How Can We Measure

the Earth's Deformation?

Duncan Carr Agnew

(in semi-absentia)

IGPP/Scripps Institution

UC San Diego

Basic Principles

We want to to measure how the Earth deforms.

To do this, our measuring systems (instruments) must include:

- Some kind of **reference**; we assume this is fixed.
- An **instrument frame**.
- Some way of measuring the **displacement** of the frame relative to the reference.
- Some way of **attaching** the frame to the ground.

Basic Principles: Developments

The development of these has been:

- It keeps getting easier to measure small displacements.
- Space geodesy has created a whole new class of references.
- The reference for most ground-based systems hasn't changed in decades.
- Attachment to the ground remains the biggest challenge:
  - It cannot be engineered completely.
  - Every site is different.

Positioning with Space Geodesy: References

(These systems give actual position, not just displacement.)

- **Reference I:** For satellite-based systems, the reference is the Earth's center of mass, and inertial space.
  - The difference from inertial space is given by the unmodeled accelerations, which can be as low as as $10^{-13}\,g$: 4 mm in a day, or 500 m in a year.

- **Reference II:** For radio-astronomy systems, really distant radio sources; these appear to match inertial space.

Space Geodesy: Displacement Sensors

These all use some variant of **time-of-flight of electromagnetic radiation**:

- Light pulses for **S**atellite **L**aser **R**anging)
- Correlatable random signals (for **V**ery **L**ong **B**aseline **I**nterferomtry)
- Phase (fraction of cycle) of a sine wave (for the **G**lobal **P**ositioning **S**ystem).

All measurements using radiation are affected by **path-length variations** (index of refraction not equal to 1), also known as **propagation effects**.

Vertical Motion from Gravity Changes

Vertical motion changes the value of g, because of the gravity gradient; since $g = \frac{GM_E}{r^2}$ the gradient is $\frac{dg}{dr} = \frac{-2GM_E}{r^3} = \frac{g}{r} \approx 1.5 \times 10^{-6} s^{-2}$

A 6 mm vertical motion changes g by one part in 10^{-9}.

But g can also change because of mass variations (air, water, magma): maybe good, maybe not.

Measuring Gravity I: Absolute Gravity

- Use a freely-falling mass, which is, briefly, a *very*-low-orbiting satellite.
- In this case the reference is inertial space; various forces on the mass make the errors about $10^{-9}g$.
- The sensor is **laser interferometery**:
    - Like GPS: measure the phase of a sinusoid to within 90°; wavelength is $\lambda \approx 6.3 \times 10^{-7}$m.

Measuring Gravity II: Vertical accelerometer

A mass on a spring, either a metal spring or magnetic levitation superconducting gravimeter)

The rference is the non-gravitational force on the mass.

Depending on the period of motion this is called either a **gravimeter** or a **vertical seismometer**:

> These are the **same thing**: both measure (apparent) acceleration in the vertical.

Displacement u produces acceleration $\omega^2 u$, and change in g of $2gu/r$; these are equal at periods of about an hour.

Sensor: Some sort of small-displacement sensor.

Small Displacement Sensors

These are used to measure diaplacements < 1 mm.

- Capacitor: 10^{-10} to 10^{-14} m resolution (nuclear dimensions).
- Inductor (LVDT): 10^{-10} m and readily available commercially, but applies more force.
- Moving-coil velocity (only useful for higher frequencies).
- Optical interferometry; calibration reproducible, but the wavelength of light ($\lambda \approx 10^{-7}$m) is large.

Deformation Measurements

So far, looked at systems that sense actual displacement.

Now look at instruments that sense the **spatial gradient** of displacement: how much does it vary with location?

These gradients are dimensionless.

Sensors are

- **Tiltmeters**: spatial gradient of **vertical** displacement (and more).
- **Strainmeters**: one part of spatial gradient of **horizontal** displacement (mostly).
- **Rotation meters**: the other part of spatial gradient of **horizontal** displacement (mostly).

