Observing Transient Deformations:

Some General Considerations

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What Are We Doing?

We aim to **detect** and **analyze** transient deformations in our observations.

We hope to use these to reveal new physics in parts of the earthquake cycle otherwise unobservable.

Specifically, sources that are:

- **Slow enough** that the deformations are a quasi-static elastic response.
- **Inside the Earth** (surface loads are easier to observe in other ways)
- **At seismogenic depths.**
- From **slip on faults**, or inelastic response.

What Can We See?

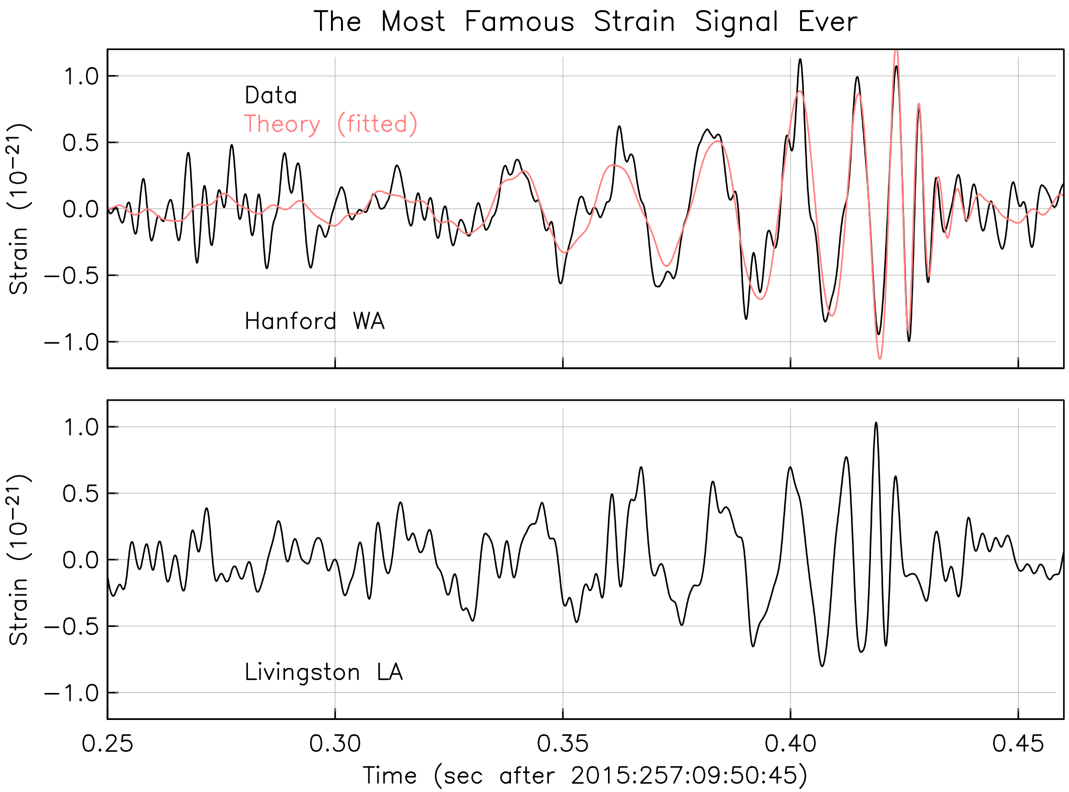
What size of sources we can see depends on three things:

- A. How signals from that source **decay with distance**.
- B. The **time history** of the source.
- C. **Instrument noise levels**: if the signal is less than the noise, we have nothing.

B and C are part of **detection**, a well-developed statistical theory that can be applied to any time series.

