Introduction to Strainmeters and Strain

2018 UNAVCO Science Workshop

Evelyn Roeloffs

March 26, 2018

USGS Earthquake Science Center

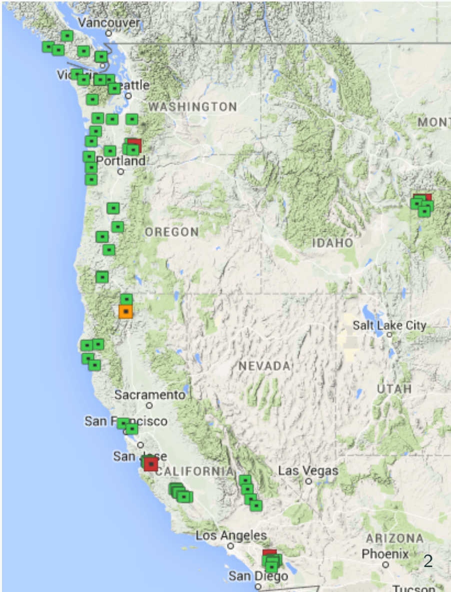
Topics

1. Why use strainmeters?
2. What a PBO borehole strainmeter measures
3. How borehole strainmeter output is related to strain
4. Strain terminology, notation, math
5. Deriving the horizontal strain tensor from strainmeter output

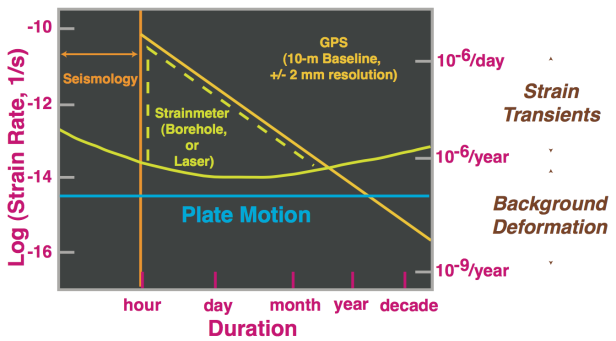
Why use strainmeters?

Plate Boundary Observatory (PBO) borehole strainmeter network

- Funded by NSF as part of the Earthscope iniative
- 78 Gladwin Tensor Strainmeters (GTSMs)
- Installed 2004-2013
- Depths 500-800 feet (150-250 m)
- Strain resolution 10^{-10}
- Sampling rate 20 sps



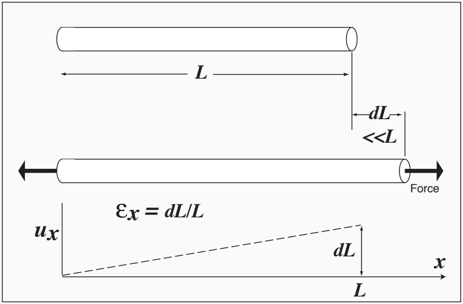
Strainmeters fill "gap" between seismology and GPS



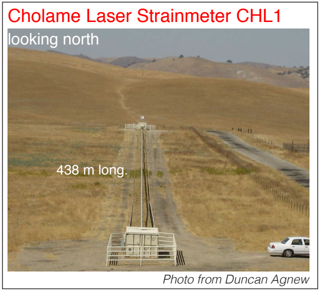
- For periods of hours to ∼10 days, strainmeters can detect time-varying crustal deformation that does not produce displacements large enough to measure with GNSS or InSAR
- Strainmeters measure strain: a tensor quantity derived from spatial derivatives of displacement

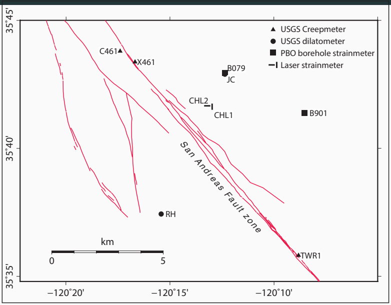
What is "strain"?

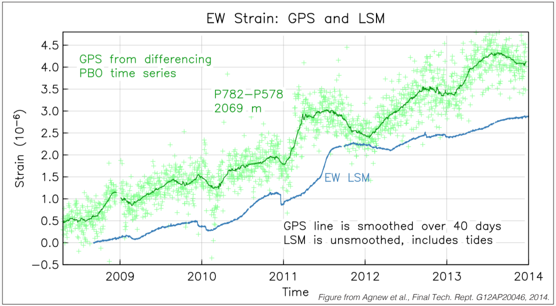
- "Strain" is a change in one or more dimensions of a solid body, relative to a reference state
  - Size may change
  - Shape may change
- We assume here that strains are small, so "infinitesimal strain theory" applies
- In 1 dimension, strain can be quantified as (change in length)/(original length)



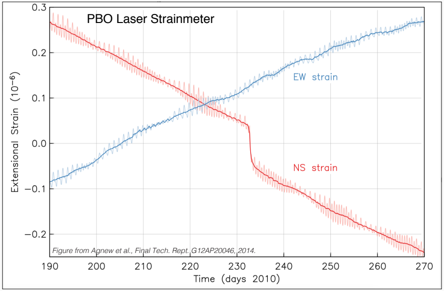
Example: Single component of a Laser Strainmeter (LSM)