Length: The Most Important Parameter

The **baseline length** *L* of an instrument that senses differential displacement is its most important characteristic:

- The displacement is *L* times the (dimensionless) gradien; for extension, strain is $\varepsilon = \frac{\Delta L}{L}$.
- There are two length classes:
  - **Short-base** (0.1 to 1 m): strain (tilt) of 10^{-9} is 1-10 atomic diameters: so seismic strains are ~size of atomic nucleus. Annual tectonic (10^{-7}) is 0.00001 mm.
  - **Long-base** (10 to 1000 m): strain (tilt) of 10^{-9} is 0.01 to 1 wavelengths of light (largest would be 0.001 mm). Annual tectonic is 0.1 mm.

Length and Ground Attachment

How stabily the frame has to be attached to the ground equals the strain (or tilt) times L.

How instruments can be sited depends on L:

- Short-base: in borehole, or tunnel.
- Long-base: in tunnel (for L 10-100 m), and on the surface.

Best results from **short-base in a deep borehole**, and **long-base on the surface** (or in tunnel).

Tiltmeters

Have a **vertical reference**, which points along **g**, the *apparent* direction of gravity.

So tiltmeters *also* measure horizontal acceleration,

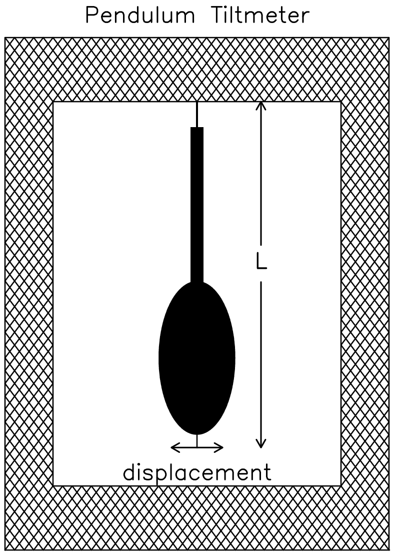
> A tiltmeter, a horizontal seismometer, and a horizontal accelerometer all measure the same quantities.

Pendulum Tiltmeter

Mass on a pivot (think of this as a “movable density interface”).

Measure displacement relative to frame.

Length of pendulum L is 0.05 to 1 m.

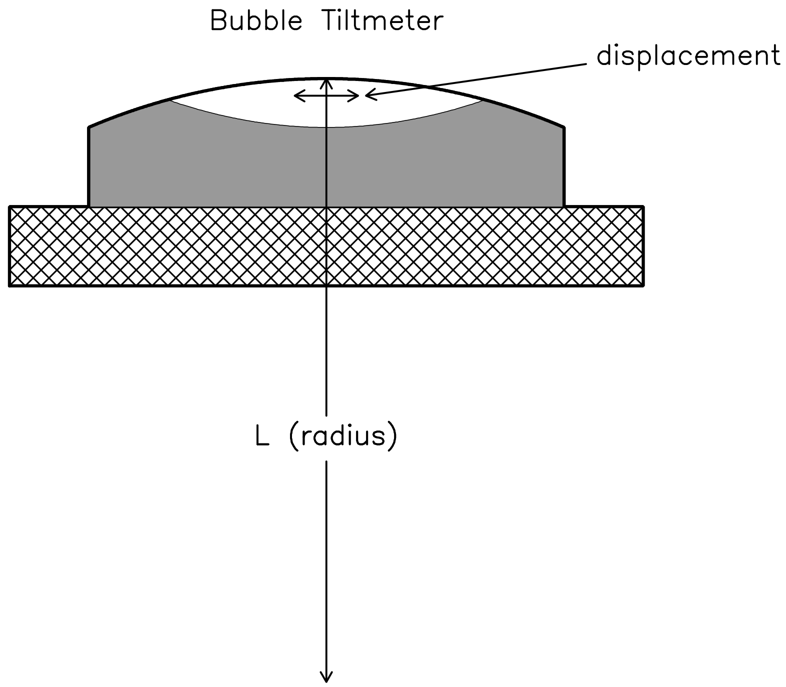


Current example: tiltmeters in Japan’s borehole seismic network.