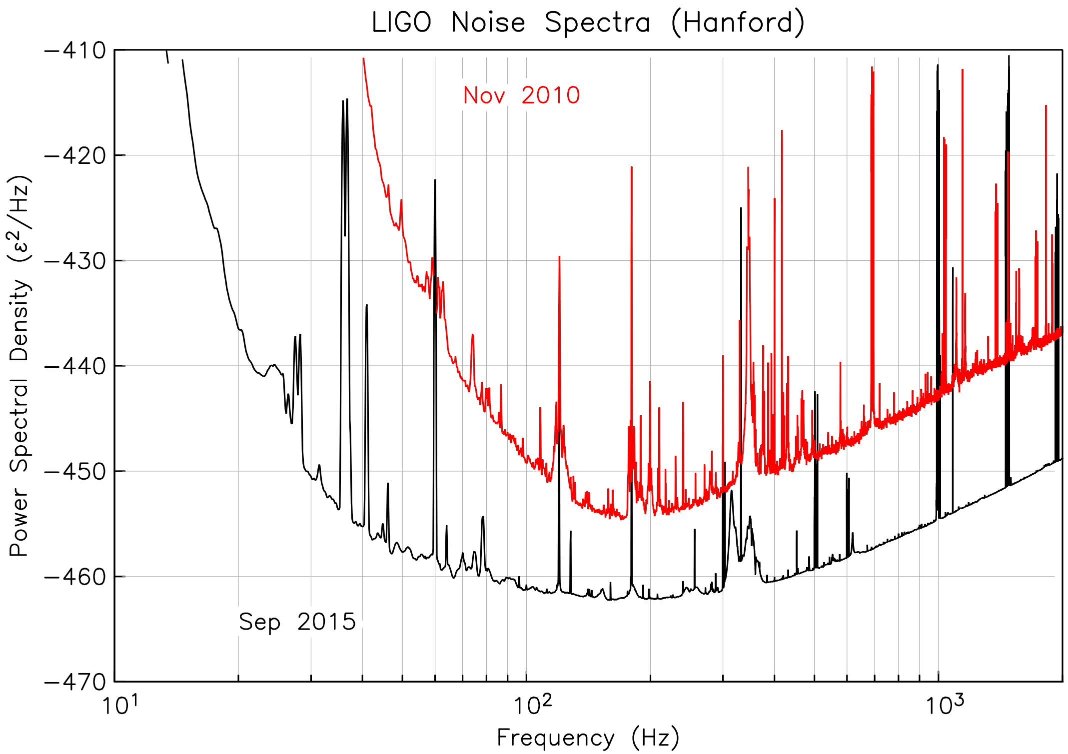
A will be more specifically geophysical.

An Astrophysical Strain



Data from two *really good* laser strainmeters (LIGO), showing deformations of the space-time continuum by gravitational radiation from two black holes that "inspiraled" and merged during the early Neoproterozoic.

Making Black-Hole Inspirals Detectable



The decrease in instrument noise level from 2010 to 2015 was what made the observation possible.

Signal Detection: General Theory

A time series can be modeled as a sum of some or all of

- **A. Periodic** variations (e.g. tides)
- **B. Random** variations not varying with time (stationary)
- **C. Transient** variations (whether "signal" or "noise")
- **D. Secular** variations, aka "drift", usually modeled by simple functions

Detection theory applies to detecting C when B is present, and can best be done in terms of the relevant **spectra**.

Kinds of Spectra I: Transients

A transient $u(t)$ has an **amplitude spectrum** $U(f)$:

$$U(f) = \int_{-\infty}^{\infty} u(t) e^{-2\pi i f t}\, dt$$

which has dimensions of signal $\times$ time, e.g. strain/Hz

Kinds of Spectra II: Stochastic

A stochastic variation $n(t)$ has a **power spectrum** $N(f)$:

$$N(f) = \int_{-\infty}^{\infty} c(u) e^{-2\pi i f u}\, du$$

where $c(u)$ is the **autocorrelation** of $n(t)$: where $c(u) = E[c(t)c(t+u)]$.

This has dimensions of signal squared $\times$ time, e.g. $(\text{strain})^2/\text{Hz}$

Often the power spectrum is given in $10\log_{10}(N(f))$, or **decibels**

Kinds of Spectra III: Periodic

A periodic signal (not necessarily sinusoidal) just has amplitudes:

$$p(t) = \sum_{m=1}^{M} a_m \cos(2\pi f_m t + \phi_{m)}$$

This has the dimension of the signal e.g. strain.