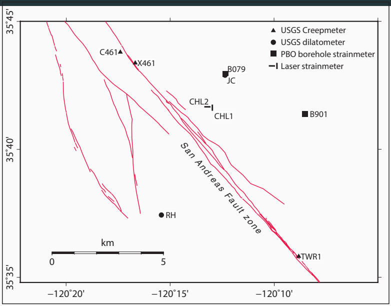


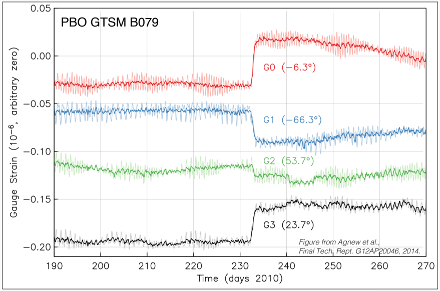




Example: A strain event on LSM and GTSM



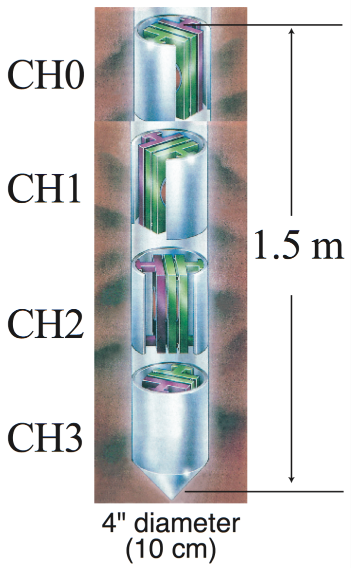




What a PBO borehole strainmeter measures

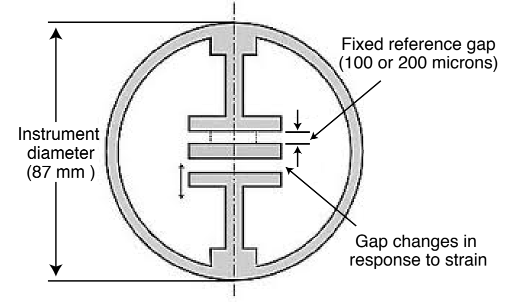
Gladwin Tensor Strain Meter (GTSM)

- Developed in Australia by Michael Gladwin
- Four "gauges" measure inner diameter of steel housing
  - All types of borehole strainmeter measure housing or borehole diameter
- Three gauges (CH0, CH1, and CH2) are 120° apart around the borehole axis
- The fourth gauge (CH3) is perpendicular to CH1



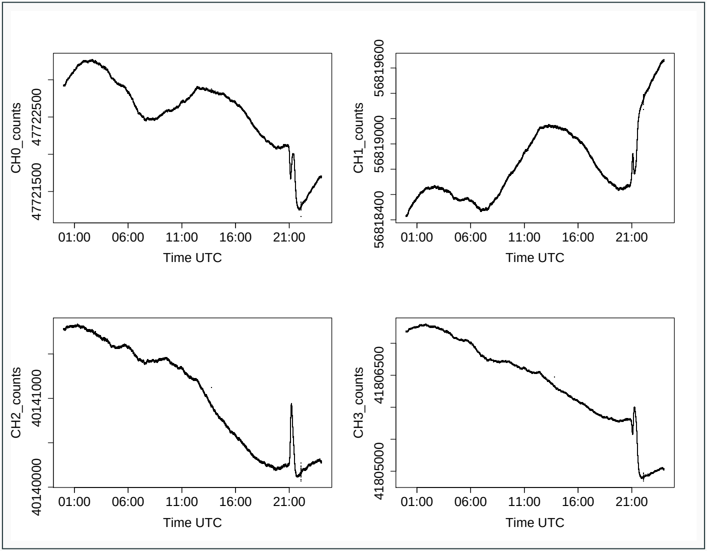
Gladwin Tensor Strain Meter: Capacitive sensing element

- Instrument diameter changes in response to strain
- Reference gap is fixed
- Strain changes capacitance of moveable gap
- Capacitance changes are measured using a bridge circuit whose other arms are at the surface
- Raw GTSM data are capacitance bridge readings in counts

