Earth Tides

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Outline

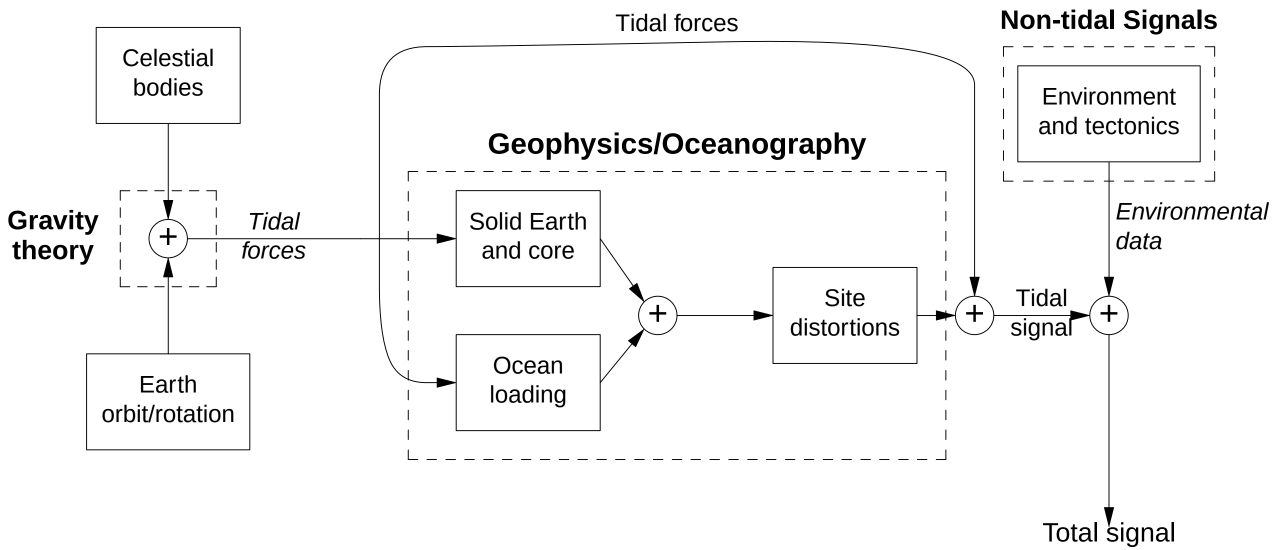
- What are the Earth Tides?
- How do We Estimate/Remove Tides in a Series?
- How do we Model the Earth Tides?

Why Do We Care About Tides?

- We want to remove them to see other things.
- So we can use them as a calibration signal for the BSM's.
  - To look at other signals, we want to to analyze (and predict) the tides as well as we can.
  - Earth-tide studies (and calibrations) compare an analysis with model tides to understand the Earth.

Fortunately, accurate modeling of the tides is – mostly – not too difficult.

From the Moon/Sun to Our Instruments

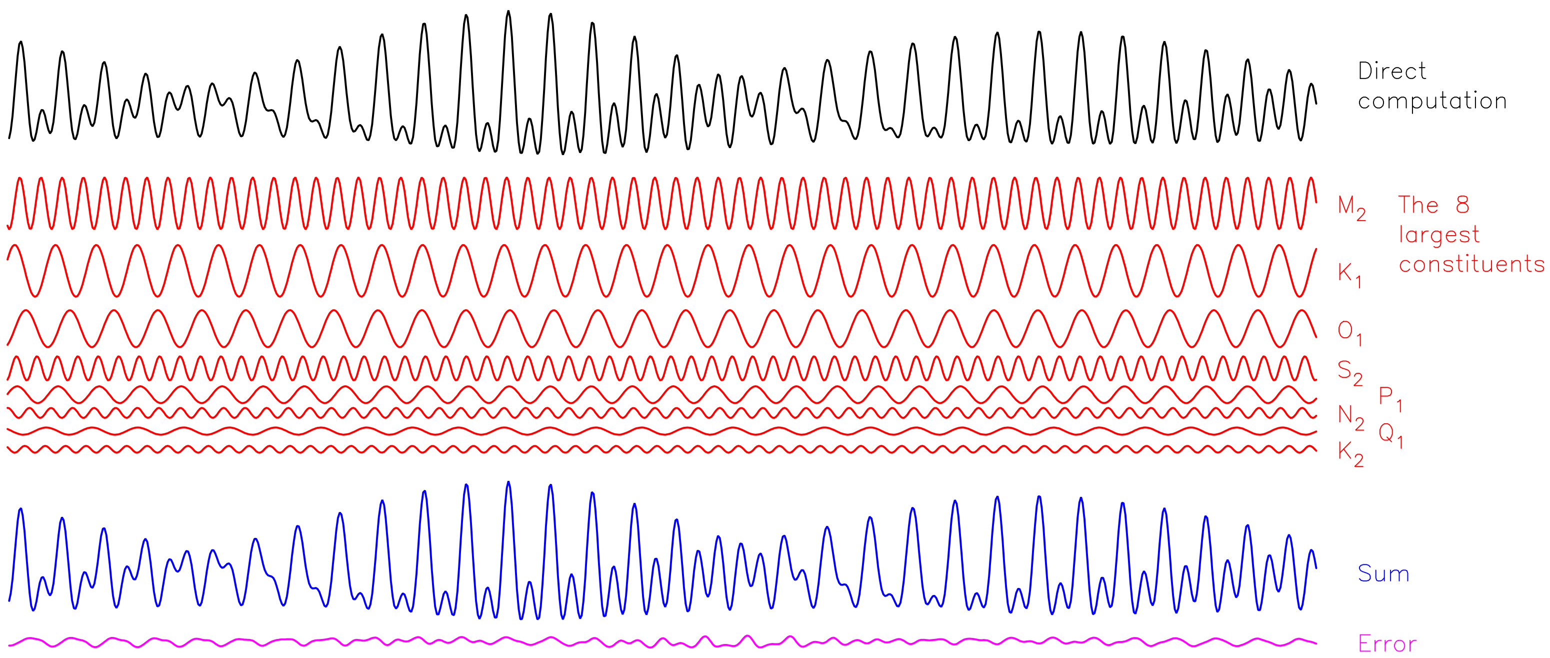


We can best look at

- How the tides actually occur (their physics)
- How we model them

by breaking the phenomenon into several pieces. We have excellent models for some pieces, pretty good ones for some, and none for some.

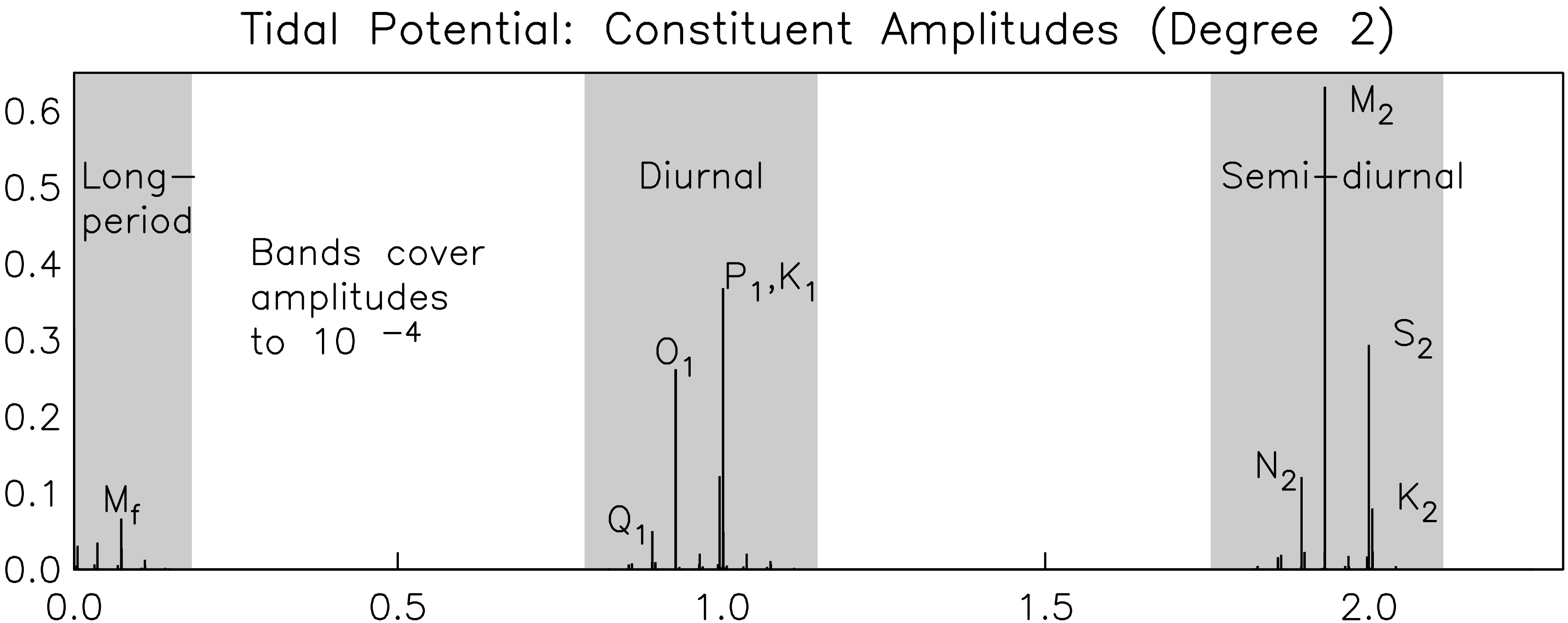
Time Domain vs Constituents



For calibrations we use sinusoids, since the largest constituents have the best signal-to-noise ratio.