Detecting A Transient in Noise

What governs detection is the **signal-to-noise ratio**:

$$\mathrm{SNR} = \left[\int_0^\infty \frac{|U(f)|^2}{N(f)}\, df \right]^{\frac{1}{2}}$$

If u has a time constant t_S, we can approximate the SNR by $\frac{u_{RMS}}{B(t_S)}$, where

$B(t_S)$ is the RMS noise over a one-octave band: $B(t_S) = \left[\int_{\frac{1}{\sqrt{2}t_S}}^{\frac{\sqrt{2}}{t_S}} N(f)\, df \right]^{\frac{1}{2}}$

The LIGO noise from 50 to 100 Hz gives $B = 7 \times 10^{-23}$, so the SNR is about 10. A proper calculation, for two detectors, gives 24.

Why Black Holes are Easier

- **More than one observation** (probably essential).
- **Well-characterized noise** (including possible outside sources).
- **Good a-priori model** (from well-established physics).

Possible slip signals often lack these features.

Let's look at the physics of a signal driven by a moment tensor in the Earth.

Geophysical Source Theory

Displacement at a distance r from a moment-tensor source $\mathbf{M}(t)$ is

$$u = u_N + u_F = \frac{G_N(\theta,\phi)}{4\pi\rho c^2}\frac{1}{r^2}M(t-r/c) + \frac{G_F(\theta,\phi)}{4\pi\rho c^3}\frac{1}{r}\frac{dM(t-r/c)}{dt}$$

where c is the wave speed, ρ the density, and the G's give radiation patterns.

So u combines a **near-field** term decaying as r^{-2}, and a **far-field** (radiation) term decaying as r^{-1}.

Seismic waves are the far-field term – and they are, in this model, always present.

Decay with Distance

If the time constant of the moment release is t_S, that the magnitude of $\frac{dM}{dt}$ is roughly M/t_S. The ratio of far-field to near-field displacement is, also roughly,

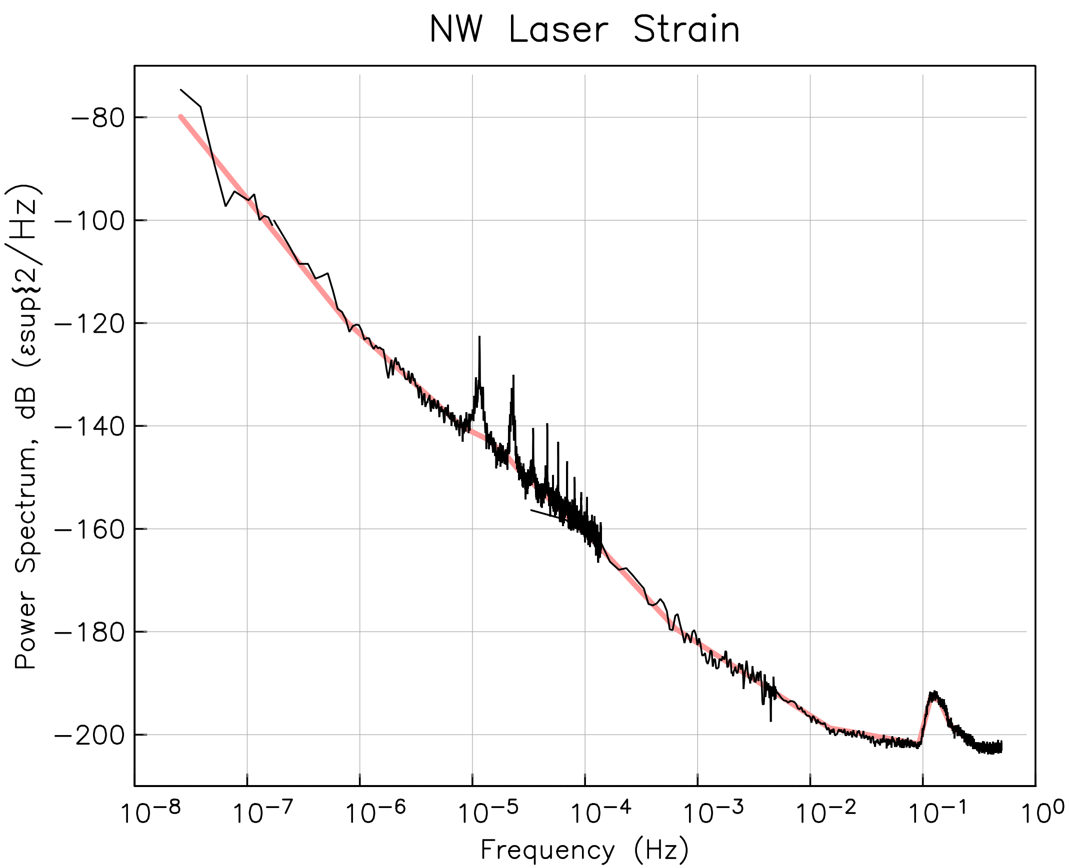
$$\frac{u_F}{u_N} = \frac{G_F \dot{M}}{G_N M} \frac{r}{c} \approx \frac{r}{t_S c} = \frac{T}{t_S}$$