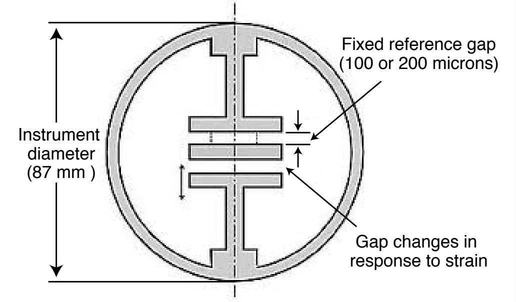


Fractional gauge elongation is obtained by "linearizing" raw GTSM data

One day of 1 sps raw gauge data from B073

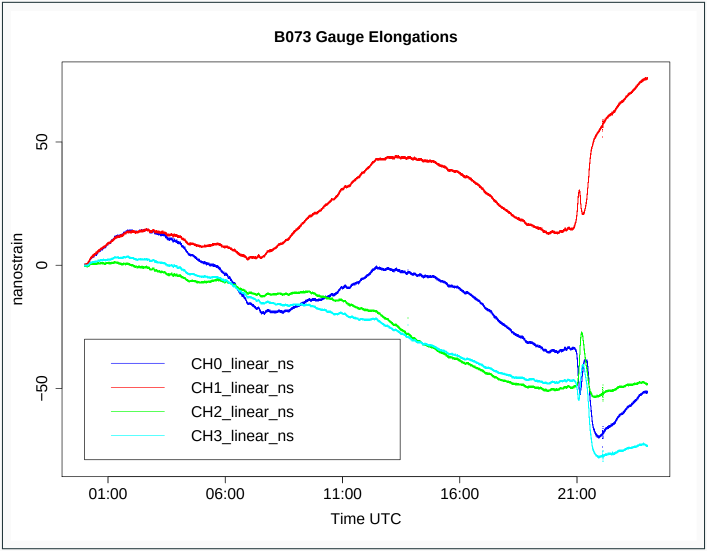


Linearizing GTSM gauge data



- $R(t)$ denotes raw gauge data in counts at time t
- $R(t_0)$ denotes raw gauge data at some reference time t_0
- e denotes fractional elongation of gauge
- $e = \left[\left(\frac{R(t)/1E8}{1-R(t)/1E8} \right) - \left(\frac{R(t_0)/1E8}{1-R(t_0)/1E8} \right) \right] \times \frac{ReferenceGap}{Diameter}$

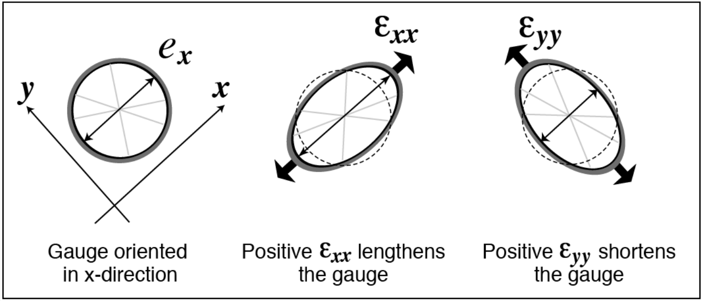
1 sps data after linearizing



How borehole strainmeter output is related to strain

Elongation of a single ideal gauge in response to strain

Gauge elongation, e_i, is a linear combination of strain parallel and perpendicular to the gauge



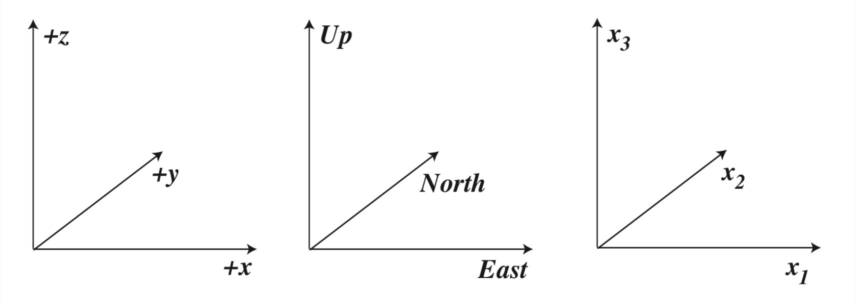
If x and y are parallel and perpendicular to the gauge, then

$$e_x = A\epsilon_{xx} - B\epsilon_{yy}$$

A and B are positive scalars with $A > B$

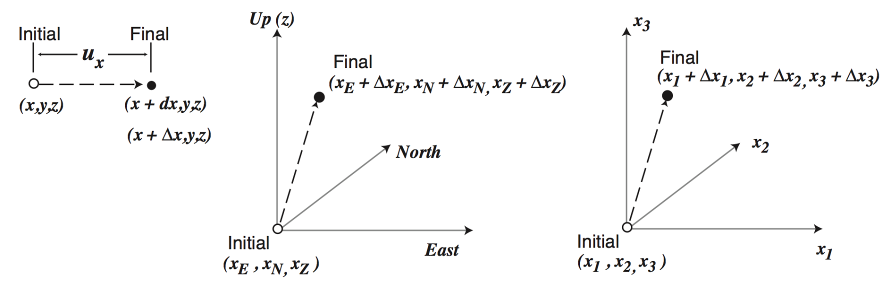
Strain terminology, notation, math

Notation: Coordinate systems



- Right-handed Cartesian coordinate system
- Various sets of names for coordinate axes (examples above)
- Horizontal axes will not always be East and North
- Strainmeters do not care about:
    - Curvature of the earth
    - Geodetic reference frames

