Tides

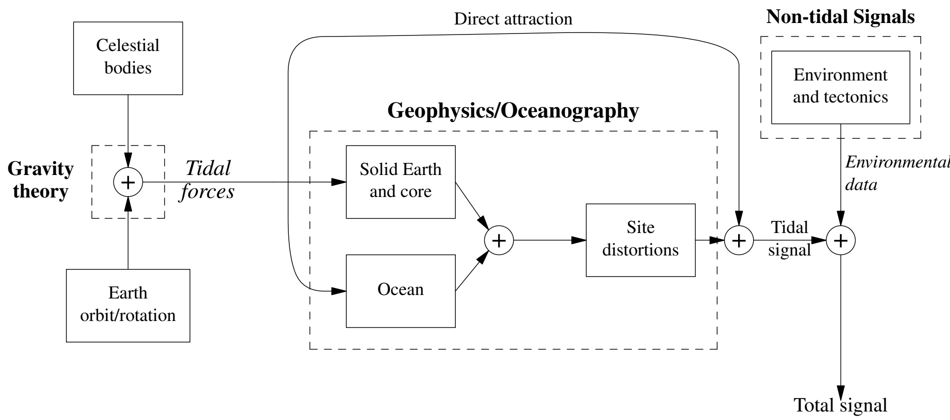
Q: What do we want to know about tides for?

A: So we can use them as a calibration signal for the BSM's.

The aim is thus to **model** the tides as well as we can, rather than to **analyze** them to understand the Earth (the usual goal of Earth-tide studies).

Fortunately, accurate modeling of the tides is not too difficult—up to a point.

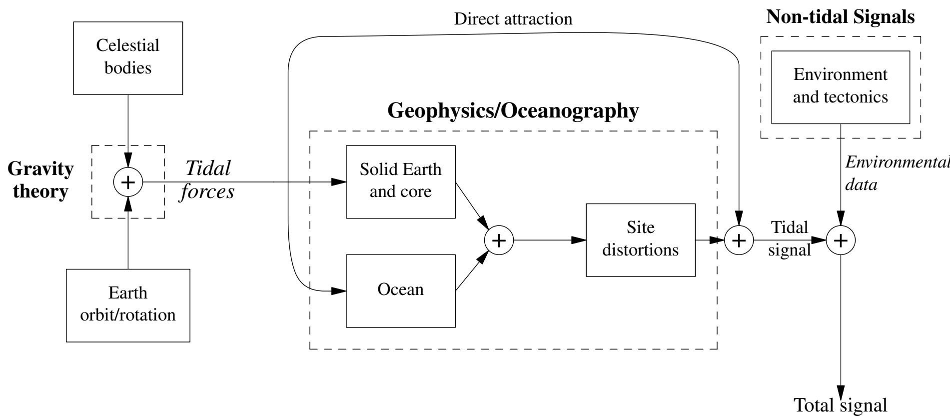
An Overview



A. We start with the **tidal forces**, which can be computed in two ways:

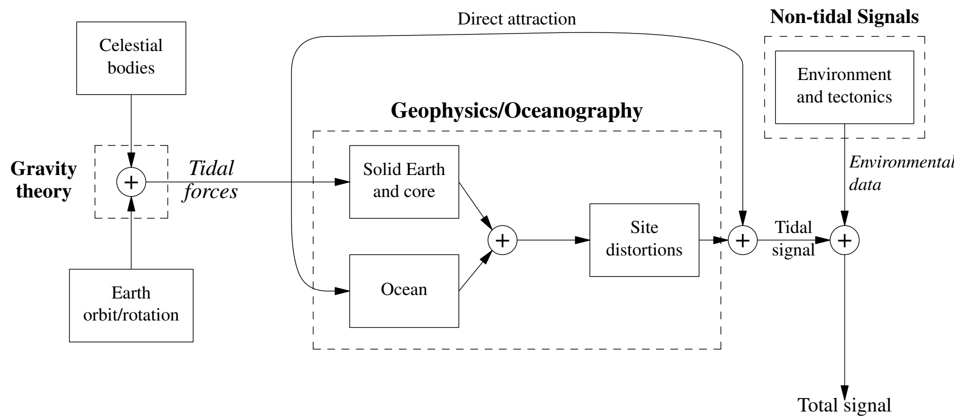
1. By computing the locations of the Moon and Sun and computing the forces actually, the potential) directly: this is what is done by `ertid` (in the PIASD and SPOTL packages).
2. By using a set of harmonic constituents: or, for one constituent, its amplitude.

For calibrations we are always interested in a few constituents only: the ones with good signal-to-noise ratio. Effectively, we do the calibrations at a few frequencies; since we expect the response to be frequency-independent, this is OK.



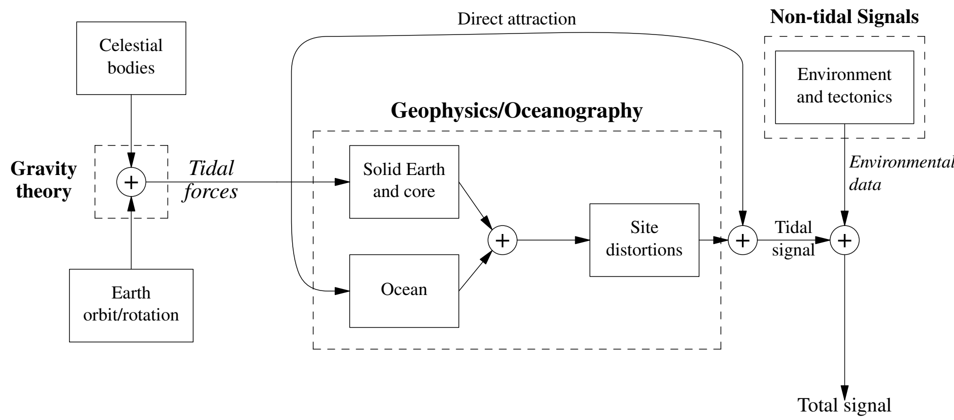
B. Next, we find the response of the Earth to the potential— assuming it to be elastic and otherwise idealized. This response is given by the **Love [Shida] numbers**, which are well-known from seismic models (1%). These are built into `ertid`, which thus computes a time-history of the tides on an elastic Earth.

- This elastic-Earth tide is called the **body tide**. For a single constituent it can be found from the amplitude of that constituent in the potential, times a combination of Love numbers and some trigonometric functions of the latitude.



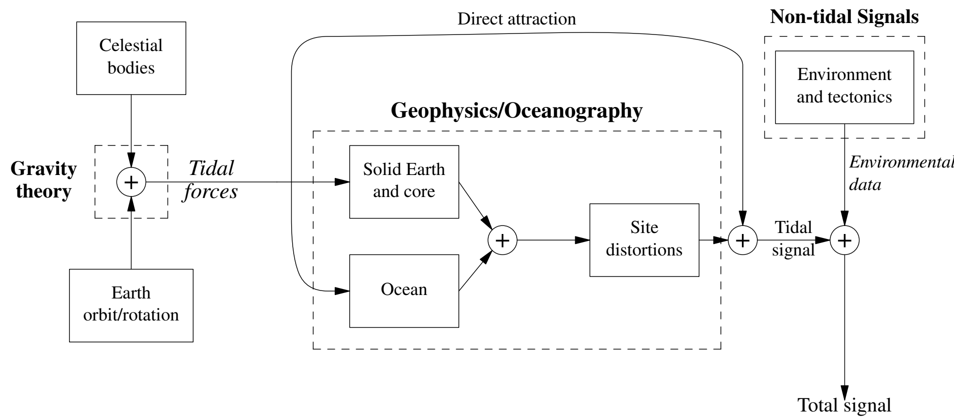
C. We also find the response of the ocean. Actually, “we” don’t do this: we just take over someone else’s models for the ocean-tide height, since this is a specialized area.