where T is the travel time.

For a typical geodetic network, the spacing gives a distance corresponding to travel times $T \approx 1$–10^2s.

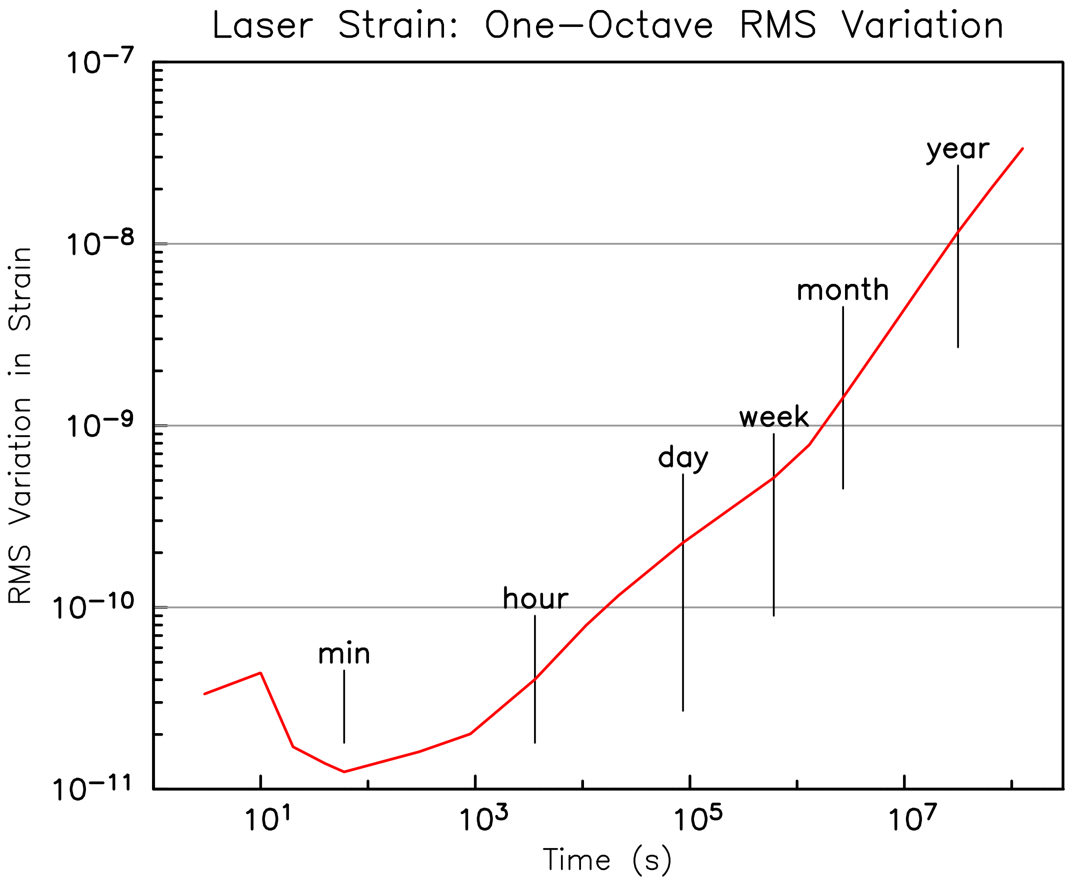
The near-field term dominates for any source slower than that.