Notation: Displacements



- Material at a point can move in three directions, e.g. (u_x, u_y, u_z) or (u_1, u_2, u_3)
- Various sets of names for components of displacement
  - e.g., 1,2,3 or x, y, z
- Strain is a result of spatially varying displacement

Spatial derivatives of displacement: Strain and rotation

- Displacement has 3 components, e.g. (u_1, u_2, u_3)
- Deformation gradient $= \begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{bmatrix} = [\frac{\partial u_i}{\partial x_j}]$, $i, j = 1, 2, 3$
- Strain and rotation are the symmetric and antisymmetric parts of the deformation gradient:
  - $[\frac{\partial u_i}{\partial x_j}] = \frac{1}{2}[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}] + \frac{1}{2}[\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}]$
- Strain components: $\epsilon_{ij} = \frac{1}{2}[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}]$
- Deformation gradient, strain, and rotation matrices all represent tensor quantities
- Strainmeters respond only to strain, not rotation
  - Rotating a body does not change its shape or size, so strainmeters do not detect rotation

The strain tensor in 3 and 2 dimensions

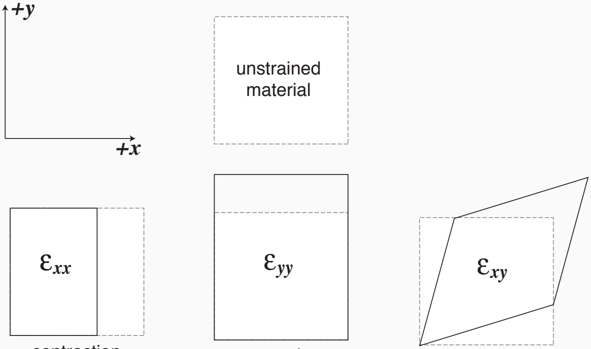
- Strain components: $\epsilon_{ij} = \frac{1}{2}[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}]$
- Strain as 3x3 matrix: $\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}[\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}] & \frac{1}{2}[\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}] \\ \frac{1}{2}[\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}] & \frac{\partial u_2}{\partial x_2} & \frac{1}{2}[\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}] \\ \frac{1}{2}[\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}] & \frac{1}{2}[\frac{\partial u_3}{\partial x_2} + \frac{\partial u_2}{\partial x_3}] & \frac{\partial u_3}{\partial x_3} \end{bmatrix}$
- "Normal" strains have $i = j$: $\epsilon_{ii} = \frac{\partial u_i}{\partial x_i}$ (no summation implied)
- "Shear" strains have $i \neq j$, note that $\epsilon_{ij} = \epsilon_{ji}$
- 2-D strain, 2x2 matrix, 3 strain components:

$$\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}[\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}] \\ \frac{1}{2}[\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}] & \frac{\partial u_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{12} & \epsilon_{22} \end{bmatrix}$$

The tensor nature of strain

- 2-D strain, 2x2 matrix: $\begin{bmatrix} \epsilon_{11} & \epsilon_{12} \\ \epsilon_{12} & \epsilon_{22} \end{bmatrix}$ or $\begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} \\ \epsilon_{xy} & \epsilon_{yy} \end{bmatrix}$ or $\begin{bmatrix} \epsilon_{EE} & \epsilon_{EN} \\ \epsilon_{EN} & \epsilon_{NN} \end{bmatrix}$
- Like any rank-2 tensor, strain can be represented as a matrix, but not every matrix is a rank-2 tensor
- The numerical values of the strain tensor‘s matrix representation depend on the coordinate system
- We will express the strain tensor in various coordinate systems, e.g.:
    - Parallel and perpendicular to a strainmeter gauge
    - East and North
    - Parallel and perpendicular to a fault
    - Radial and tangential to a volcano

Horizontal (2D) strain components; sign conventions



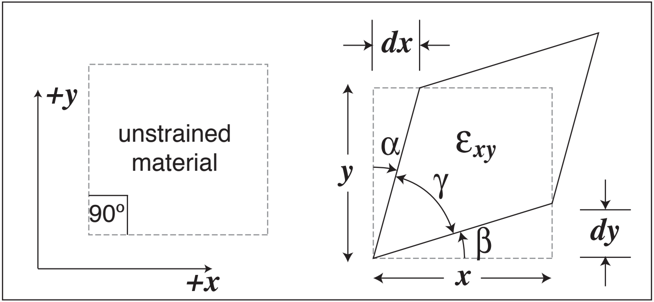
contraction in the x-direction (a negative strain)

extension in the y-direction (a positive strain)

xy shear (a positive strain because y-displacement increases with increasing x)

$$\epsilon_{xx} = \frac{\partial u_x}{\partial x} \qquad \epsilon_{yy} = \frac{\partial u_y}{\partial y} \qquad \epsilon_{xy} = \frac{1}{2}\left[\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right]$$