- These ocean models are given for specific frequencies (constituents): another reason for doing calibrations in the frequency domain.



D. Given an ocean-tide model, or set of models, we find the strains produced at our location by the loading of the Earth. This is done using the `spotl` package, to give the **load tides**.

- Like the ocean models, the load tides are for particular constituents.

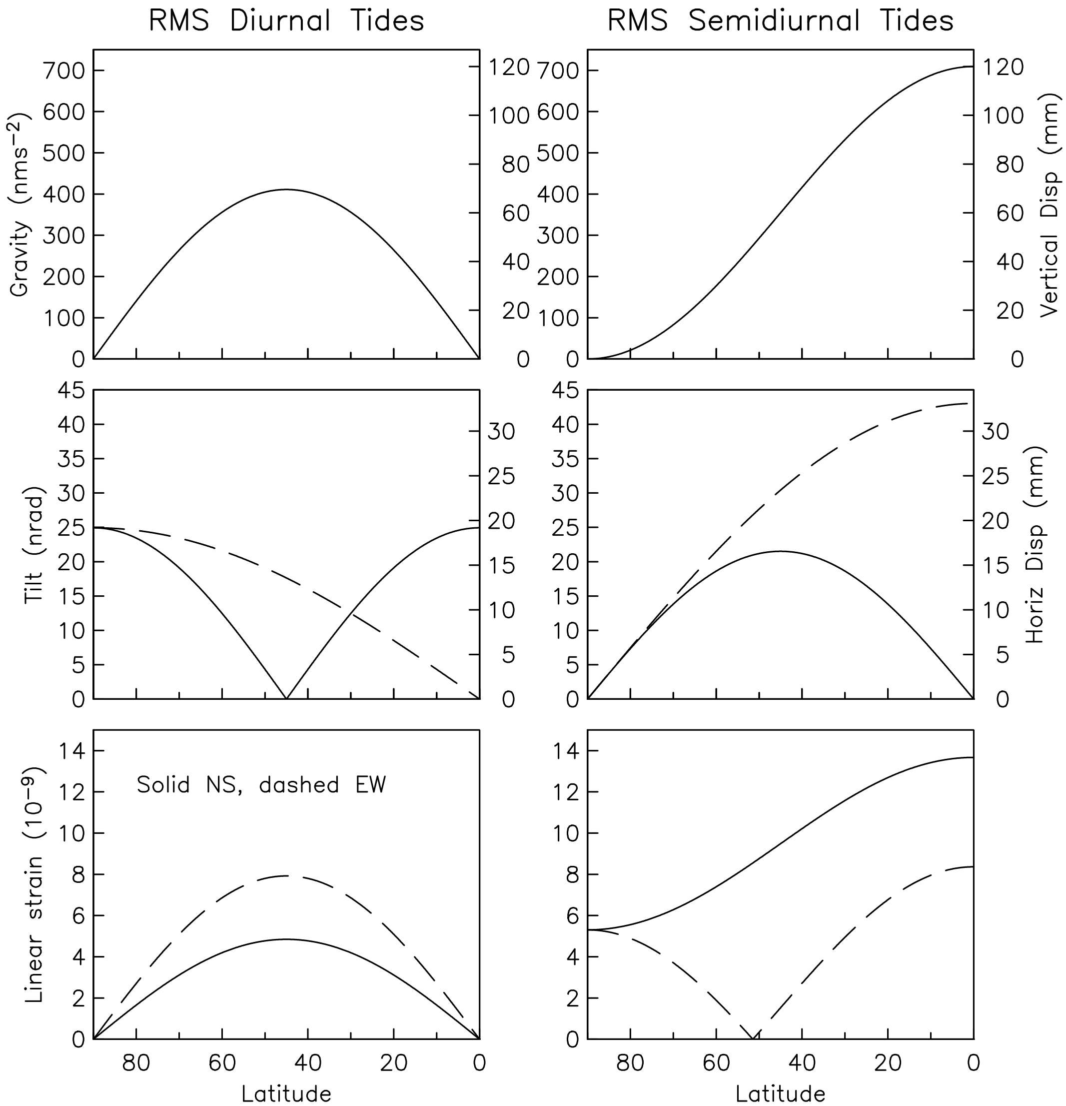


E. Finally, the body tide plus the load tide is the **theoretical tide**, which we can compare the observations to.

The diagram shows two additional features, one relevant to strain tides and one not:

1. For some tides there is a contribution even if the Earth is assumed rigid. (Obviously, not true for strain).
2. For the strain tides, and others, the tides (and other strains) may be distorted by local topography and geology. This is by far the biggest source of inaccuracy in the theoretical strain tides; unfortunately the amount of inaccuracy (systematic error) is very difficult to quantify.

Elastic-Earth Tides



The plot above shows the latitude dependence of the rms tides for several types of tide.

Areal strain has the same latitude dependence as vertical displacement and gravity.

Extensional strains depend on the azimuth as well.

More on Ocean Loading

To compute loading, we find the integral of the (complex) tide height H over the sphere:

$$\int_0^{\pi} d\theta \int_0^{2\pi} d\phi G_l(\theta, \phi, \theta', \phi') \rho g H(\theta, \phi) \sin\theta$$

G_L is the **Green function** for an effect at θ', ϕ' from a point load (δ-function) applying a force $\rho g H d\theta \sin\theta d\phi$ at θ, ϕ.

For a spherically symmetric Earth, G_L depends only on the distance Δ between θ, ϕ and θ', ϕ', so we have

$$\int_0^{\pi} d\Delta \int_0^{2\pi} d\theta G_L(\Delta, \theta) \rho g H(\Delta, \theta) \sin\Delta$$

where G_L depends on the azimuth θ only through trigonometric expressions for vector or tensor quantities.

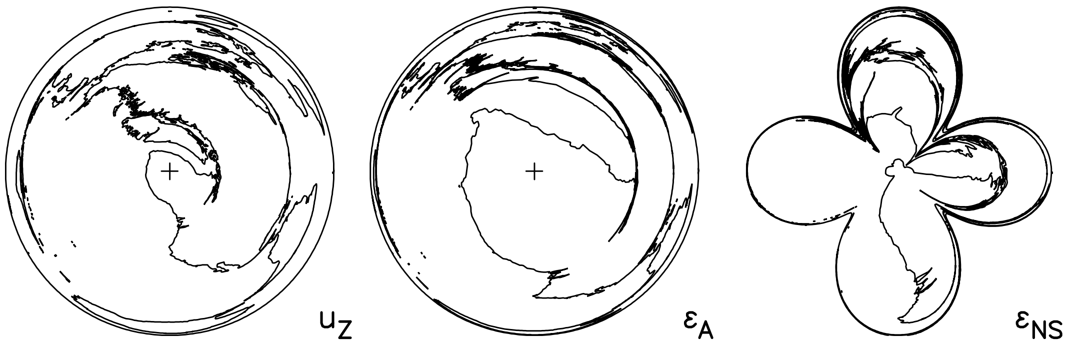
The SPOTL package does this computation; it includes