Liquid Tiltmeters I: Bubble

In this case, the "movable density interface" is a liquid surface.

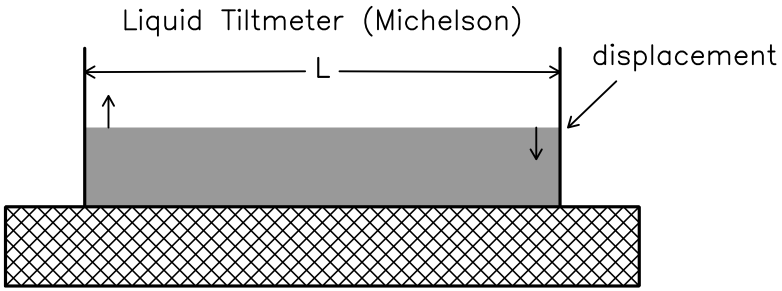
For sensing purposes, L is the radius of the surface the bubble is under; the baselength may be much shorter.

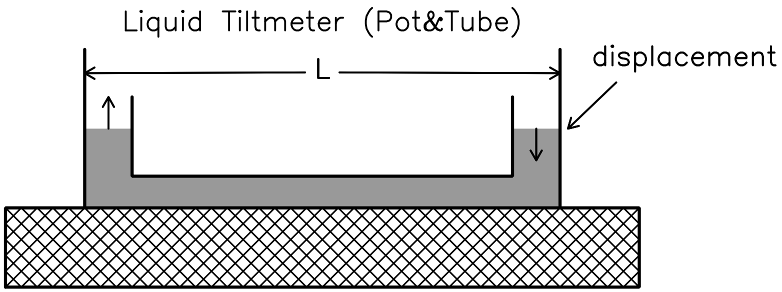


Current example: PBO borehole tiltmeters (from Applied Geomechanics).

Liquid Tiltmeters II: Long-base

The “Michelson” design (developed in 1916!), has an unbroken free surface; it is harder to build, but can be hundreds of meters long. The pot and tube design is sensitive to temperature, and must be installed in a tunnel.



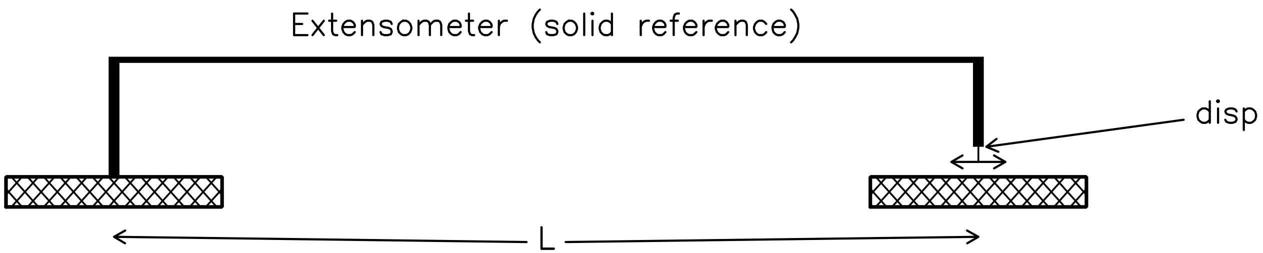


Current examples (Michelson) in Pacific NW, Mammoth, Pinon Flat; pot-and-tube in Japan and Europe.

Strainmeters I: Extensometers

These measure linear strain ε.

- **Instrument frame** is two endpoints a distance L apart.
- **Reference length** is L: either a physical length standard, or laser light.
- Measure the **relative displacement** ΔL between the two ends; then $\varepsilon = \frac{\Delta L}{L}$



We will say more about these later.