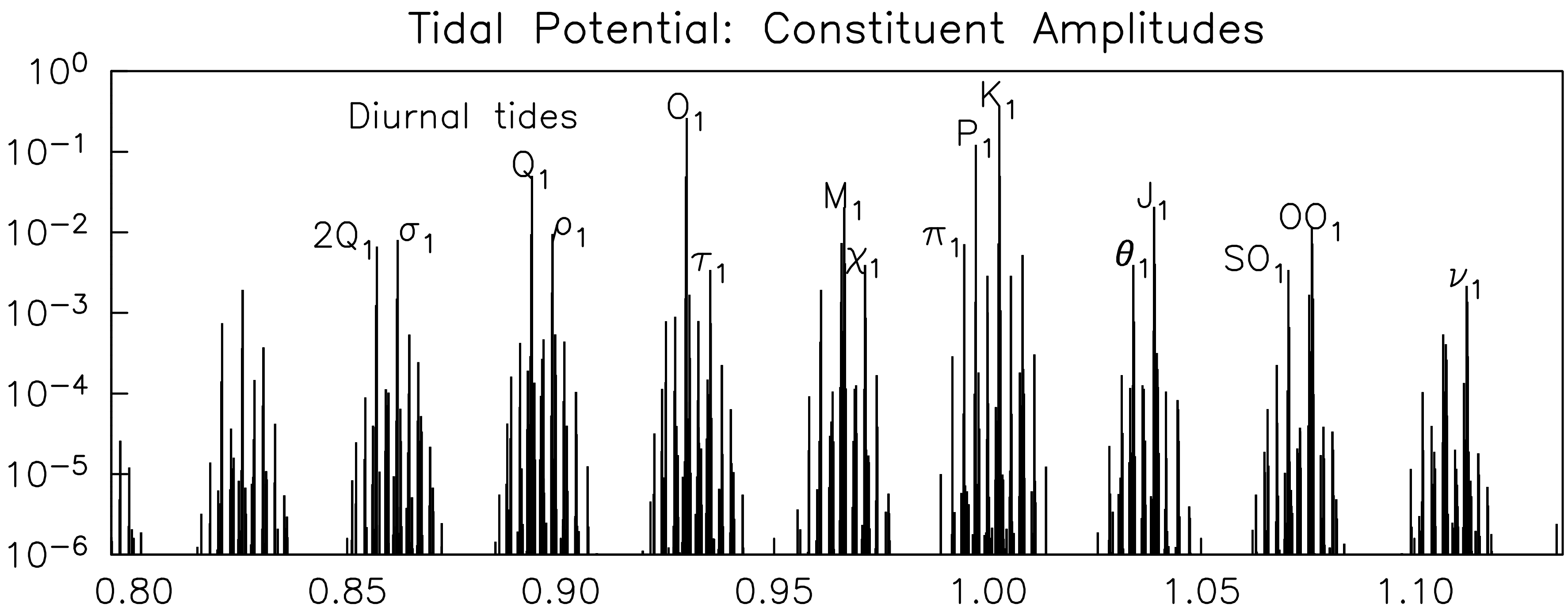
And, the sinusoidal representation has a long history, so you need to know it.

Tidal Constituents (I)



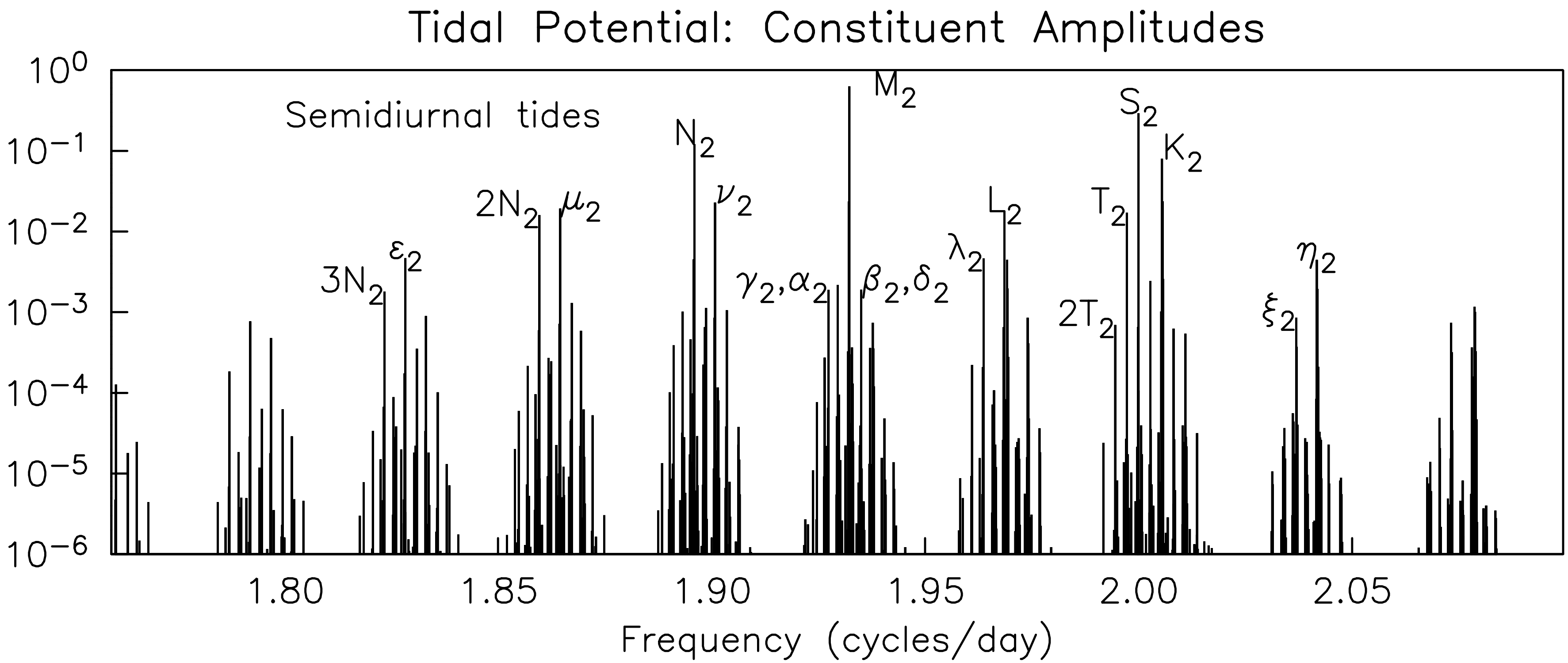
- All constituents are clustered around 0, 1, 2 cycles/day (different species).
- A few large constituents dominate
- Names (Darwin/Kelvin): not arbitrary, but not obvious.

Tidal Constituents: Diurnal

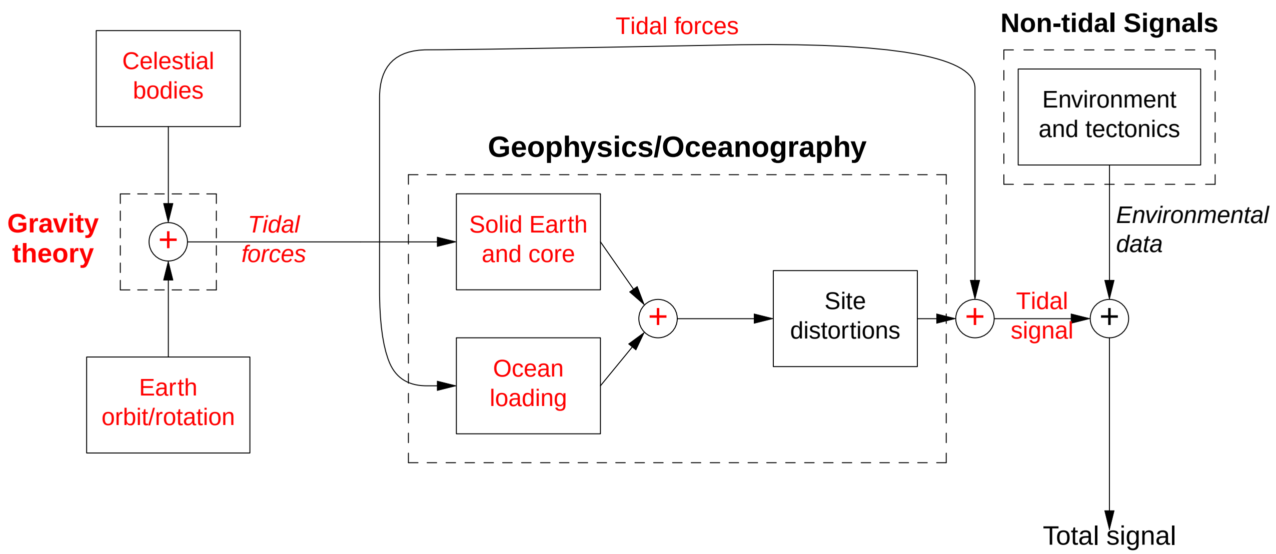


- Within each species, constituents cluster into groups spaced 1 cycle/month apart,
- Within groups there is finer spacing of 1 cycle/year or less.