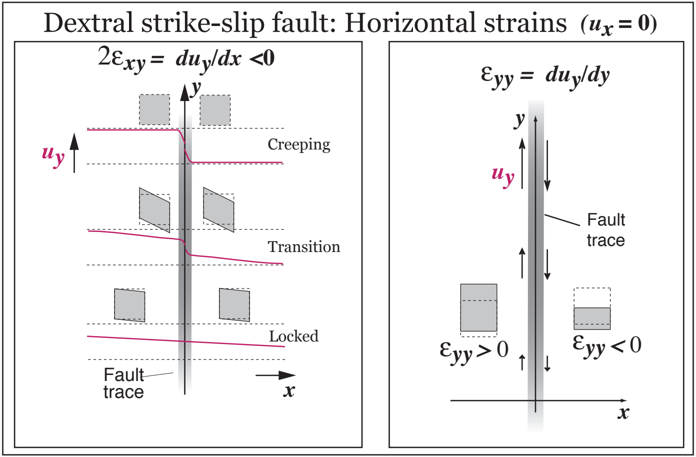
Engineering shear in terms of angle change



$$\epsilon_{xy} \approx \frac{1}{2}\left[\frac{dx}{y} + \frac{dy}{x}\right]$$
$$= \frac{1}{2}[\tan\alpha + \tan\beta]$$
$$\approx \frac{1}{2}[\alpha + \beta] \text{ for small } \alpha \text{ and } \beta$$
$$\epsilon_{xy} \approx \frac{1}{2}[90^\circ - \gamma]$$

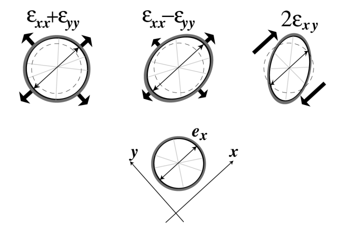
Sketch shows a positive shear strain

Example: Locked vs. creeping strike-slip fault



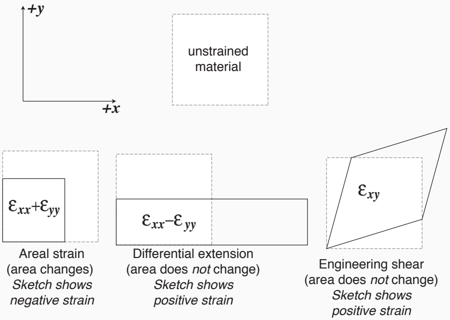
Map view

Elongation of ideal gauge: Areal strain and differential extension



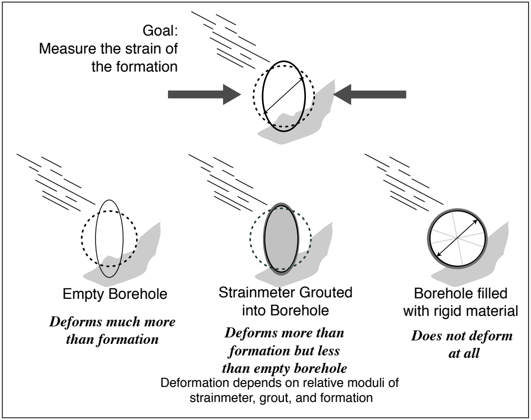
- $e_x = A\epsilon_{xx} - B\epsilon_{yy} = 0.5(A - B)(\epsilon_{xx} + \epsilon_{yy}) + 0.5(A + B)(\epsilon_{xx} - \epsilon_{yy})$
- We refer to $\epsilon_{xx} + \epsilon_{yy}$ as "areal strain" and $\epsilon_{xx} - \epsilon_{yy}$ as "differential extension"
- Define $C = 0.5(A - B)$, the "areal strain response coefficient" and $D = 0.5(A + B)$, the "shear strain response coefficient"
- $e_x = C(\epsilon_{xx} + \epsilon_{yy}) + D(\epsilon_{xx} - \epsilon_{yy})$
- NOTE: ϵ_{xy} doesn‘t change length of an ideal gauge parallel to x or y

Areal strain, differential extension, engineering shear



- Areal strain $\epsilon_{xx} + \epsilon_{yy}$ does not depend on coordinate system
- We refer to differential extension $(\epsilon_{xx} - \epsilon_{yy})$ and engineering shear $2\epsilon_{xy}$ as shear strain components
    - Neither shear strain component changes area
    - Both shear strains depend on coordinate system

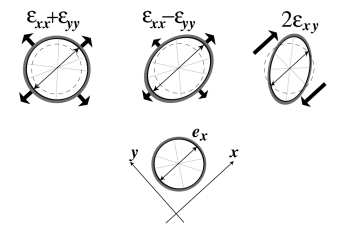
Gauge elongation depends on elastic moduli of the formation



- The stiffer the formation, the larger are C and D
- Nominal values are $C = 0.75$ and $D = 1.5$ for the strain component definitions here
- C and D are estimated for each strainmeter (or gauge) based on known strain signals