A. A description of H: **ocean-tide models**.

B. A description of where the ocean ends: a **land-sea model** (finer detail than the ocean models).

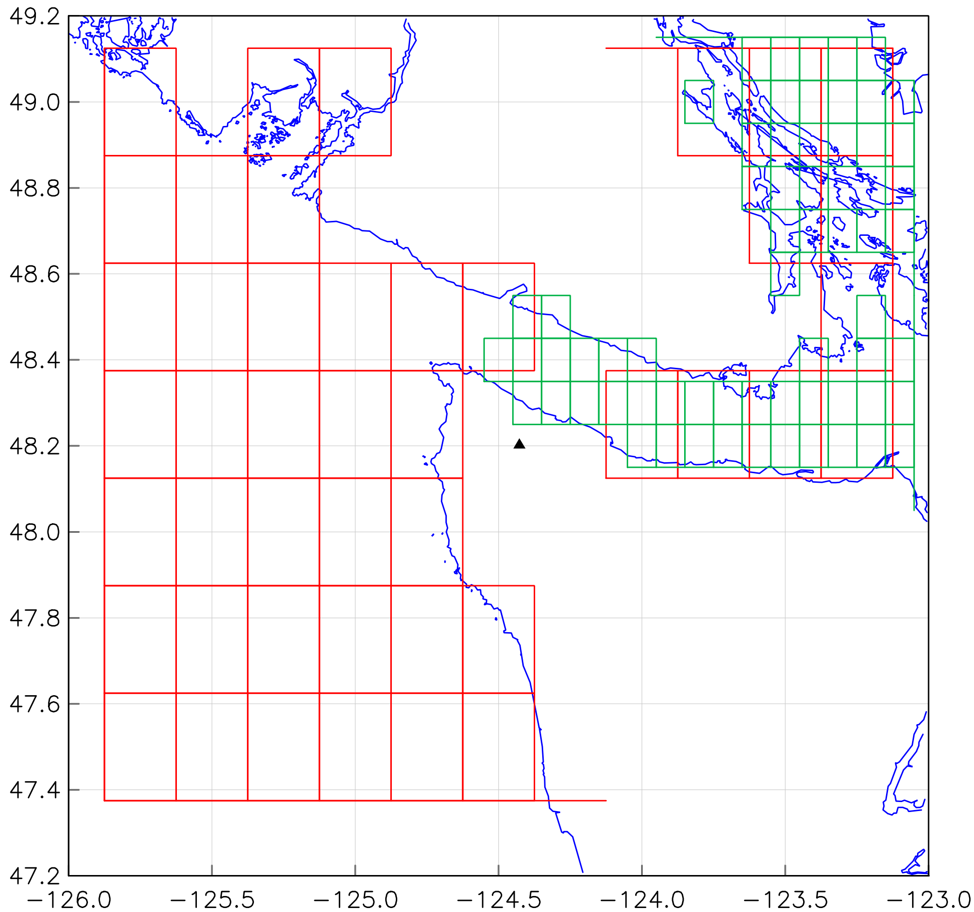
C. Green functions, for specified Earth models.

The loading viewed from Hoko Falls



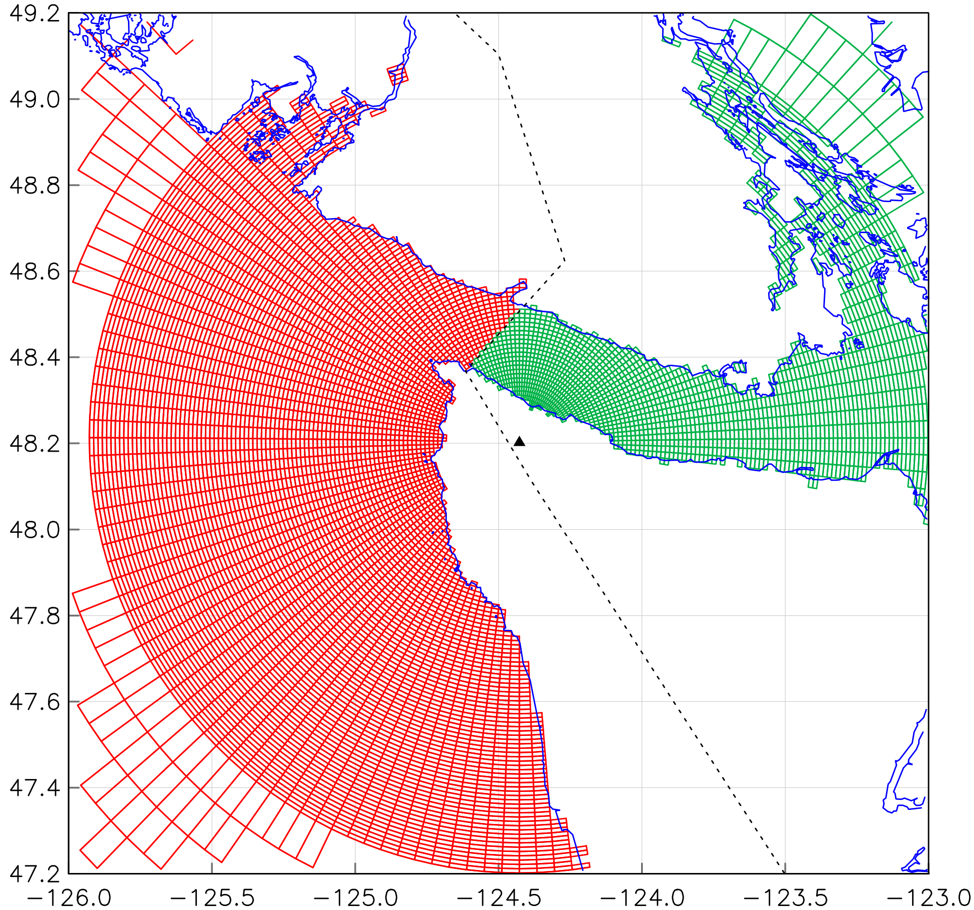
The plot above shows a map, with equal areas indicating equal effect on the computed load (assuming the same heights). Local areas dominate the picture, so we need good local tidal models, which are done separately from global models.

Ocean Models for the PNW

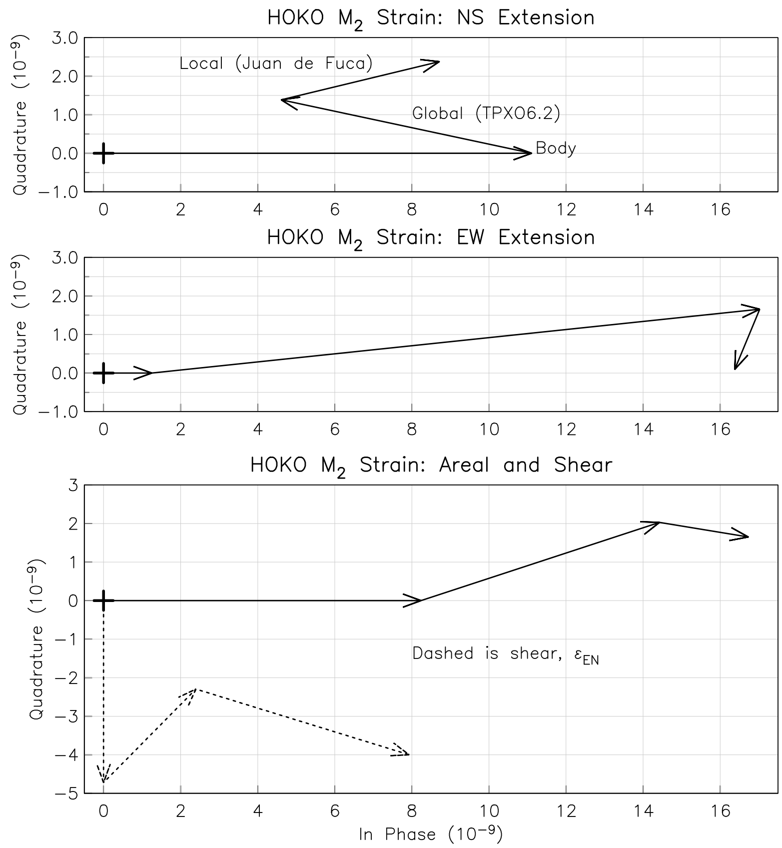


The red grid is for the global TPXO tidal model; the green is for a local model for the Straits of Georgia and Juan de Fuca.