Strainmeter Spectrum I



We use a high-quality strain record, from the NW-SE laser strainmeter at Piñon Flat Observatory, and fit a spectrum to this that consists of linear segments (in log-log space: a composite power-law spectrum).

Strainmeter Spectrum II



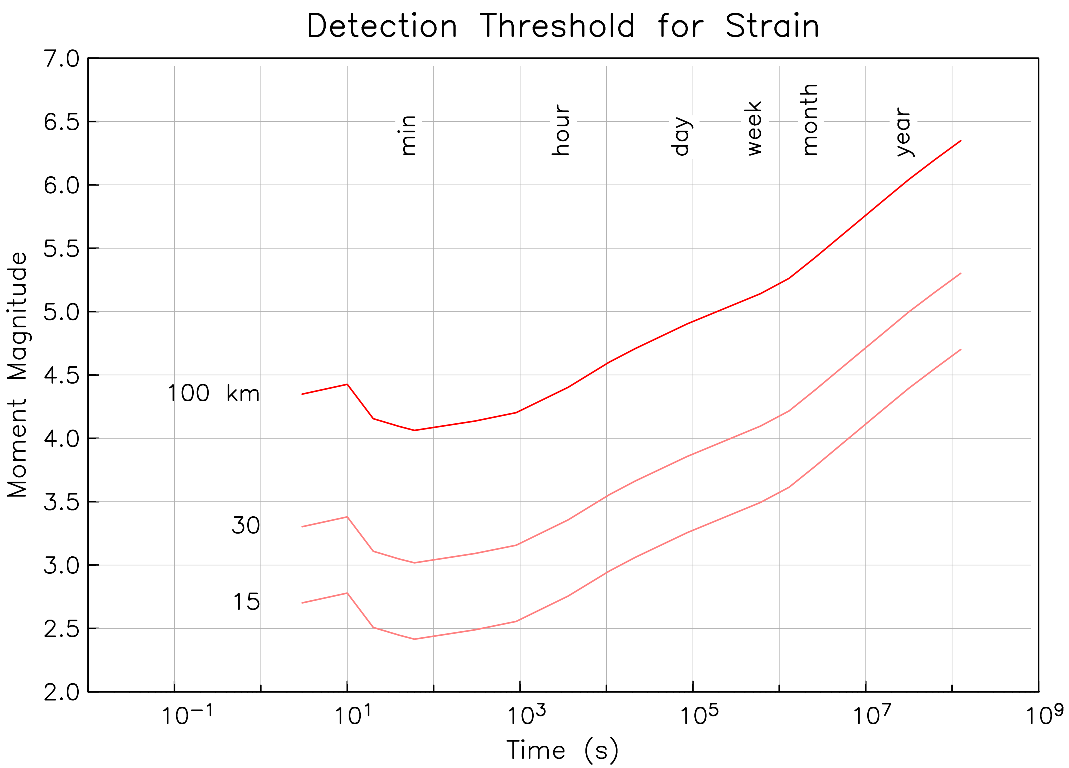
And from this we find $B(t_S)$.

Finding the Detection Level

To find out what moment we can detect, we equate the near-field strain $E = \frac{K_E M_0}{r^3}$ to $B(t_S)$. For a particular value of r, this gives a line in M_0 against t_S.

- On one side are events too small to detect.
- On the other, events we should be able to.