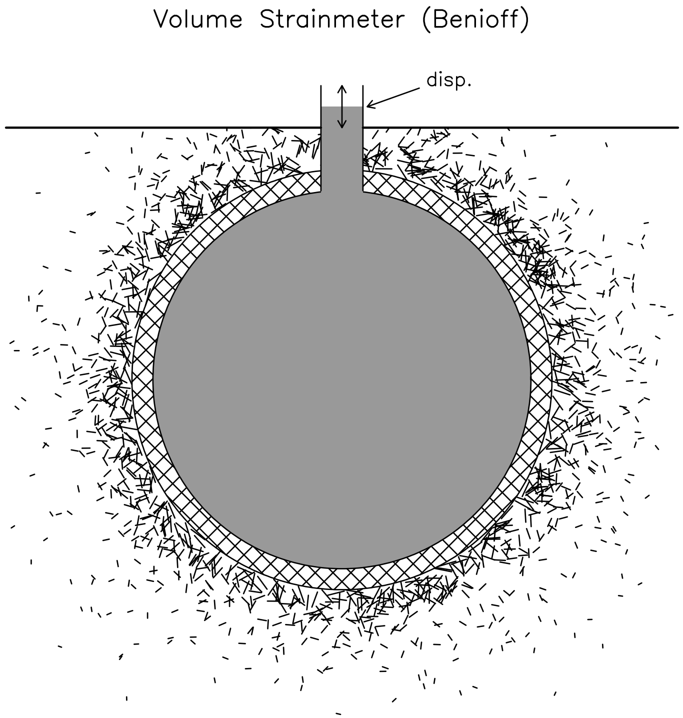
Strainmeters II: Dilatometers

These measure the **dilatation**, which is $\varepsilon_V = \frac{\Delta V}{V}$ where V is the volume and ΔV is the change in volume.

In tensor strain, this is $\varepsilon_V = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$, independent of the coordinate system used.

- **Instrument frame** is a volume containing a liquid.
- **Reference volume** is the volume of liquid.
- Measure the **fluid volume** that moves in and out of the reference volume to get ΔV.

A Dilatometer Concept



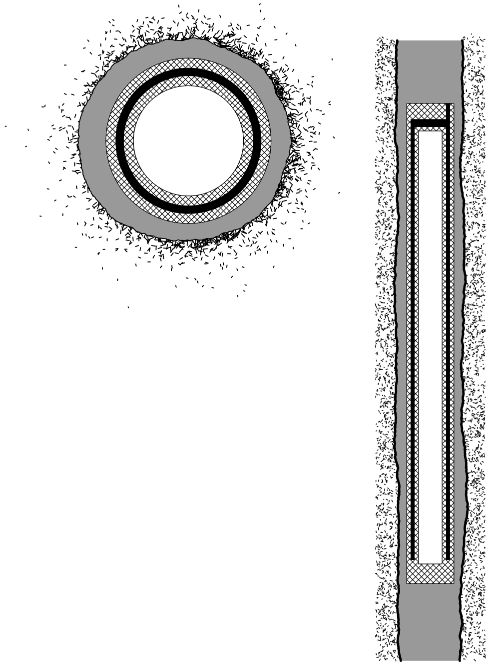
The original proposal for a volume strainmeter was by Benioff, in the same 1932 paper in which he described the first useful extensometer. However, nothing seems to have come of this.

Sacks-Evertson Dilatometer

Fluid volume is a cylindrical annulus, with the change in volume being measured through the displacment of an attached bellows. Instrument senses arealc[1] and vertical strain.

The baselength *L* is about 2 m

Hydraulic amplification makes *L* for the sensor about 50 m.