Neither grid is very detailed, so SPOTL uses its own, centered on the station.



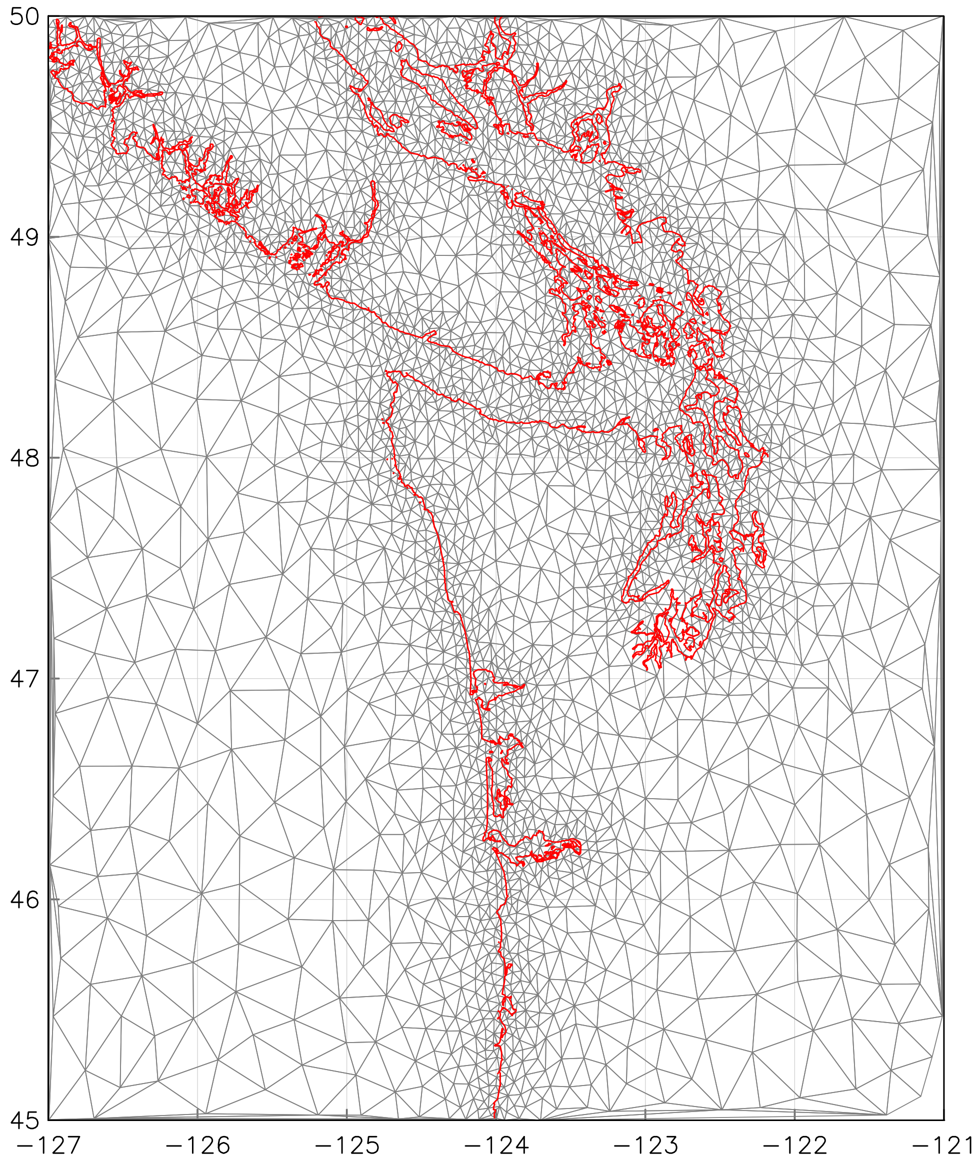
One useful feature of SPOTL is that it is possible to combine local and global models without overlap, using polygons in lat and long to define areas to be used or omitted.