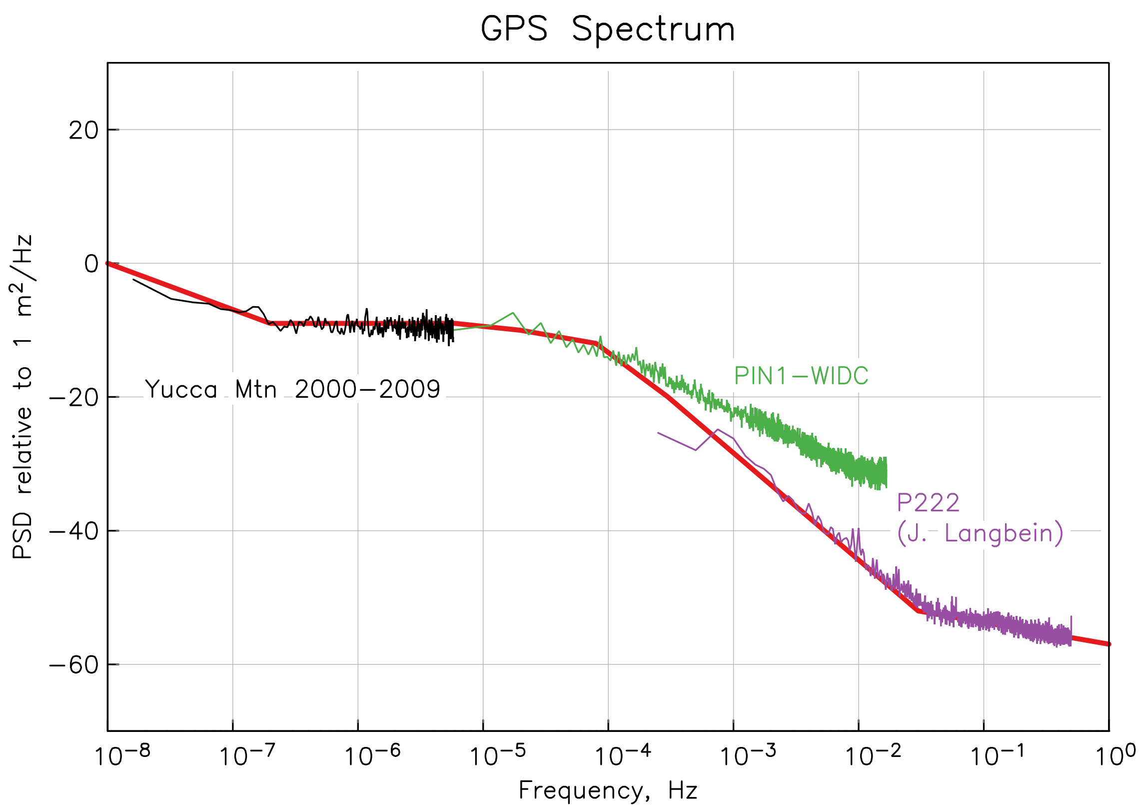
Detection Level for Laser Strainmeter



We choose distances 15 km (the minimum) to 100 km.

