empty lot. Either way, you'd end up on a street corner diagonally across the block from where you started. You could show that as two vectors adding to make a third. (Fig. 3.)

Figure 3. Walking a block on College Avenue and Mulberry Street--or taking a shortcut.

One of the beauties of vectors is that you can add them head to tail like in Figure 3, or you can move them around. Mathematical rules allow you to adjust the position of the tails so long as the vectors keep their same direction and length. This will be useful when you draw vectors to show how a GPS station moves. You'll use two vectors—one to show its velocity in a north - south direction and another to show its east - west velocity. (Fig. 4.)

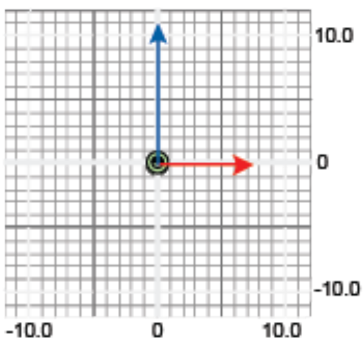

Figure 4. GPS velocity vectors. The blue vector represents about 11mm/yr to the north. The red vector represents about 7.5 mm/yr to the east.

To find the total velocity vector, move one of the vectors so that you can add the original vectors head to tail. (Fig. 5.) The direction of the new total velocity vector shows the overall direction the GPS moved, and its length gives the total velocity. The purpose is to know how far and in what direction the GPS station moved from when it was emplaced to now. You will use a protractor or directional compass to find the direction—the azimuth. You will use a scrap of paper or a ruler and the graph's scale to find the magnitude. Knowing the total velocity vectors from three GPS stations will allow you to analyze the strain in that part of Earth's crust.

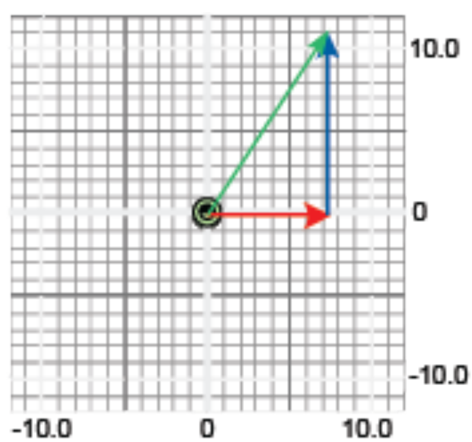

Figure 5. The green vector shows the total horizontal velocity vector--like the shortcut in Fig. 3. Its velocity is about 13.3 mm/yr to the northeast, with an azimuth of about 34 degrees.