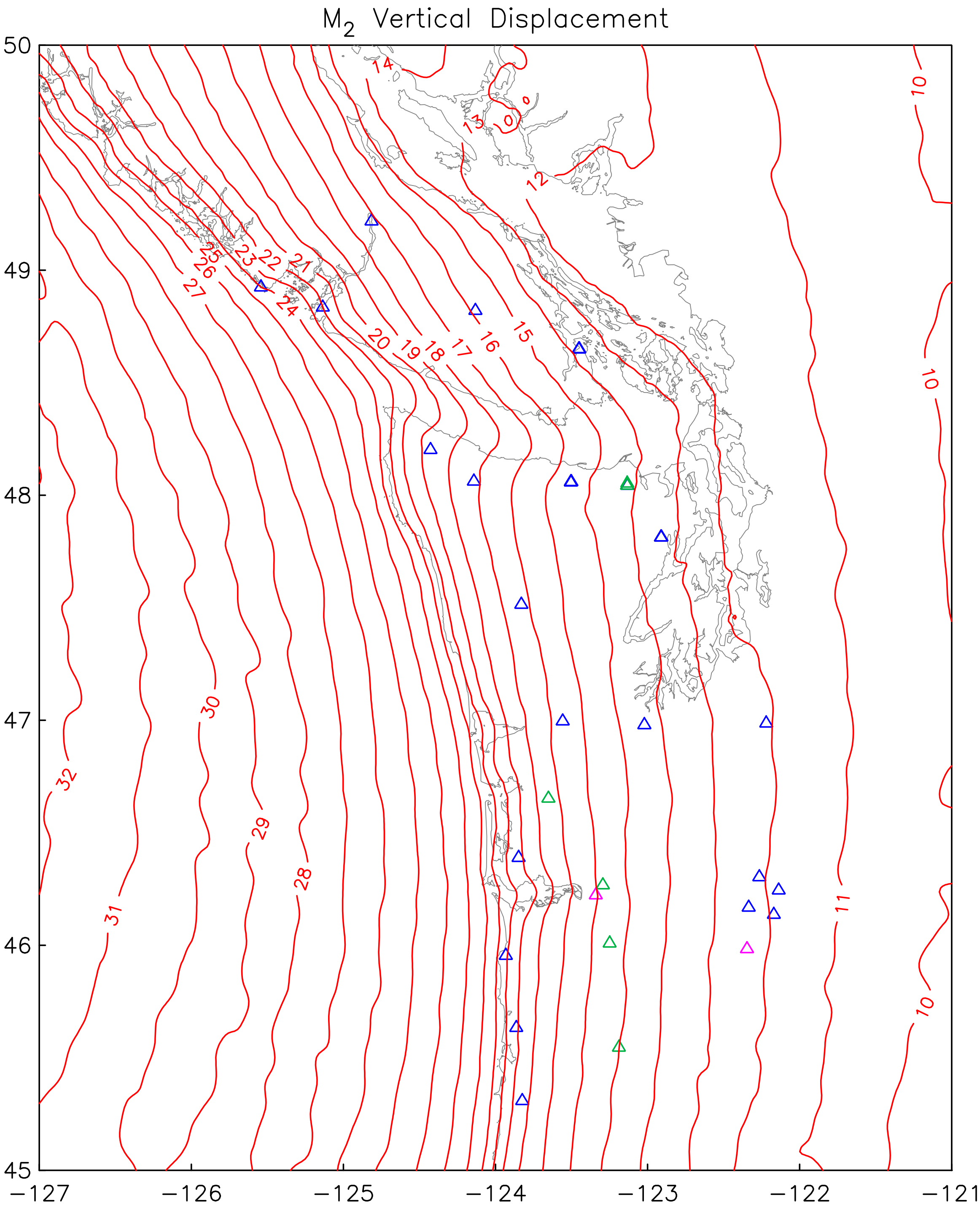


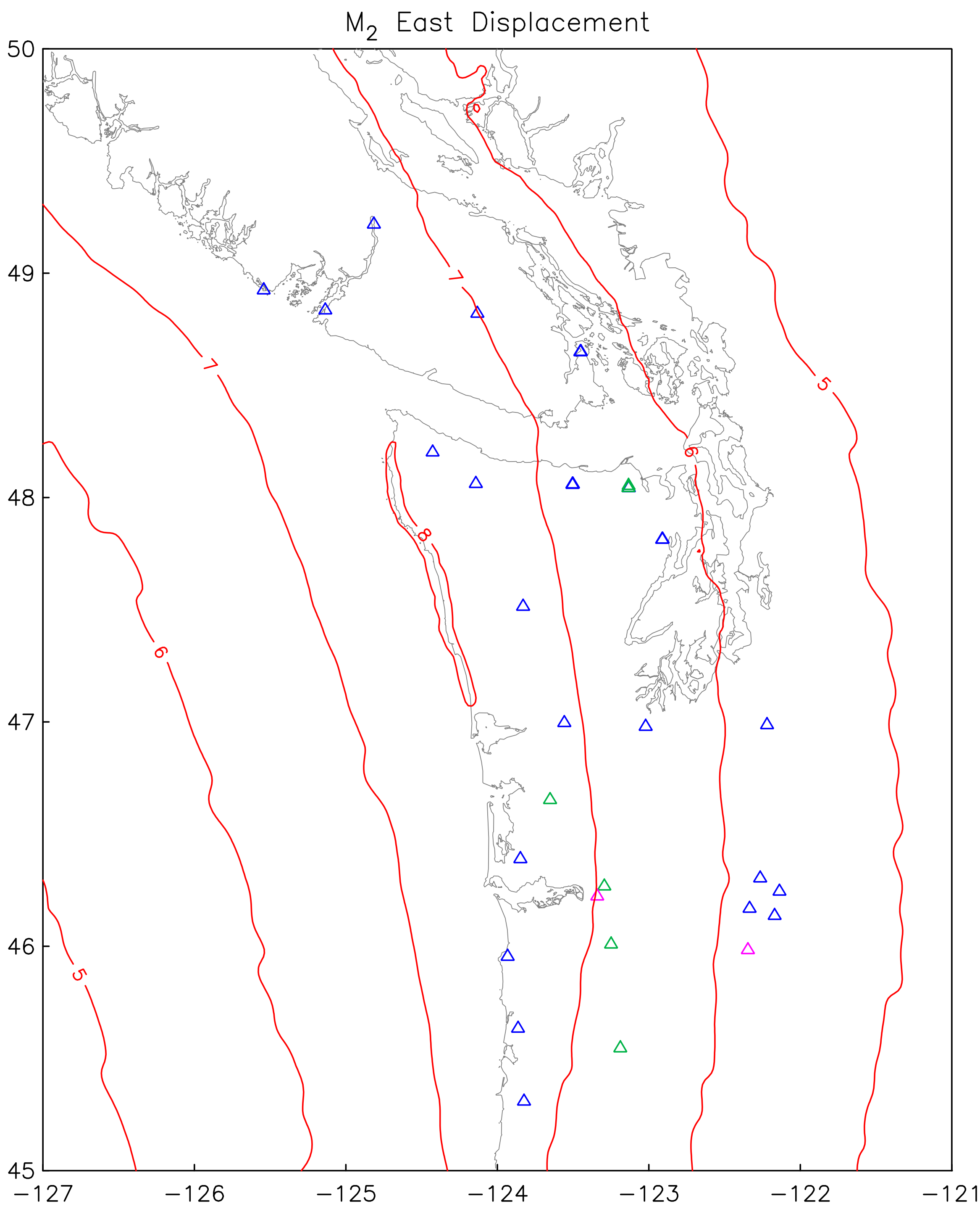
This is the final result for HOKO; note that both the global and local models are important—though the local part of the global model is probably dominating.

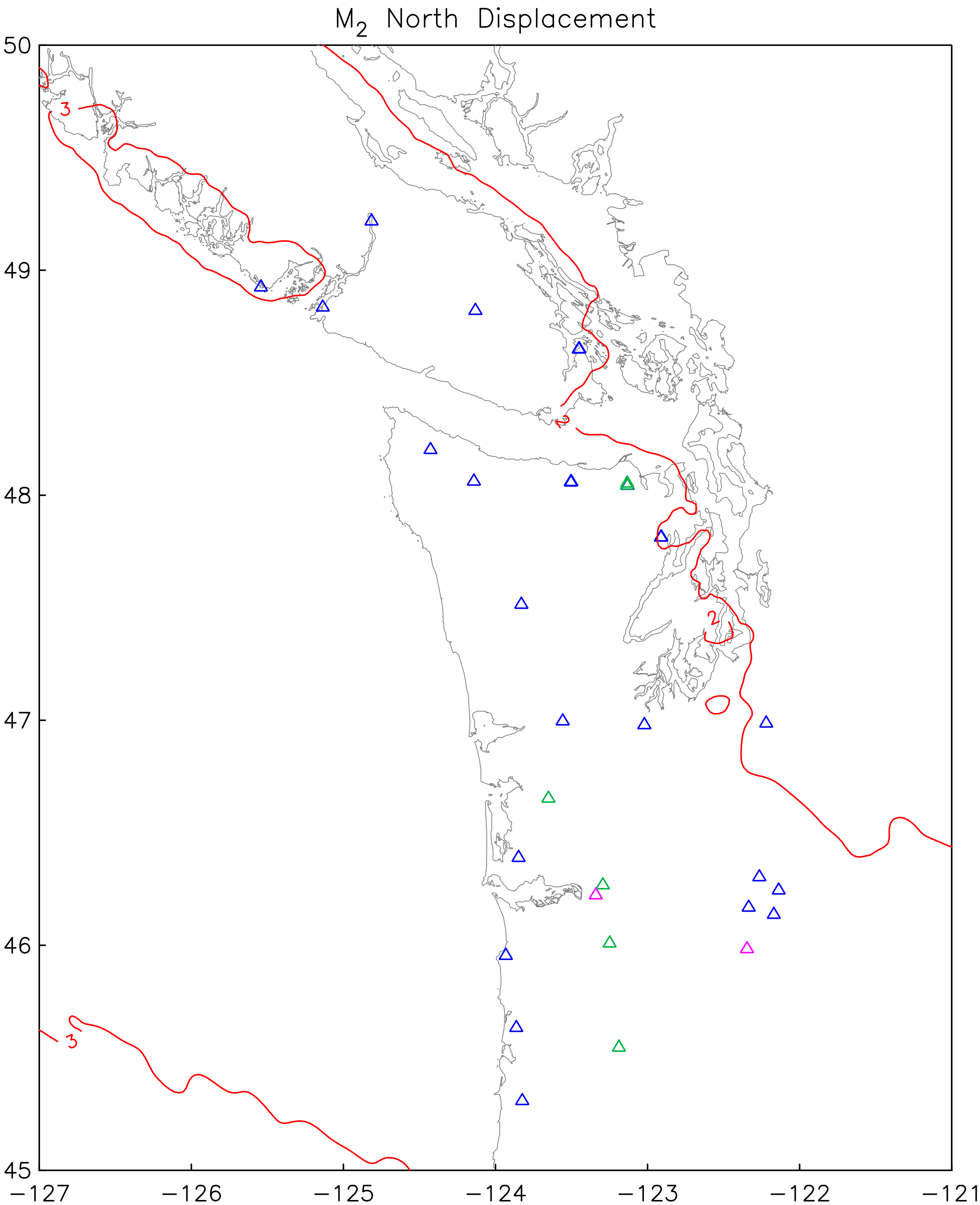
Some Examples of Ocean Loads

We show maps of loads, computed on grids that get finer near the coast, and then interpolated and contoured. We start with an overview for the PNW.