Tidal Constituents: Semidiurnal



All the Pieces Together



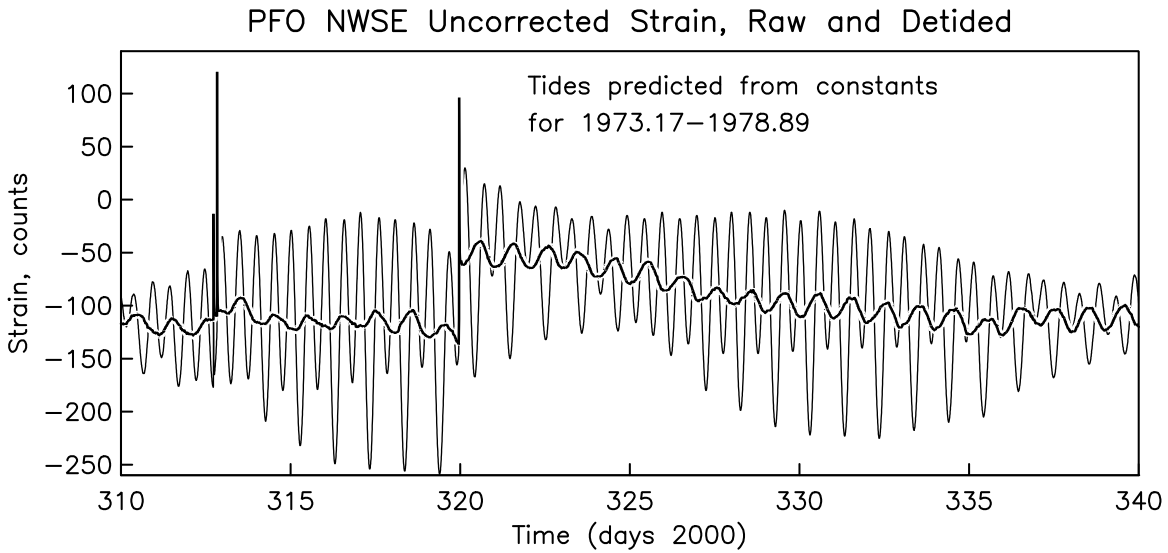
The body tide plus the load tide is the tidal signal in reality, and the theoretical tide in modeling.

How do We Estimate/Remove Tides in a Series?

Why Do We Estimate Tides

We want to extract tidal parameters from a time series so that:

- We can compare them with the theoretical tides, for calibration or geophysics.
- We can predict them so that they can be removed.



Removal of tides predicted from data 25 years earlier. Residual energy around one cycle/day is from thermal effects. a local fit can remove this.

Issues in Tidal Analysis

Nearly all procedures fit sinusoids to the data.

Complications

- The frequency separation between many constituents is $\ll$ than the inverse of the record length – we can't just "fit all the constituents".
- The larger tides vary slowly in amplitude over a nodal cycle (19 yr, up to 15% in amplitude) – we can't just "fit the largest constituents".

Simplifications

- The **frequencies** and **amplitudes** of the tidal forces are known to 10^{-6} or better.
- For nearby frequencies, the relative amplitudes and phases of the observed tides are close to those of the tidal forces.

Smoothness of the Admittance

The **admittance** is the response of the system (ocean or earth) to forcing; this does not vary rapidly with frequency.

So, suppose that the force is

$$F = A_1 \cos(2\pi f_1 t + \phi_1) + A_2 \cos(2\pi f_2 t + \phi_2)$$

and the observed is

$$O = B_1 \cos(2\pi f_1 t + \theta_1) + B_2 \cos(2\pi f_2 t + \theta_2)$$

then, if $|f_1 - f_2| \ll f_1$

$$\frac{B_1}{B_2} \approx \frac{A_1}{A_2}$$

and

$$\theta_1 - \theta_2 \approx \phi_1 - \phi_2$$

So if we have B_1 and θ_1, we can infer B_2 and θ_2.

Outline of Harmonic Analysis

- Bandpass the data to reduce noise at frequencies below and above the tides.
- Make a least-squares fit of sinusoide with the selected tidal frequencies: $\sum_{n=1}^{N} a_n \cos(\omega_n t) + b_n \sin(\omega_n t)$
- Include all the constituents in a frequency band, in proportion, for each “sinusoid”.
- The amplitude should be normalized to the mean value without nodal variations.

What Do We Get ?

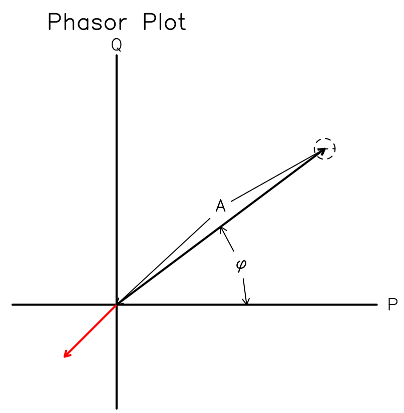
For “each tide”, considered as $A\cos(2\pi ft + \phi)$ we get

- An amplitude, A, in counts (or whatever).
- A phase, ϕ, in degrees or radians (or, ϕ/f, in time).

There are two sources of confusion about the phase:

- The sign. As written, if $\phi = 0$, the tide is largest at $t = 0$; for $\phi < 0$, largest for t positive: negative phase is a lag. This is **not** completely standard.
- What the phase is relative to.
    - **Local phase**: $\phi = 0$ when the tidal force for “that tide” is maximum at the location of the data.
    - **Greenwich phase**: $\phi = 0$ when the tidal force for “that tide” is maximum at the Greenwich meridian.

Plotting the Tidal Results



Using $A\cos(2\pi ft + \phi)$, with A and ϕ as parameters, decouples time shifts and scale factors.

Or, write

$$A\cos\phi \cdot \cos(2\pi ft) - A\sin\phi \cdot \sin(2\pi ft) = P\cos(2\pi ft) + Q\sin(2\pi ft)$$

where P and Q are the **in-phase** and **quadrature** parts of the tide.

We can plot P and Q on a **phasor plot** (same as the Argand diagram for complex numbers).

We can express the sum of sinuosids on such a plot.

Errors in P and Q are uncorrelated, unrelated to size of sinusoid. Errors in A and ϕ are more complicated.

Tidal Prediction

Given the A_n's and Φ_n's, a crude prediction of the tides would be a sum of terms of the form $A_n \cos\left[\omega_n t - \Phi_n - \Phi_n^0(t_0)\right]$ where there is a reference time t_0.

To be more precise (including the nodal variations), use more constituents, with amplitudes and phases from interpolating the ratio of the amplitude of the potential to the data. The `hartid` program (in SPOTL) does this.

Tidal Analysis Packages

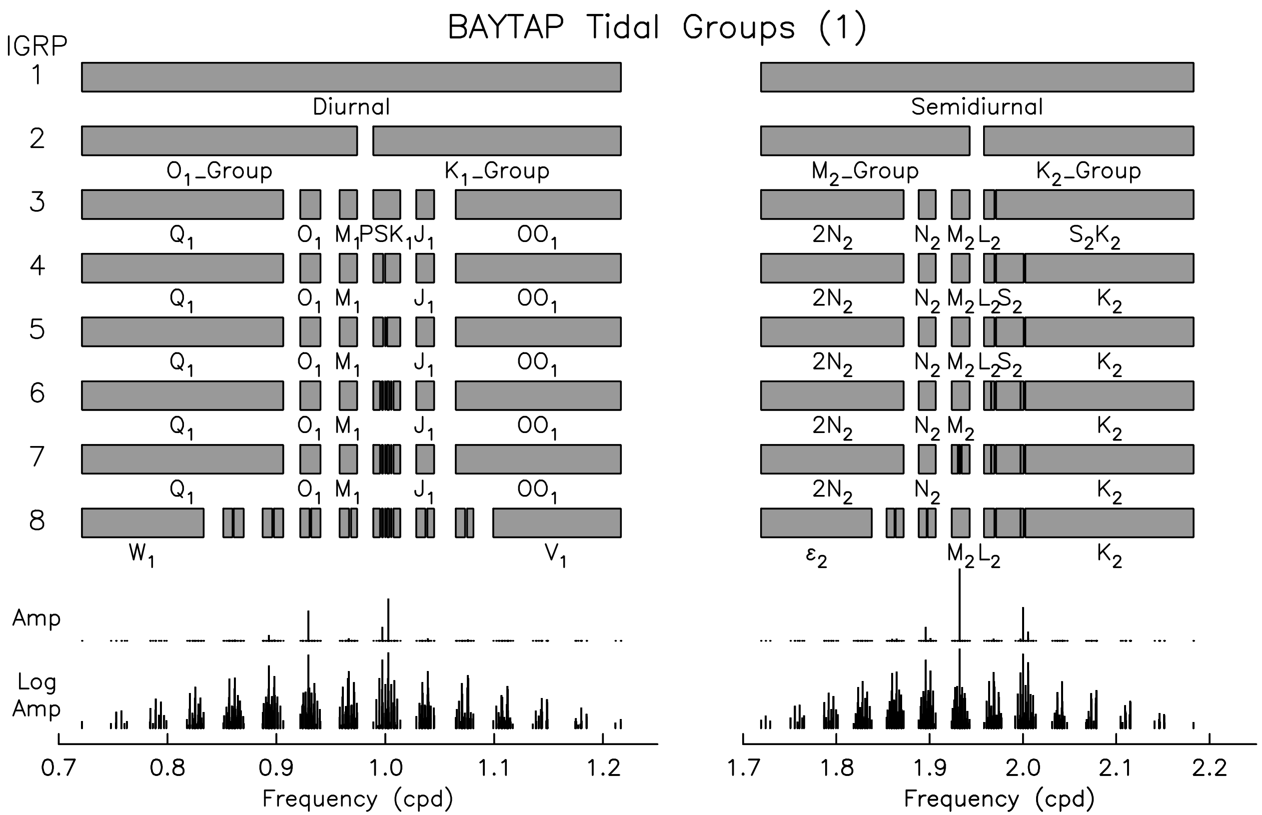
- BAYTAP08, which includes decomposition into drift, tides, short-period variations.
- ETERNA: least-squares package, designed especially for gravity tides (very high SNR).
- T_Tide: MATLAB routines, designed for ocean tides; there is a version that uses robust fitting methods.