Deriving the horizontal strain tensor from strainmeter output

Measurements from several gauges must be combined to determine the strain tensor

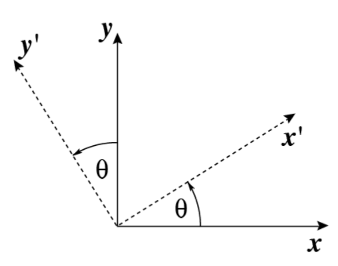


- $e_x = C(\epsilon_{xx} + \epsilon_{yy}) + D(\epsilon_{xx} - \epsilon_{yy})$
- Each gauge responds to only two strain components, if strain is expressed in gauge-parallel coordinates
- To combine measurements from different gauges, need to express them in a single coordinate system
- This requires understanding how to express the strain tensor in a rotated coordinate system

Transforming horizontal strains to rotated coordinates

Horizontal strain tensor can be expressed in a coordinate system rotated about the vertical axis

θ is the angle from the original coordinate system to the new coordinate system, measured counterclockwise (CCW)



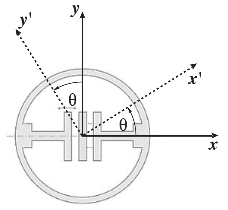
$$\begin{bmatrix} \epsilon_{x'x'} + \epsilon_{y'y'} \\ \epsilon_{x'x'} - \epsilon_{y'y'} \\ 2\epsilon_{x'y'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta \\ 0 & -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \epsilon_{xx} + \epsilon_{yy} \\ \epsilon_{xx} - \epsilon_{yy} \\ 2\epsilon_{xy} \end{bmatrix}$$

- Areal strain is invariant under rotation: $\epsilon_{x'x'} + \epsilon_{y'y'} = \epsilon_{xx} + \epsilon_{yy}$ for any θ
- Shear strains are functions of 2θ

Gauge elongations in a non-gauge-parallel coordinate system

With x parallel to the i^{th} gauge,

$$e_i = [C, D, 0] \times \begin{bmatrix} \epsilon_{xx} + \epsilon_{yy} \\ \epsilon_{xx} - \epsilon_{yy} \\ 2\epsilon_{xy} \end{bmatrix}$$



$$e_i = [C, D, 0] \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\theta & -\sin 2\theta \\ 0 & \sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \epsilon_{x'x'} + \epsilon_{y'y'} \\ \epsilon_{x'x'} - \epsilon_{y'y'} \\ 2\epsilon_{x'y'} \end{bmatrix}$$

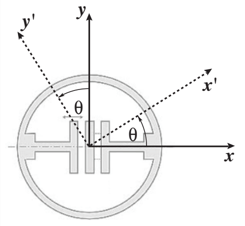
$$e_i = [C, D\cos 2\theta, -D\sin 2\theta] \begin{bmatrix} \epsilon_{x'x'} + \epsilon_{y'y'} \\ \epsilon_{x'x'} - \epsilon_{y'y'} \\ 2\epsilon_{x'y'} \end{bmatrix}$$

$$e_i = C(\epsilon_{x'x'} + \epsilon_{y'y'}) + D\cos 2\theta(\epsilon_{x'x'} - \epsilon_{y'y'}) - D\sin 2\theta(2\epsilon_{x'y'})$$

In a non-gauge-parallel coordinate system, gauge elongation depends on engineering shear

With x parallel to the i^{th} gauge,
$e_i = C(\epsilon_{xx} + \epsilon_{yy}) + D(\epsilon_{xx} - \epsilon_{yy})$

Note that e_i does not depend on $2\epsilon_{xy}$

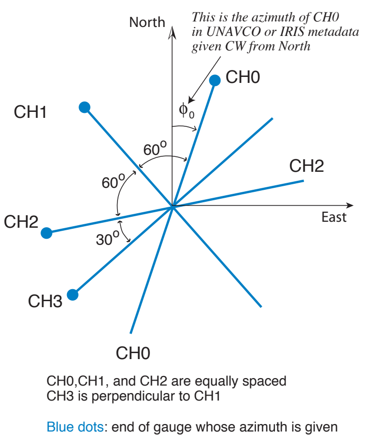


If the gauge is not aligned along the coordinate axis:

$$e_i = C(\epsilon_{x'x'} + \epsilon_{y'y'}) + D\cos 2\theta(\epsilon_{x'x'} - \epsilon_{y'y'}) - D\sin 2\theta(2\epsilon_{x'y'})$$

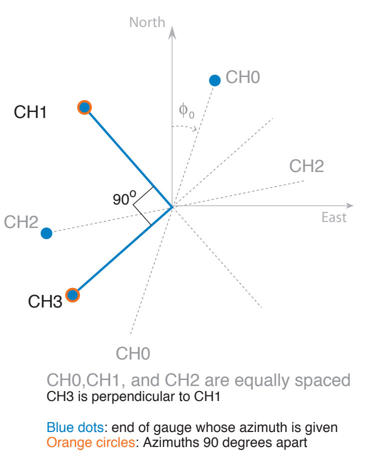
Now e_i does depend on $2\epsilon_{x'y'}$