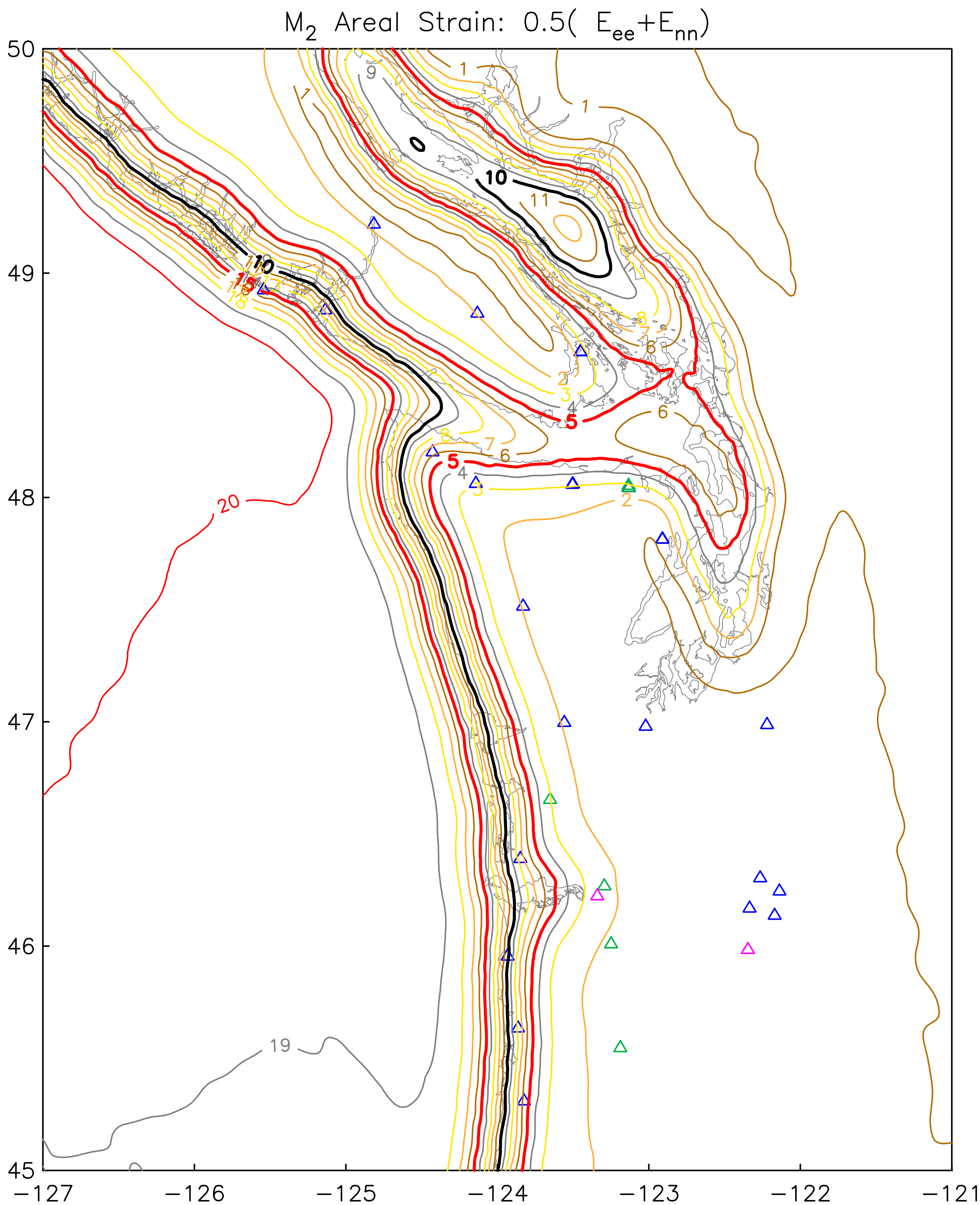



M2 Vertical Displacement
50
49
48
47
46
45
−127
−126
−125
−124
−123
−122
−121
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32

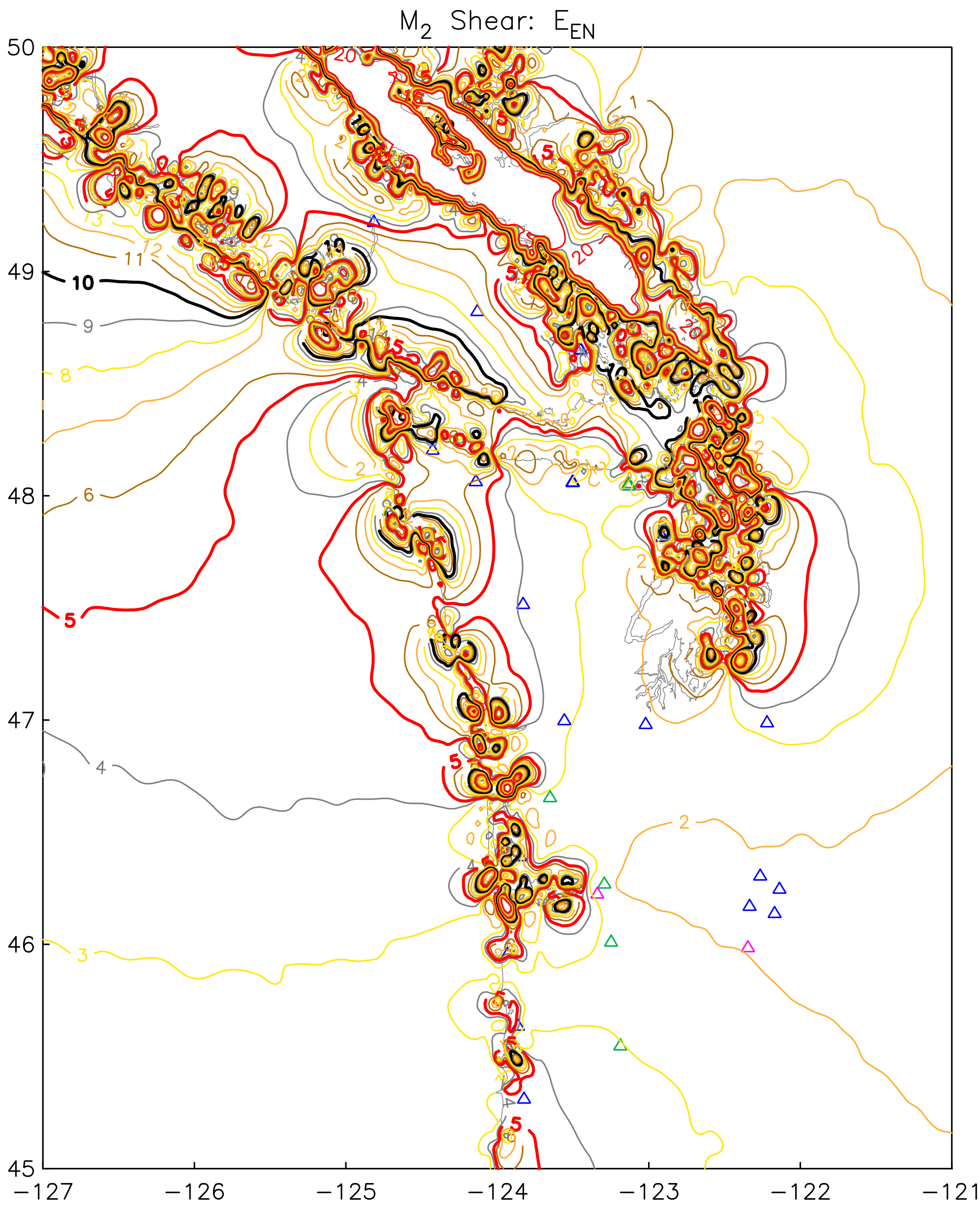

M_2 East Displacement
50
49
48
47
46
45
−127
−126
−125
−124
−123
−122
−121
5
6
7
8
7
6
5