Baytap08

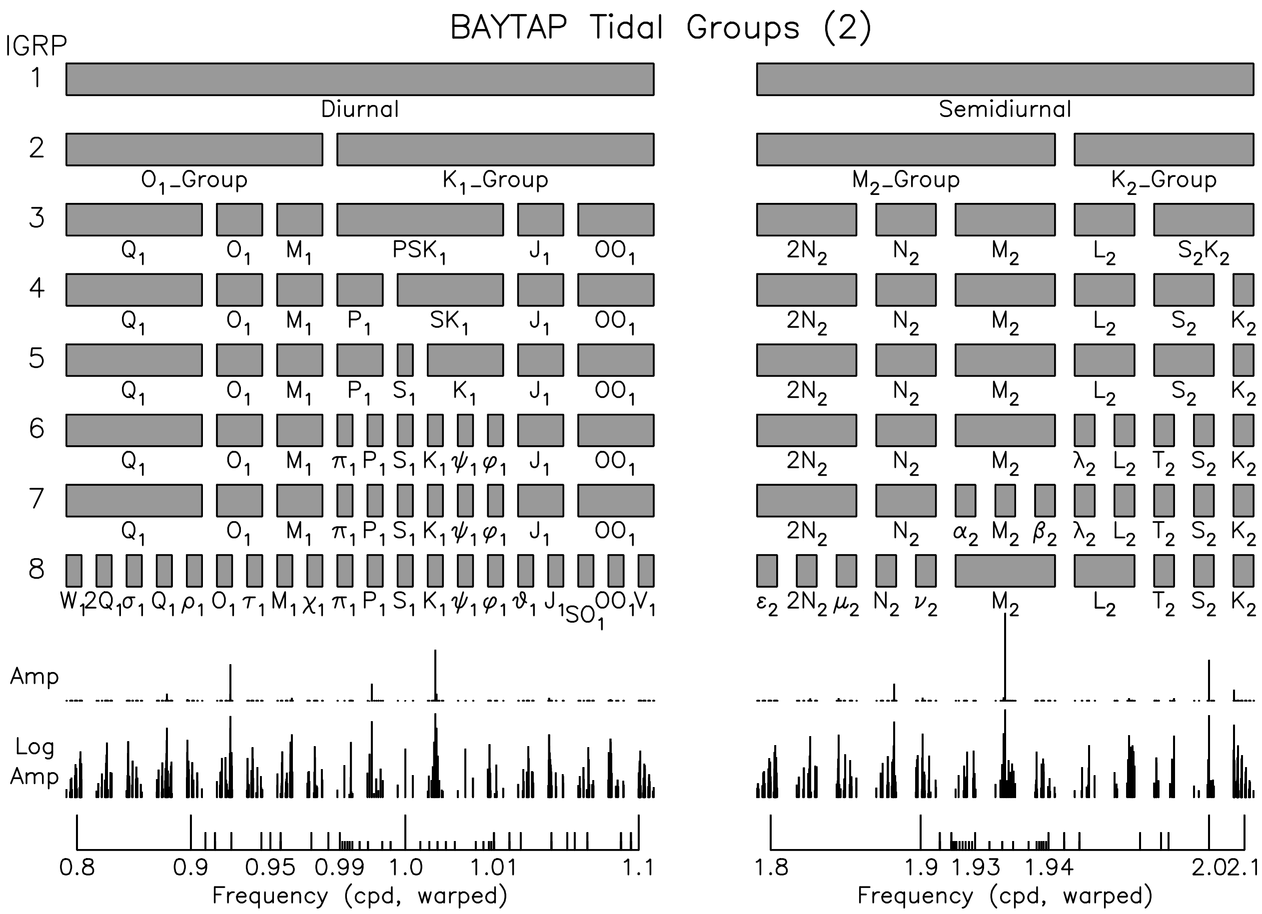
Derived from BAYTAP-G; performs a least-squares fit for:

- The **tides**
- A smooth "**trend**" or "**drift**"
- Parts of the data that are **correlated** with other series.
- The program also adjusts the smoothness of the trend for "minimum Akaike Bayesian Information Criterion".
- Tidal analysis is done by "groups" to avoid instability from fitting sinusoids with very similar frequencies – depends on data length.

Baytap Groups



Baytap Groups



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Running Baytap

Baytap08 takes as input:

- The **data file**: various formats, simplest is a title line and 1 value/line
- A **control file**: instructions to the program
- Optional, **auxiliary files**: barometric pressure etc.

Baytap08 produces:

- A **results file**: results of the analysis, especially tidal results and residual spectrum
- A **series file**: the trend, tides, correlated part, and so on, for plotting.

Syntax:

```
cat data | baytap08 control results [aux[aux[aux]]] > output
```

How do We Model Tides?

Step 1: Body Tides

These can be modeled using

- The tidal forces
- Love numbers for a elastic and spherical Earth model

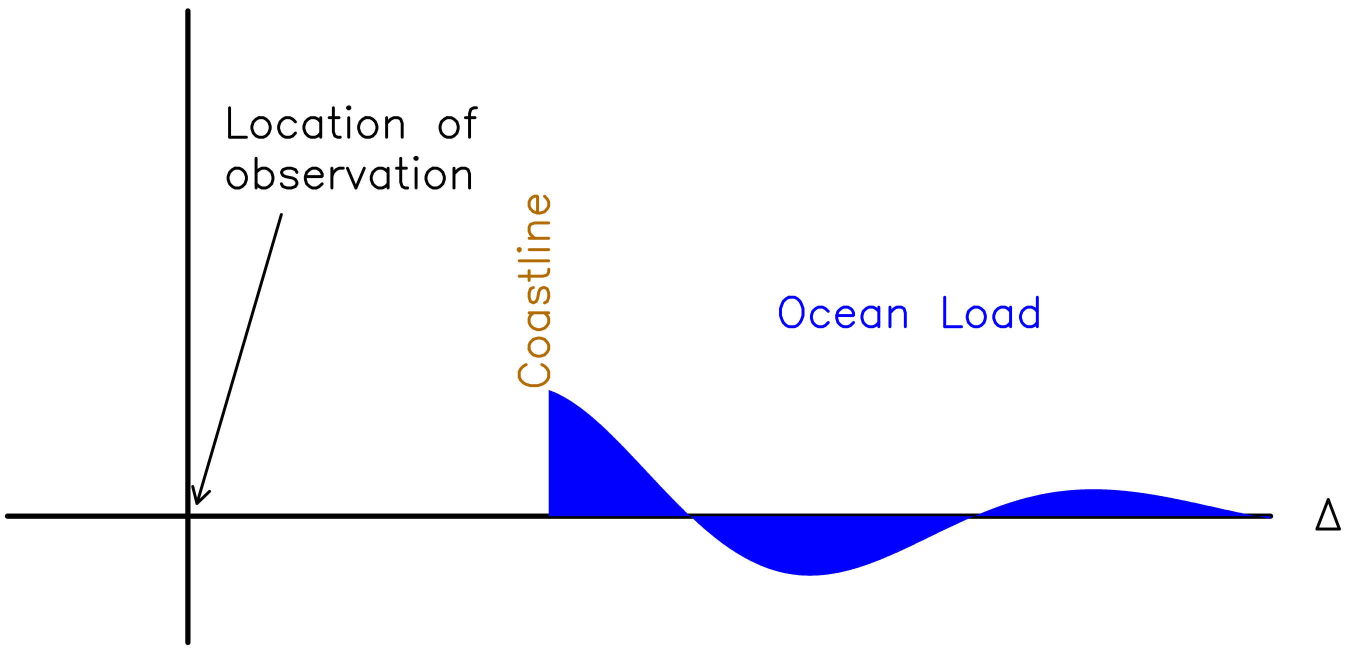
Because the Love numbers are for degree-2 spherical harmonics, they depend on whole-Earth structure, and so are known to 1% or better.