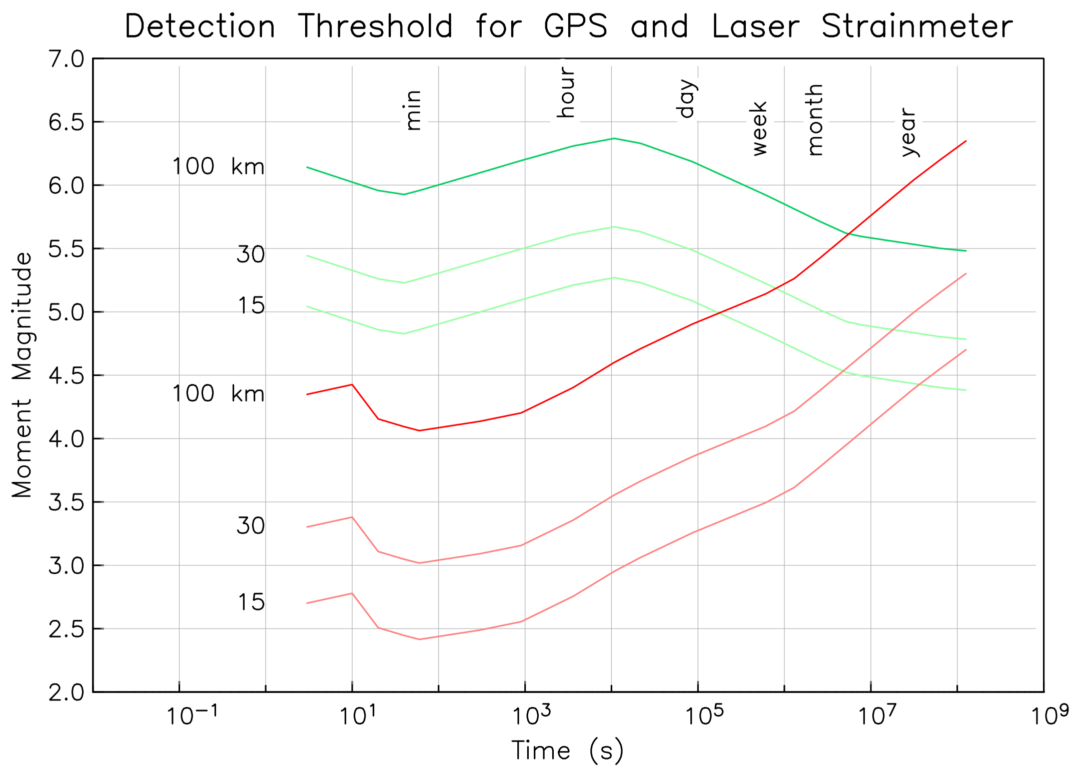
At short times and large distances the detection line would be lower (more sensitive) because the far-field term becomes important.

GPS Spectrum



Piecewise linear approximation to noise spectrum, using a range of datasets/analyses.