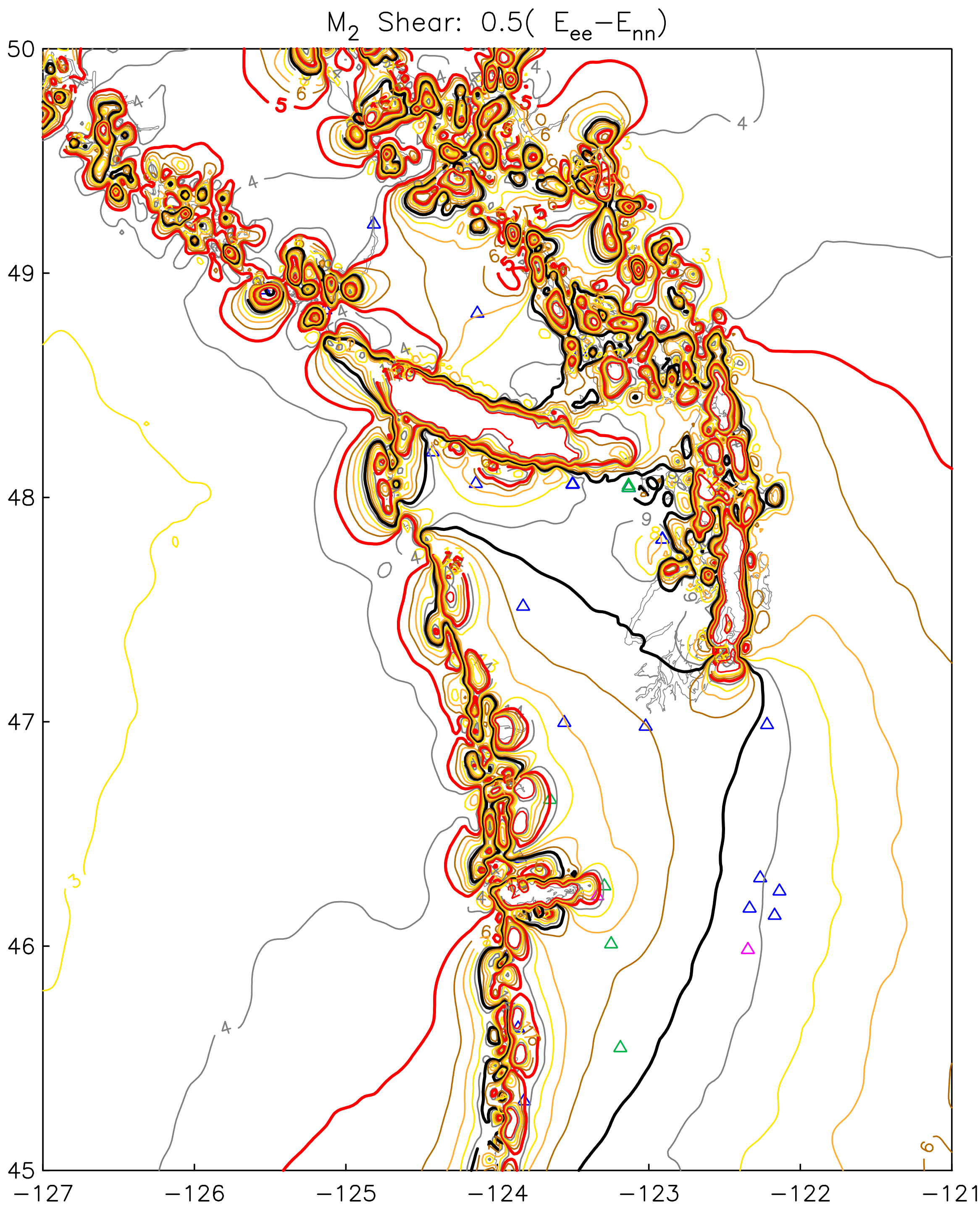
M_2 North Displacement
50
49
48
47
46
45
−127
−126
−125
−124
−123
−122
−121
3
2



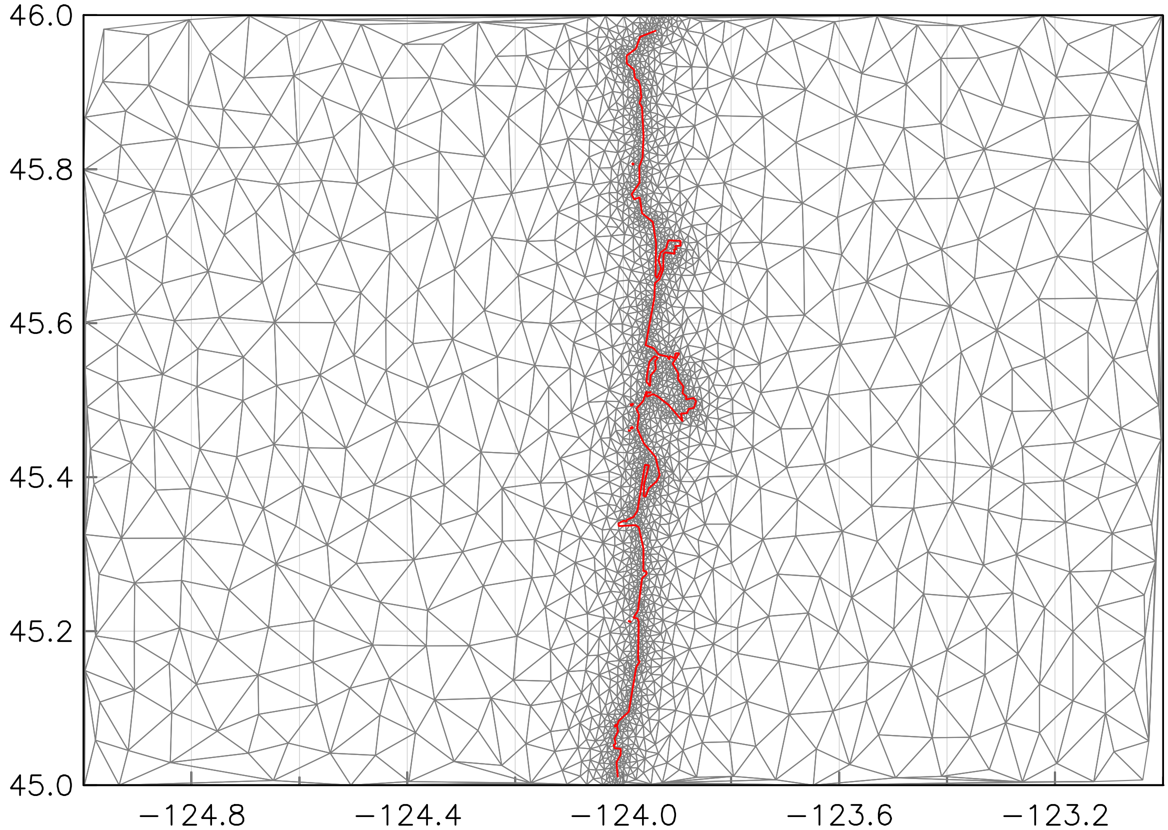
M$_2$ Areal Strain: 0.5(E$_{ee}$+E$_{nn}$)
50
49
48
47
46
45
−127
−126
−125
−124
−123
−122
−121
20
19
10
5