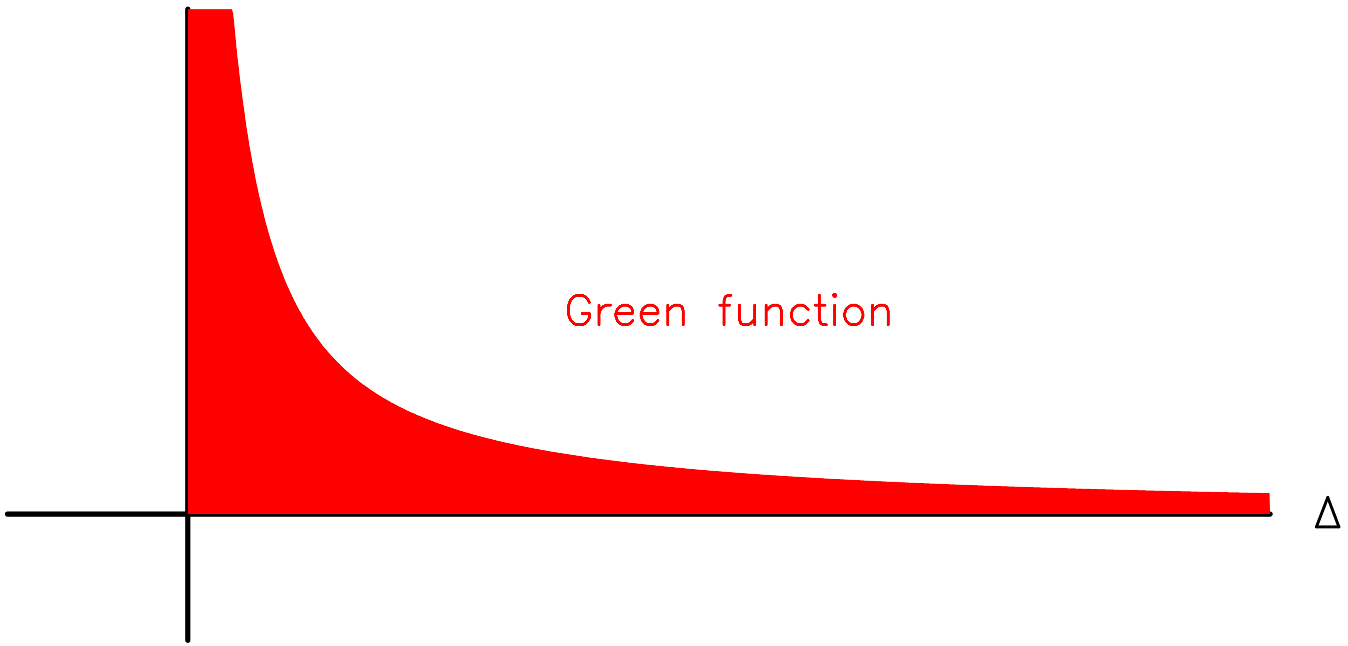
Step 2: Ocean Loading

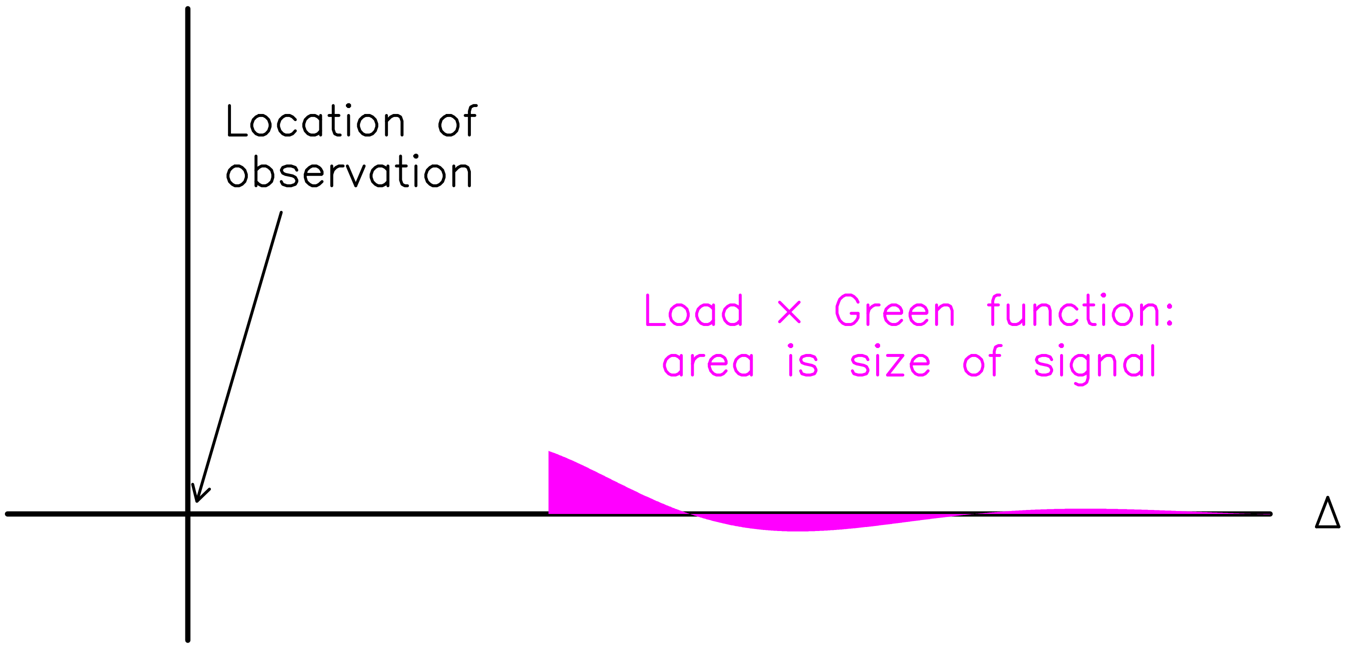
Loading at a location from a load H somewhere else is

$$\rho g H G_L(\Delta, \phi)$$

where G_L is the **Green function** for an effect at the origin (our location) from water height H at distance Δ and azimuth ϕ. We integrate this over the Earth.




Green function
Δ



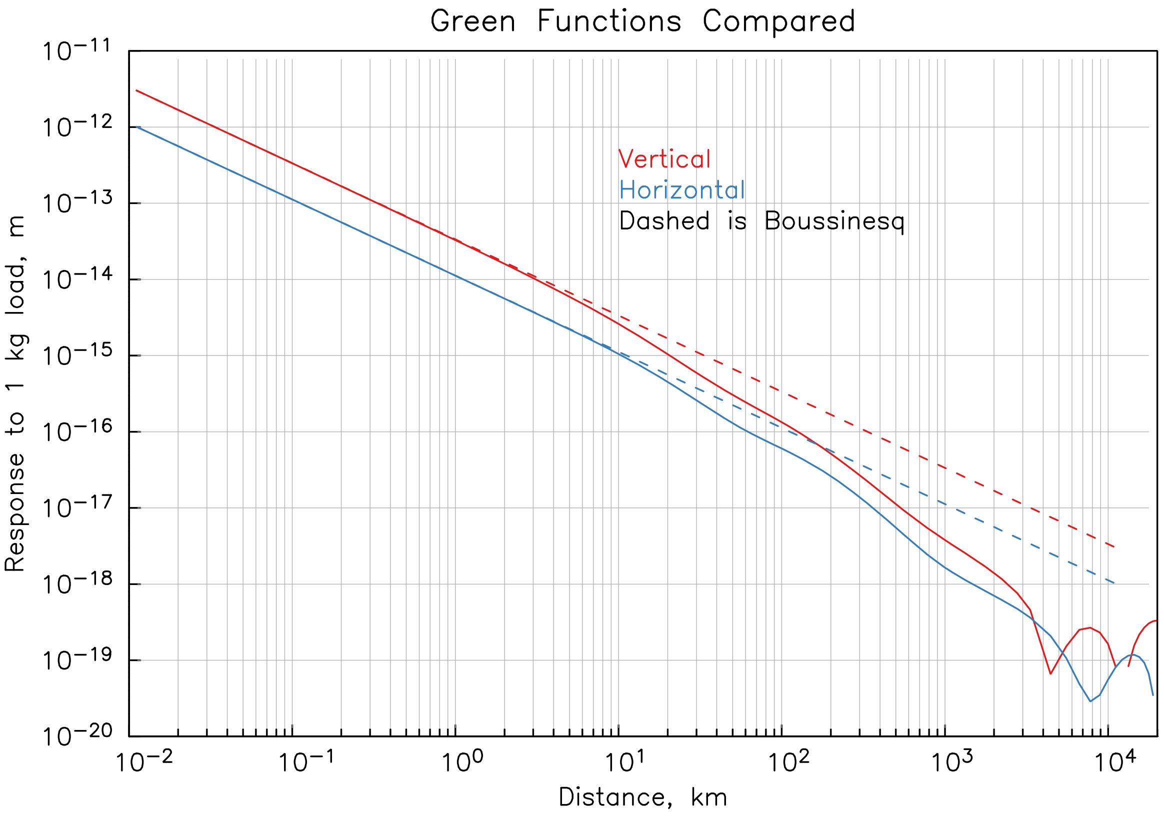
SPOTL (Some Programs for [Ocean [Tide]] Loading) finds this product and takes the integral of the product to find the loading effect at a specified location.

Green Functions

Boussinesq: Point load on an elastic halfspace.

Simple, popular, and not very good – though the best that can be done analytically.

Done properly (Farrell 1972) can include full elastic structure on a spherical Earth.

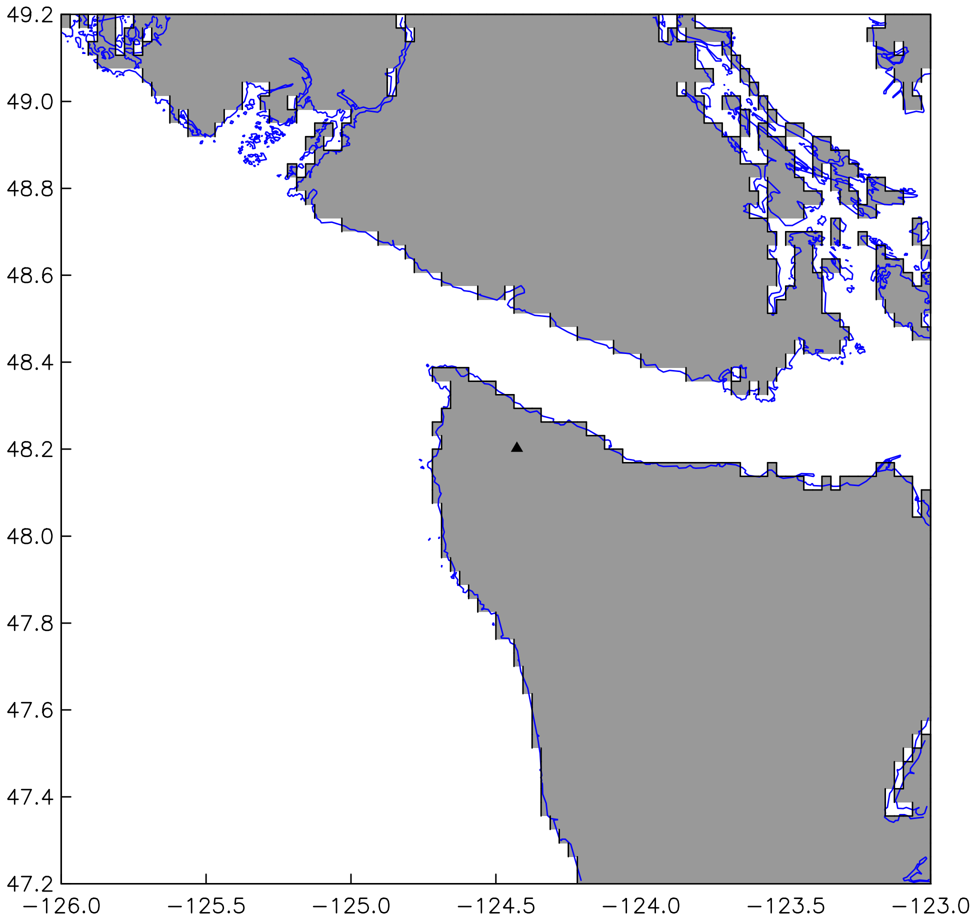


Load Computation with SPOTL

SPOTL includes

- A. Ocean-tide models giving *H* for
  - A variety of global models (low-resolution)
  - High-resolution models for selected areas, with boundaries in polygon files.
- B. A land-sea model, to describe more precisely where the ocean ends or begins.
- C. Green functions for different Earth models.