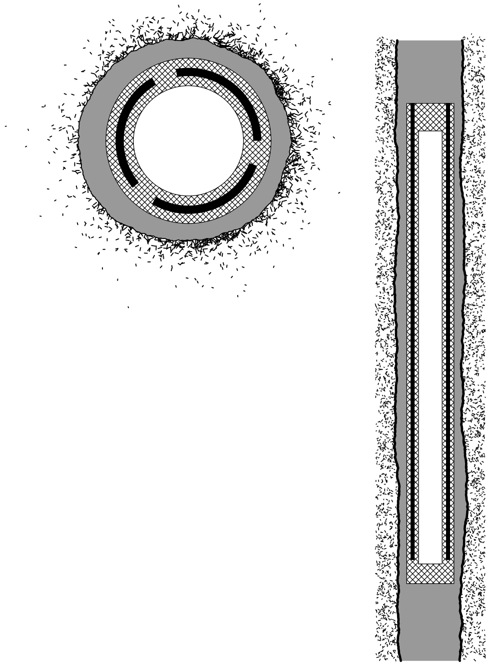


Sakata Directional Volume Strainmeter

This instrument has three fluid volumes, each sensed separately, with the relative changes in different volumes allowing the measurement of the full strain tensor.

Works because a non-cylindrical space with fluid responds to different horizontal strains with different volume changes.

A version of this was used in the “Mini-PBO”.

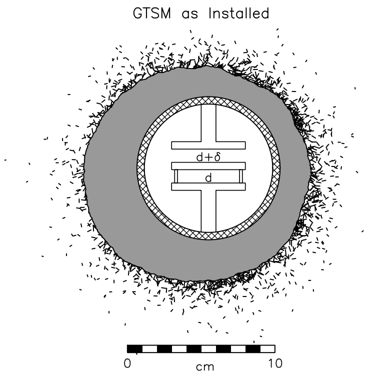


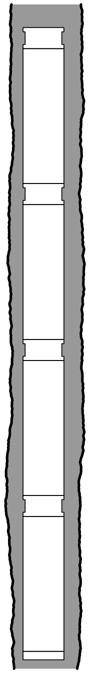
Gladwin Tensor Strainmeter

This has an *L* of 0.097 m; displacements (very small) are measured using a capacitive sensor.

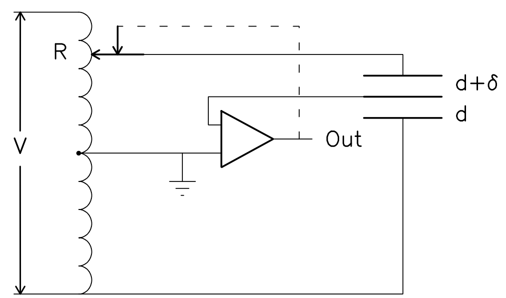
The PBO instrument has 4 sensing modules, one for each direction: the first three are at 120° to each other, and the fourth at 90° to the first.

This provides redundancy, a calibration check – and [useful] confusion.





Gladwin Tensor Strainmeter Electronics



An input voltage is divided in a ratio transformer, with the output (ratio to input good to 7 figures) applied to the capacitor plates. The ratio R is varied both to minimize the output from the center plate, and also to calibrate the system.

The two capacitances C_1 and C_2 are proportional to d^{-1} and $(d+\delta)^{-1}$; the output voltage is zero if $\frac{1-R}{R} = \frac{C_2}{C_1} = \frac{d+\delta}{d}$ so that $d+\delta = d\,\frac{R}{1-R}$; the "linearized strain" is found from R and the output voltage using this equation.

Rotation Meter

Measures the rotation around a vertical axis (tiltmeters measure around a horizontal axis).

There are three different kinds:

- **A. Reference:** inertial space
- **A. Sensor:** light (Sagnac effect) to measure *rate* of rotation.

- **B. Reference:** mass on a pivot (short periods only)
- **B. Sensor:** displacement

- **C. Reference:** gyroscope (noisy)
- **C. Sensor:** displacement

(A) is still experimental; (B) the second is only useful at high frequencies, and (C) is noisy, but could be used for strong motion.