Detection Level for GPS and Laser Strainmeter

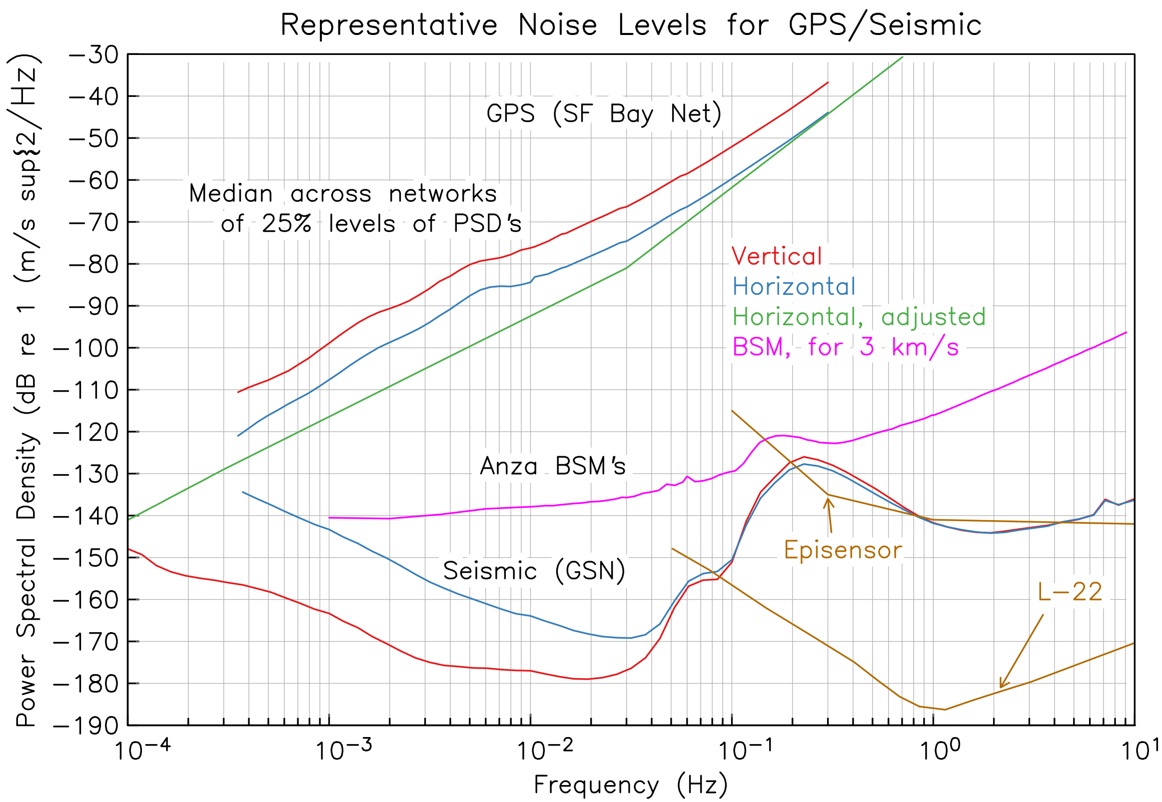


Close to the source, and at periods of less than years, the detection level can be **much** lower on the strainmeter: there may be a large class of transient deformations detectable on strainmeters but not GPS.

Some Practical Issues

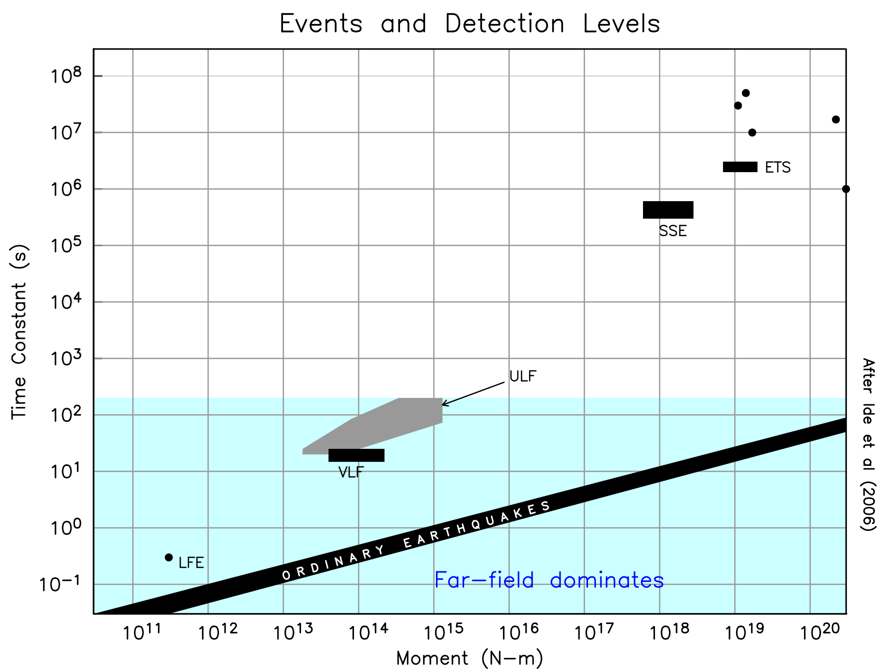
- Detection levels improved by stacking (over time or over stations).
- This is more difficult for the BSM's (than for GPS) because there can be variations in noise levels on
  - **Different stations**
  - **Different channels** at the **same station**. (So it is important to **look at channel data**.)
  - **Different times** for the **same channel**.
- Each case may have to be handled separately.

At Higher Frequencies



Broadband seismic best, especially vertical; high-rate GPS only useful for strong motion (and even then only at 10 s and longer).

How Does this Match Observations?



Lack of events on the upper left is not a scaling law, but the limits of observation.

Two Lessons

- Always look at the noise spectrum.
- Be conservative: the SNR assumes that you do know the noise, and you may not: 3σ may not be enough.