Global Land-sea Database



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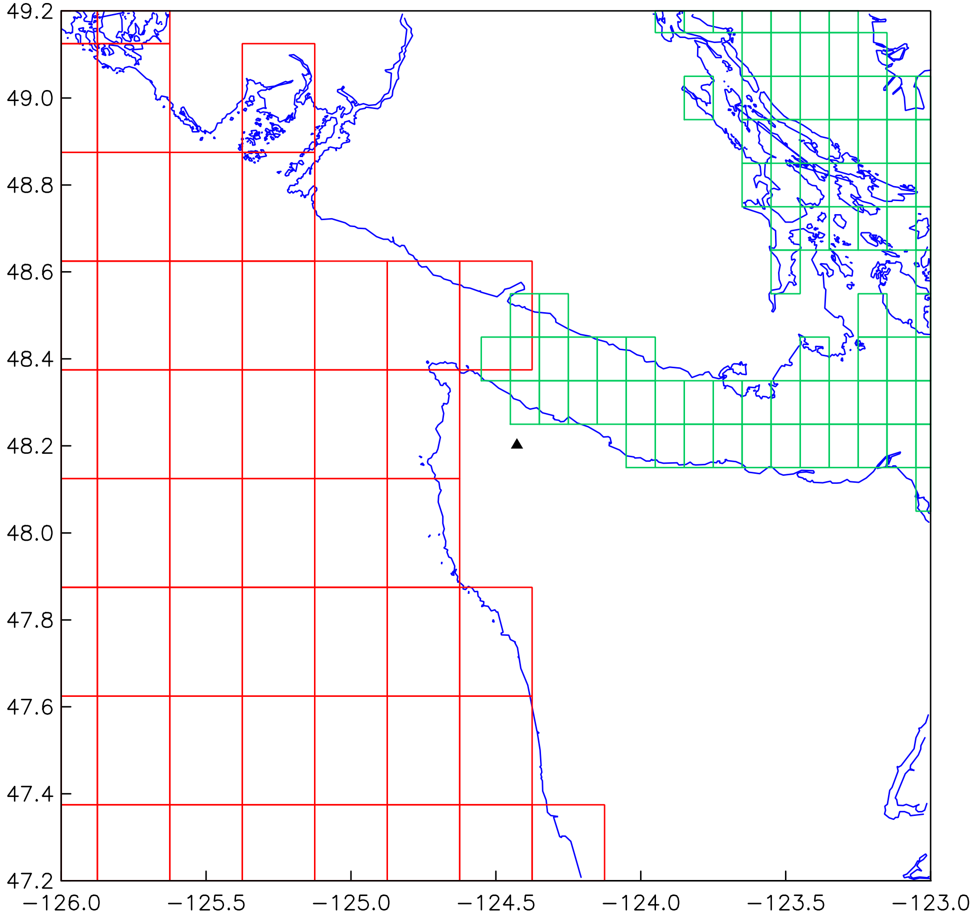
Ocean Tide Models



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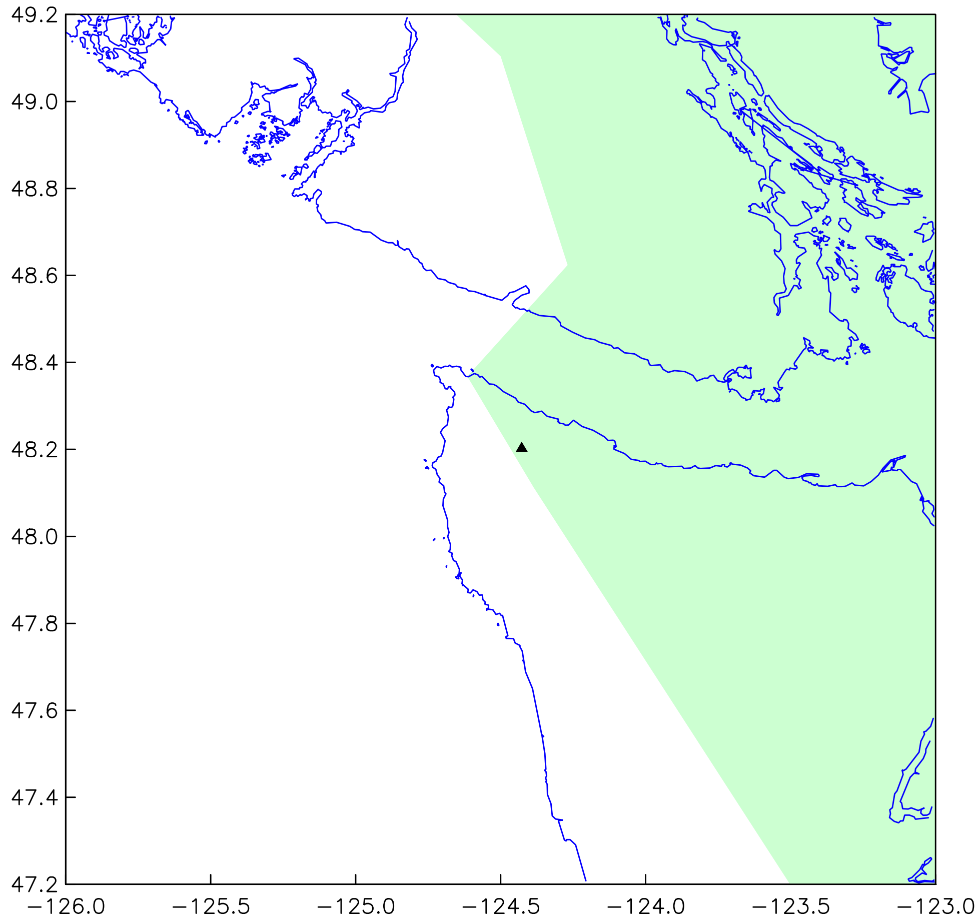
Polygon for Separating Different Models



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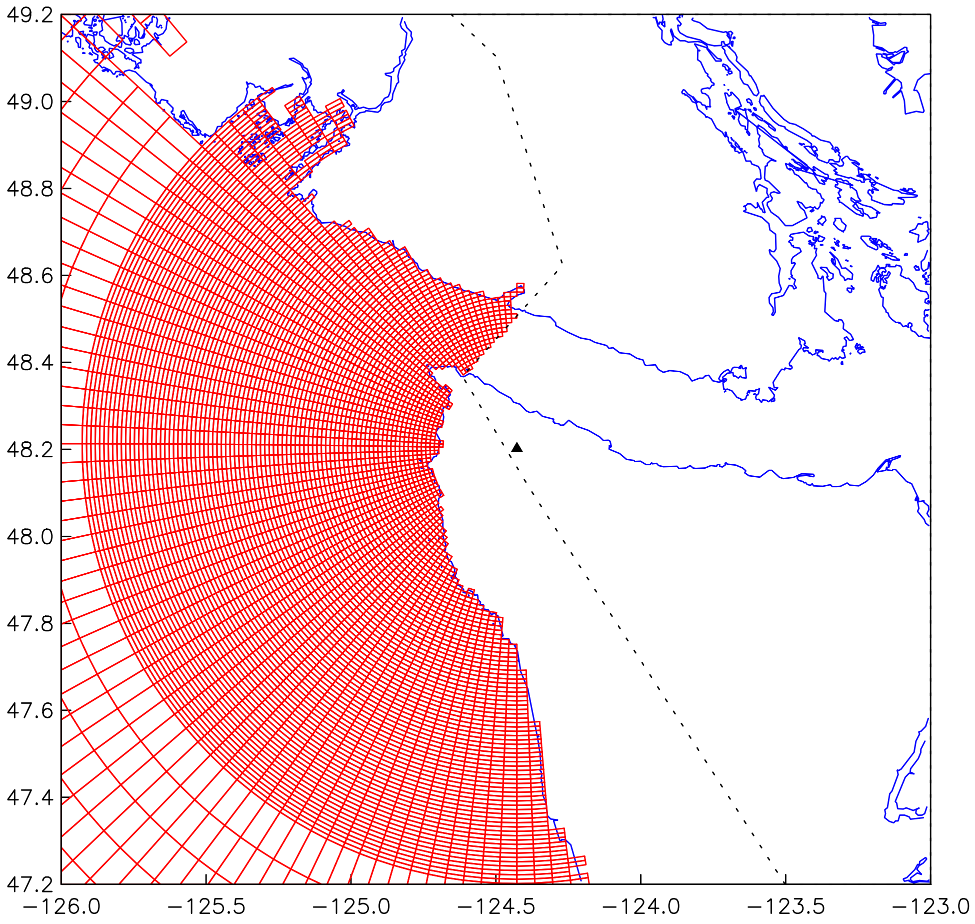
Load Grid for Global Model