PBO 4-component GTSM: Gauge configuration from metadata



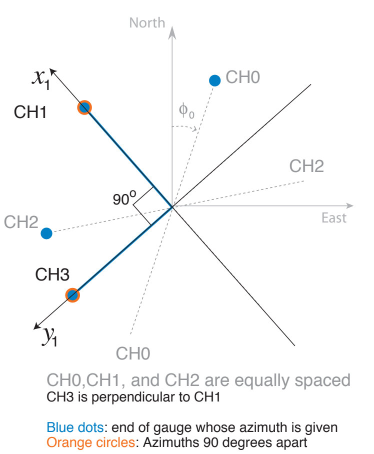
- Strainmeter orientation cannot be controlled during installation
- Orientation is measured after installation (and may be inaccurate)
- UNAVCO and IRIS metadata give measured azimuths of the 4 gauges
- It does not matter which "end" of the strainmeter gauge is referred to
- Azimuths are clockwise from north

PBO 4-component GTSM: Orthogonal CH1 and CH3



- Ignore CH0 and CH2 for now...note that CH1 and CH3 are orthogonal

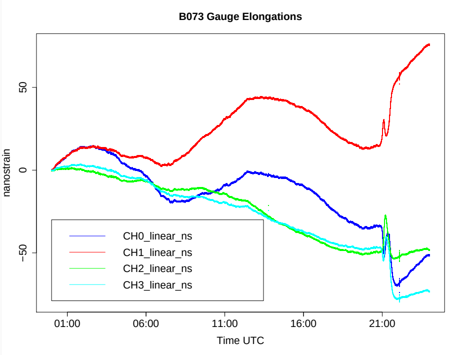
PBO 4-component GTSM: Orthogonal CH1 and CH3

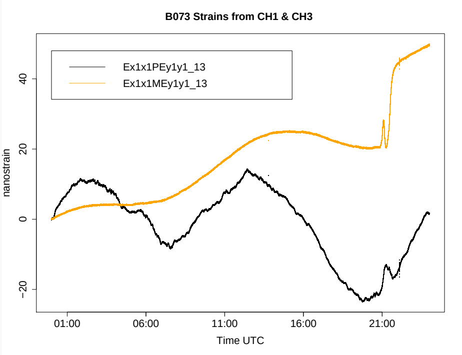


- Use right-handed coordinates x_1, y_1
- x_1 is parallel to CH1 so CH1 elongation is
  $e_1 = C(\epsilon_{x_1x_1} + \epsilon_{y_1y_1}) + D(\epsilon_{x_1x_1} - \epsilon_{y_1y_1})$
- Use formula for rotating by 90° to get e_3:
  $e_3 = C(\epsilon_{x_1x_1} + \epsilon_{y_1y_1}) - D(\epsilon_{x_1x_1} - \epsilon_{y_1y_1})$
- Solve for areal strain and differential extension:
  $(\epsilon_{x_1x_1} + \epsilon_{y_1y_1}) = (e_1 + e_3)/2C$
  $(\epsilon_{x_1x_1} - \epsilon_{y_1y_1}) = (e_1 - e_3)/2D$

- Areal strain is proportional to average of gauge elongations
- Differential extension is proportional to difference between gauge elongations

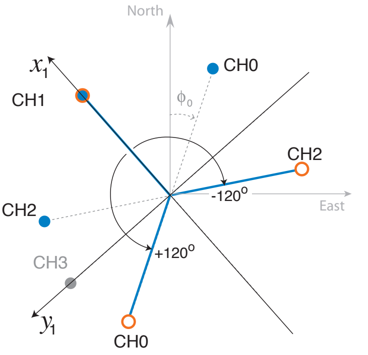
Strains from CH1 and CH3: Example





Note these strains are expressed in coordinates aligned along B073 CH1
Metadata give this as N210°E, equivalently, N30°E

PBO 4-component GTSM: Equally spaced CH0,CH1,CH2



CH0,CH1, and CH2 are equally spaced

Blue dots: end of gauge whose azimuth is given
Orange circles: Azimuths 120 degrees apart

- At least 3 gauges are needed to get the three components of the horizontal strain tensor
- Now ignore CH3...note that CH0,CH1,CH2 are equally spaced in azimuth
  - Need to use the opposite "ends" of CH1 and CH2
- Use same coordinates with x_1 parallel to CH1
- CH0 is $+120^\circ$ from CH1 and CH2 is -120° from CH1
  - Note these angles are positive CCW, using polar coordinate math convention

3 gauge elongations to 3 strain components

- 3 identical gauges 120° apart $(CH0, CH1, CH2) = (e_0, e_1, e_2)$
- Express elongations in CH1-parallel coordinates:

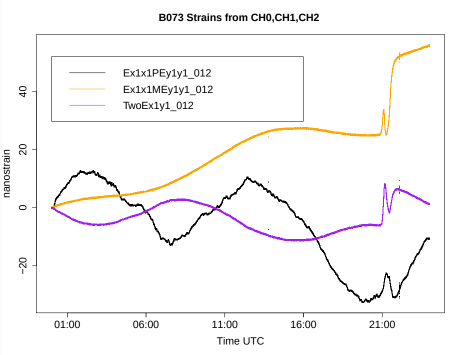
$$e_0 = C(\epsilon_{x_1x_1} + \epsilon_{y_1y_1}) + Dcos(240^\circ)(\epsilon_{x_1x_1} - \epsilon_{y_1y_1}) + Dsin(240^\circ)(2\epsilon_{x_1y_1})$$
$$e_1 = C(\epsilon_{x_1x_1} + \epsilon_{y_1y_1}) + D(\epsilon_{x_1x_1} - \epsilon_{y_1y_1})$$
$$e_2 = C(\epsilon_{x_1x_1} + \epsilon_{y_1y_1}) + Dcos(-240^\circ)(\epsilon_{x_1x_1} - \epsilon_{y_1y_1}) + Dsin(-240^\circ)(2\epsilon_{x_1y_1})$$

- Solve for strain components:
  $(\epsilon_{x_1x_1} + \epsilon_{y_1y_1}) = (e_0 + e_1 + e_2)/3C$
  $(\epsilon_{x_1x_1} - \epsilon_{y_1y_1}) = [(e_1 - e_0) + (e_1 - e_2)]/3D$
  $2\epsilon_{x_1y_1} = (e_2 - e_0)/[2(0.866D)]$
- Areal strain is proportional to average of outputs from equally spaced gauges
- Shear strains are proportional to differences among gauge outputs

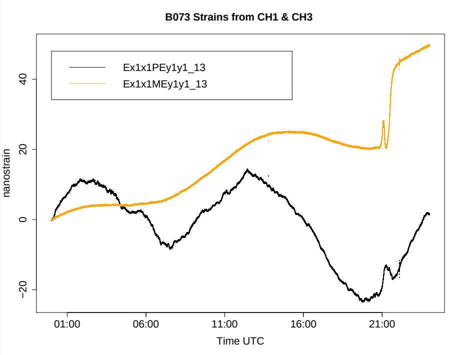
Strains from different gauge subsets: Example

Any subset of 3 gauges can be used to obtain the horizontal strain tensor

CH0, CH1, CH2



CH1 and CH3

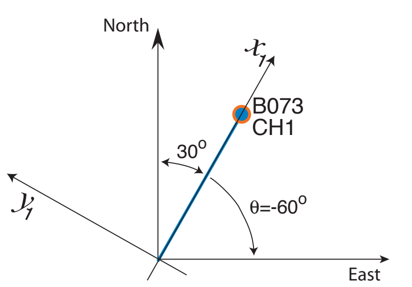


These strains are expressed in coordinates aligned along B073 CH1= N210°E, equivalently, N30°E

The rotation formula can be used to express them in E-N coordinates

Transforming horizontal strains to E-N coordinates

Apply the rotation formula with $\theta = -60^\circ$



$$\begin{bmatrix} \epsilon_{EE} + \epsilon_{NN} \\ \epsilon_{EE} - \epsilon_{NN} \\ 2\epsilon_{EN} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-120^\circ) & \sin(-120^\circ) \\ 0 & -\sin(-120^\circ) & \cos(-120^\circ) \end{bmatrix} \begin{bmatrix} \epsilon_{x_1x_1} + \epsilon_{y_1y_1} \\ \epsilon_{x_1x_1} - \epsilon_{y_1y_1} \\ 2\epsilon_{x_1y_1} \end{bmatrix}$$

Note that the areal strain is unchanged.

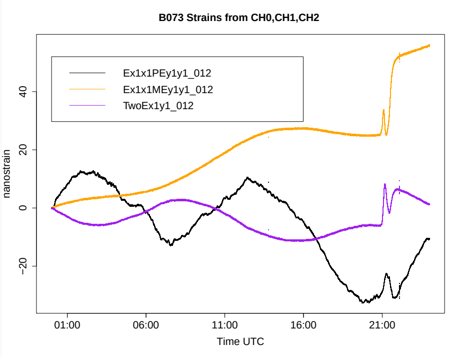
$$\epsilon_{EE} - \epsilon_{NN} = 0.5(\epsilon_{x_1x_1} + \epsilon_{y_1y_1}) - 0.866(2\epsilon_{x_1y_1})$$

$$2\epsilon_{EN} = 0.866(\epsilon_{x_1x_1} + \epsilon_{y_1y_1}) + 0.5(2\epsilon_{x_1y_1})$$

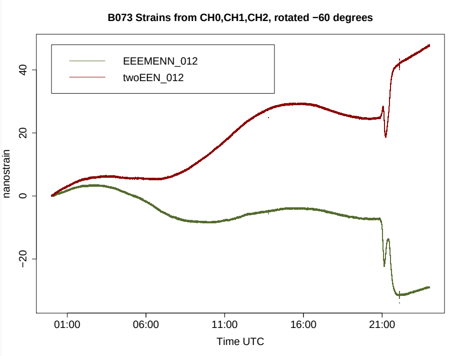
Strains in E-N coordinate system: Example

East is 60° CW from B073 CH1

x_1 parallel to B073 CH1



Shear strains in E-N coordinates



Time for questions...