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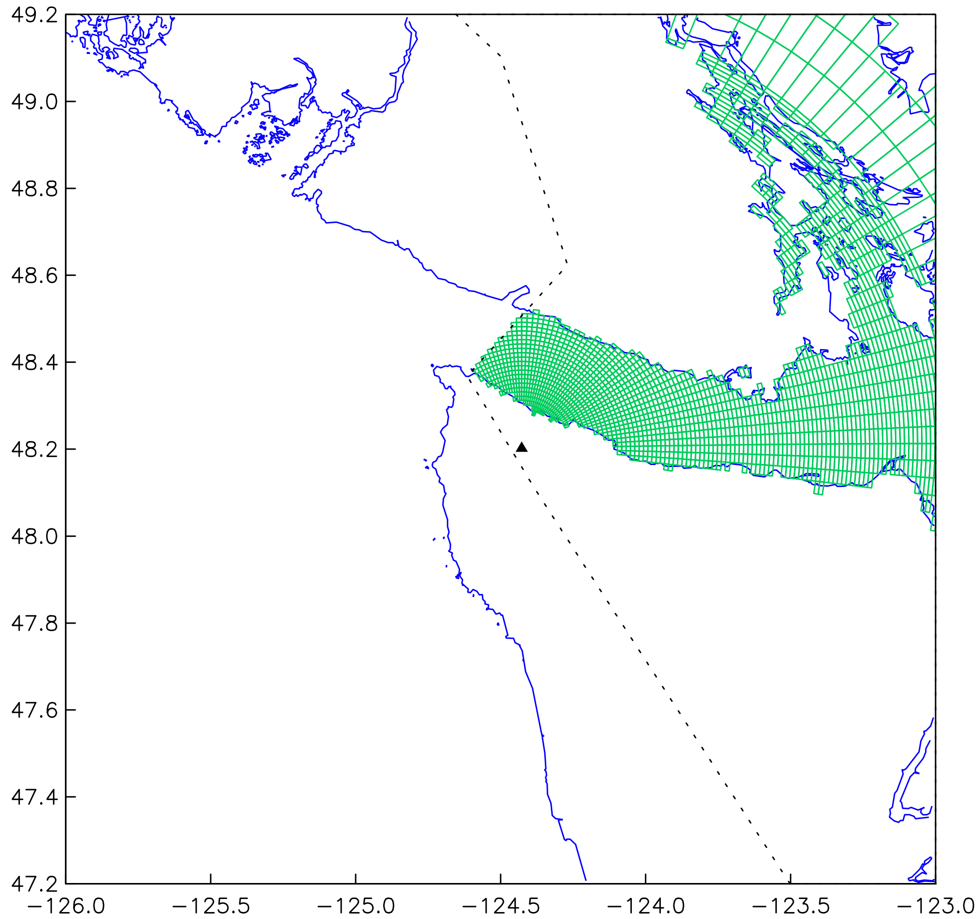
Load Grid for Local Model



Programs in SPOTL

The programs are:

- 1. `nloadf` for finding the load, at one location, for a particular constituent.
- 2. `loadcomb` for combining loads from different models, adding the body tide, and adjusting for different azimuths.
- 3. `harprep` and `hartid` for using the results for many constituents to produce a time series
- 4. `ertid` for computing the body tide directly.

To compute loads but not tidal time series, need only `nloadf`.

Computing a Load

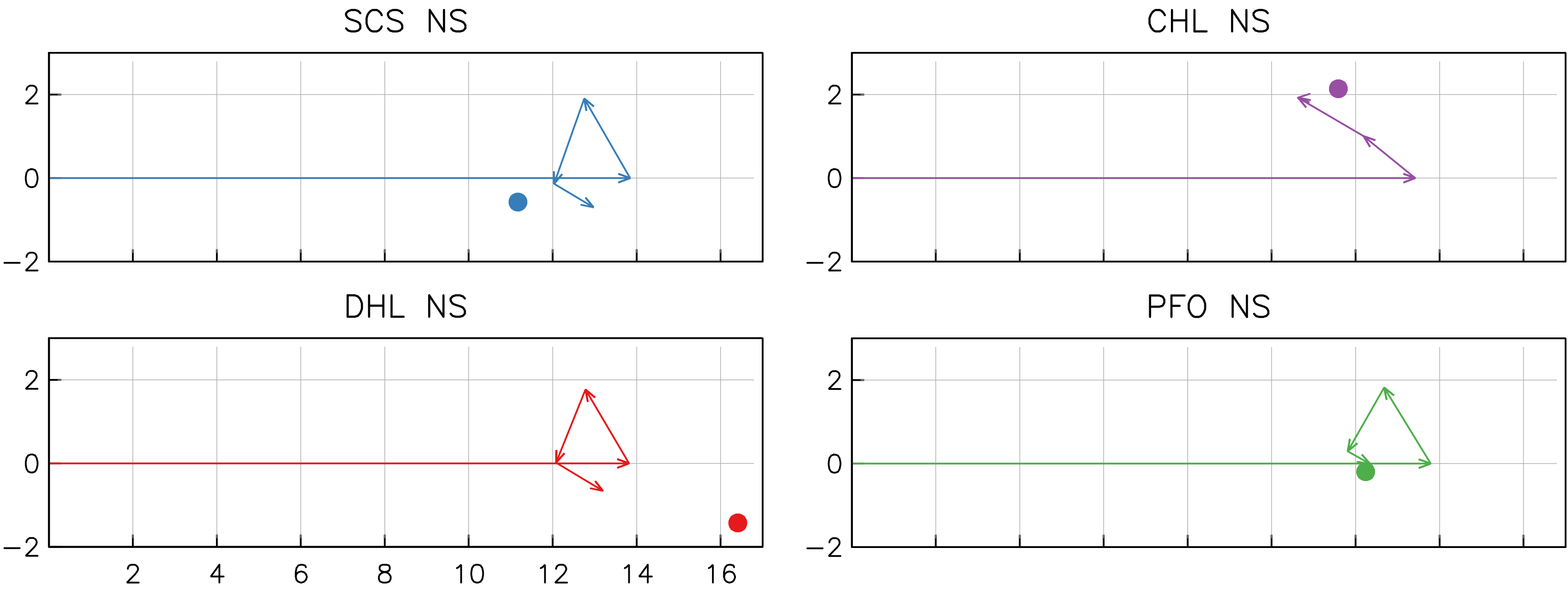
```
nloadf HOKO 48.202 -124.427 100. m2.gefu green.gbavap.std l poly.gefu + > tmp.m2.l
```

The command line includes (the order is required):

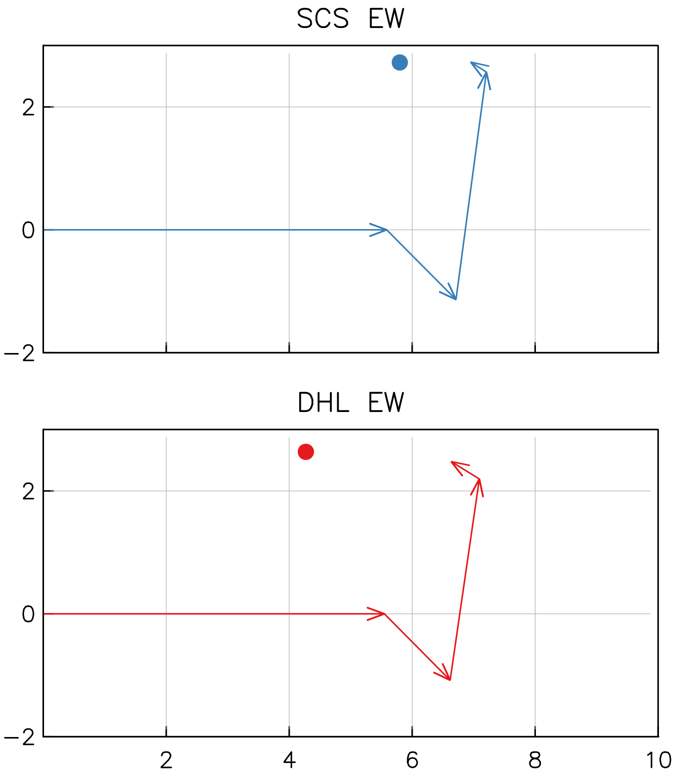
- Station information (name and position)
- Name of ocean model file
- Name of Green function file file
- The phase convention (don’t ask)
- Information on use of a polygon file (optional)

and the result is sent to standard output, which can be redirected to a file.

Example: LSM NS



Example: LSM EW



Questions?

Computing and Combining Loads

```
nloadf HOKO 48.202 -124.427 100. m2.gefu   green.gbavap.std l poly.gefu + > tmp.m2.l

nloadf HOKO 48.202 -124.427 100. m2.tpxo70 green.gbavap.std l poly.gefu - > tmp.m2.g

cat tmp.m2.l tmp.m2.g | loadcomb c > tmp.m2.load
```

These three commands

- Put loading results from two models (with the polygon used for the boundary) into two files
- Combine these two files to give the total load (vector addition of the loads).

Sample SPOTL Output: `tmp.m2.l`

```
S  HOKO                                          48.2020 -124.4270      100.
O M2        2 0 0 0 0 0     Straits of Georgia and Juan de Fuca
G                 GUTENBERG BULLEN GREENS FUNCTIONS  JOBO2Q 10/19/71
G    Rings from   0.03 to   1.00 with spacing 0.01 - detailed grid used
G    Rings from   1.05 to   9.95 with spacing 0.10 - detailed grid used
G    Rings from  10.25 to  89.75 with spacing 0.50 - ocean model grid used
G    Rings from  90.50 to 179.50 with spacing 1.00 - ocean model grid used
P Polygon to include the Straits of Georgia and Juan de Fuca
P   all polygon areas included
C   Version 3.2 of load program, run at Wed Aug 25 19:39:56 2010
C   closest nonzero load was   0.09 degrees away, at   48.28 -124.39
C       23 zero loads found where ocean present, range   0.78-   3.05 deg
L l          Phases are local, lags negative
X
g              0.2297  107.9319
p              1.2406  -95.4251
d              0.2350  -87.3167    0.1802  -97.2629    0.8072   96.1769
t             22.2885  -30.4658   30.6713  -12.8066
s              1.6887 -112.2628    4.2123   13.6132    5.7803  -17.1302
```

Last lines are amp and local phase of gravity (g), potential (p), displacement (d: ENU), tilt (EN) and strain (ε_{EE}, ε_{NN}, ε_{EN})

Sample SPOTL Output: `tmp.m2.g`

```
S  HOKO                                     48.2020 -124.4270      100.
O M2        2 0 0 0 0 0     OSU TPXO 7.0
G                 GUTENBERG BULLEN GREENS FUNCTIONS  JOBO2Q 10/19/71
G    Rings from   0.03 to   1.00 with spacing 0.01 - detailed grid used
G    Rings from   1.05 to   9.95 with spacing 0.10 - detailed grid used
G    Rings from  10.25 to  89.75 with spacing 0.50 - ocean model grid used
G    Rings from  90.50 to 179.50 with spacing 1.00 - ocean model grid used
P Polygon to include the Straits of Georgia and Juan de Fuca
P   all polygon areas excluded
C   Version 3.2 of load program, run at Wed Aug 25 19:39:58 2010
C   closest nonzero load was   0.17 degrees away, at   48.21 -124.69
C       39 zero loads found where ocean present, range   0.83-   9.85 deg
L l          Phases are local, lags negative
X
g             5.5521 -178.6621
p            34.7121   -1.0729
d             7.3550  178.3120    2.0137 -103.0084   19.4539  178.7385
t           146.5715 -169.9169   30.0543 -144.0923
s            16.0916    6.5470    6.7084  168.4417    3.2678   47.7671
```

When we combine two files, the results part (the last 5 lines) are added; the other lines are concatenated to give a complete record of what was done.

Sample SPOTL Output: `tmp.m2.load`

```
S  HOKO                                      48.2020 -124.4270      100.
O M2         2 0 0 0 0 0     Straits of Georgia and Juan de Fuca
G                  GUTENBERG BULLEN GREENS FUNCTIONS  JOBO2Q 10/19/71
G    Rings from   0.03 to   1.00 with spacing 0.01 - detailed grid used
G    Rings from   1.05 to   9.95 with spacing 0.10 - detailed grid used
G    Rings from  10.25 to  89.75 with spacing 0.50 - ocean model grid used
G    Rings from  90.50 to 179.50 with spacing 1.00 - ocean model grid used
P Polygon to include the Straits of Georgia and Juan de Fuca
P   all polygon areas included
C   Version 3.2 of load program, run at Wed Aug 25 21:42:17 2010
C   closest nonzero load was   0.09 degrees away, at   48.28 -124.39
C       23 zero loads found where ocean present, range   0.78-   3.05 deg
L l          Phases are local, lags negative
O M2         2 0 0 0 0 0     OSU TPXO 7.0
G                  GUTENBERG BULLEN GREENS FUNCTIONS  JOBO2Q 10/19/71
G    Rings from   0.03 to   1.00 with spacing 0.01 - detailed grid used
G    Rings from   1.05 to   9.95 with spacing 0.10 - detailed grid used
G    Rings from  10.25 to  89.75 with spacing 0.50 - ocean model grid used
G    Rings from  90.50 to 179.50 with spacing 1.00 - ocean model grid used
P Polygon to include the Straits of Georgia and Juan de Fuca
P   all polygon areas excluded
C   Version 3.2 of load program, run at Wed Aug 25 21:42:18 2010
C   closest nonzero load was   0.17 degrees away, at   48.21 -124.69
C       39 zero loads found where ocean present, range   0.83-   9.85 deg
L l          Phases are local, lags negative
X
g             5.6220  179.0940
p            34.6401   -3.1194
d             7.3408 -179.8589    2.1931 -102.5371   19.5748  176.3950
t           130.4428 -163.5393   25.0510  -77.1638
s            15.3493    1.0151    3.4055  136.6995    7.7535    5.3059
```

This is what is produced by `loadcomb c`, combining files.

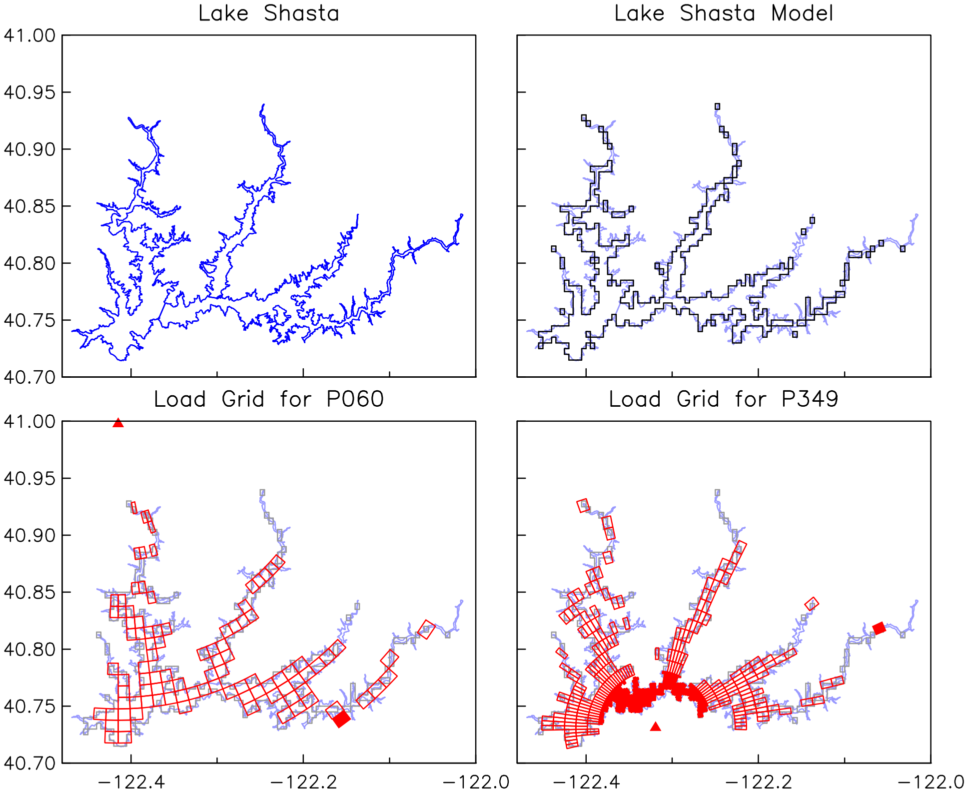
Using SPOTL for Other Loads

Any load can be gridded and used in place of one of the ocean-tide models – though usually, the load then is purely real, and the output phases are either 0° or 180° (plus or minus).

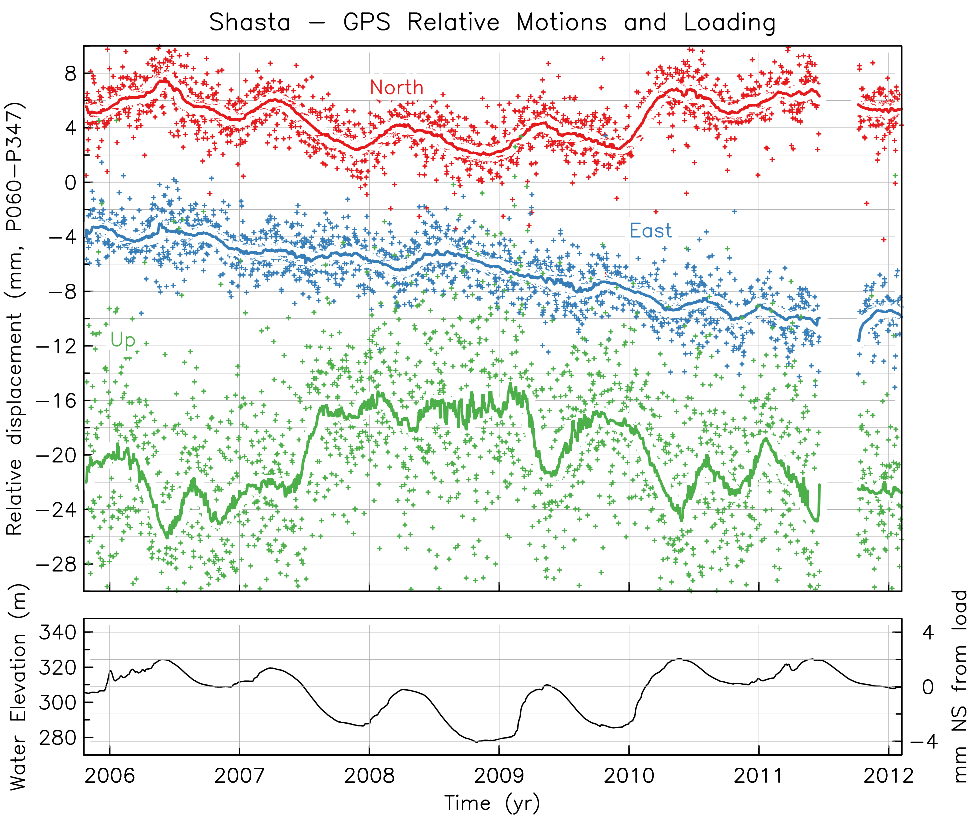
To get loads on land, the land-sea database must be disabled; this is done (admittedly clumsily) using flags in the Green-function file:

- F: Land-sea database determines if point on ocean or not; if not, assumes no load. Invokes bilinear interpolation.
- C: Source grid determines if point on land; if not, assumes no load. No interpolation, load is that of ocean cell.
- L: Land-sea database determines if point on ocean; if so assumes no load. Invokes bilinear interpolation. This mode is useful for air-pressure loads, which are compensated over the ocean.
- G: Source grid determines scope of integration; if no cell, assumes no load. This is the same as C, but the density is assumed to be 1000 kg/m, not the density of ocean water.

An Example: Lake Shasta



Lake Shasta Time Series



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