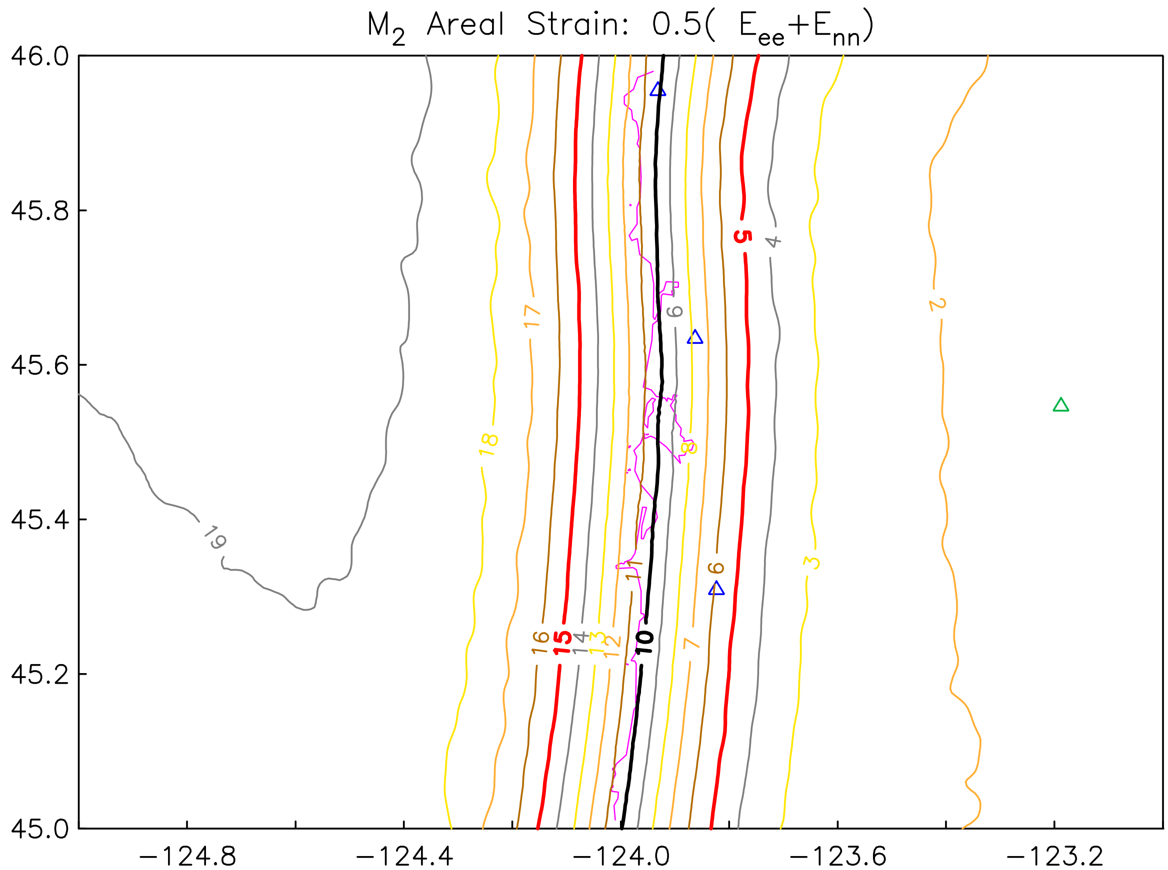


M_2 Shear: E_{EN}
50
49
48
47
46
45
−127
−126
−125
−124
−123
−122
−121

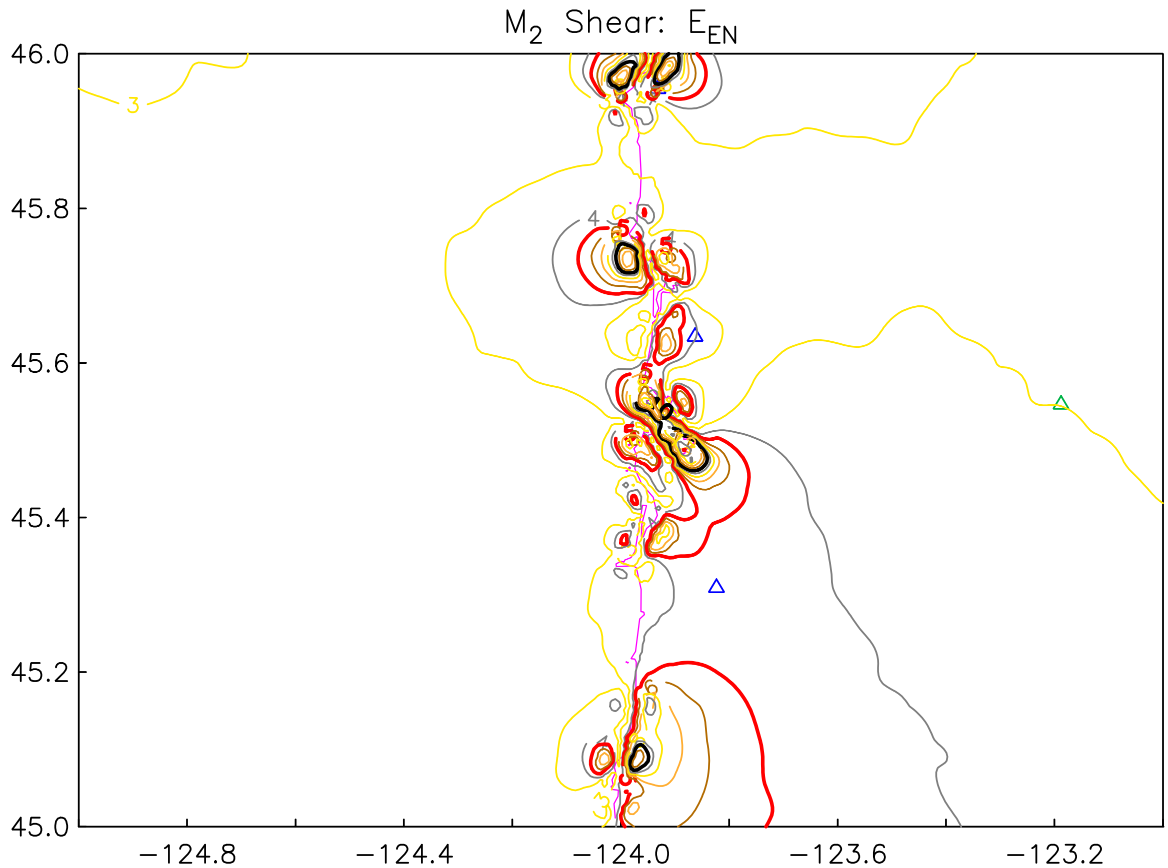

M2 Shear: 0.5(Eee−Enn)
50
49
48
47
46
45
−127
−126
−125
−124
−123
−122
−121

A finer spacing (300 m near the coast):

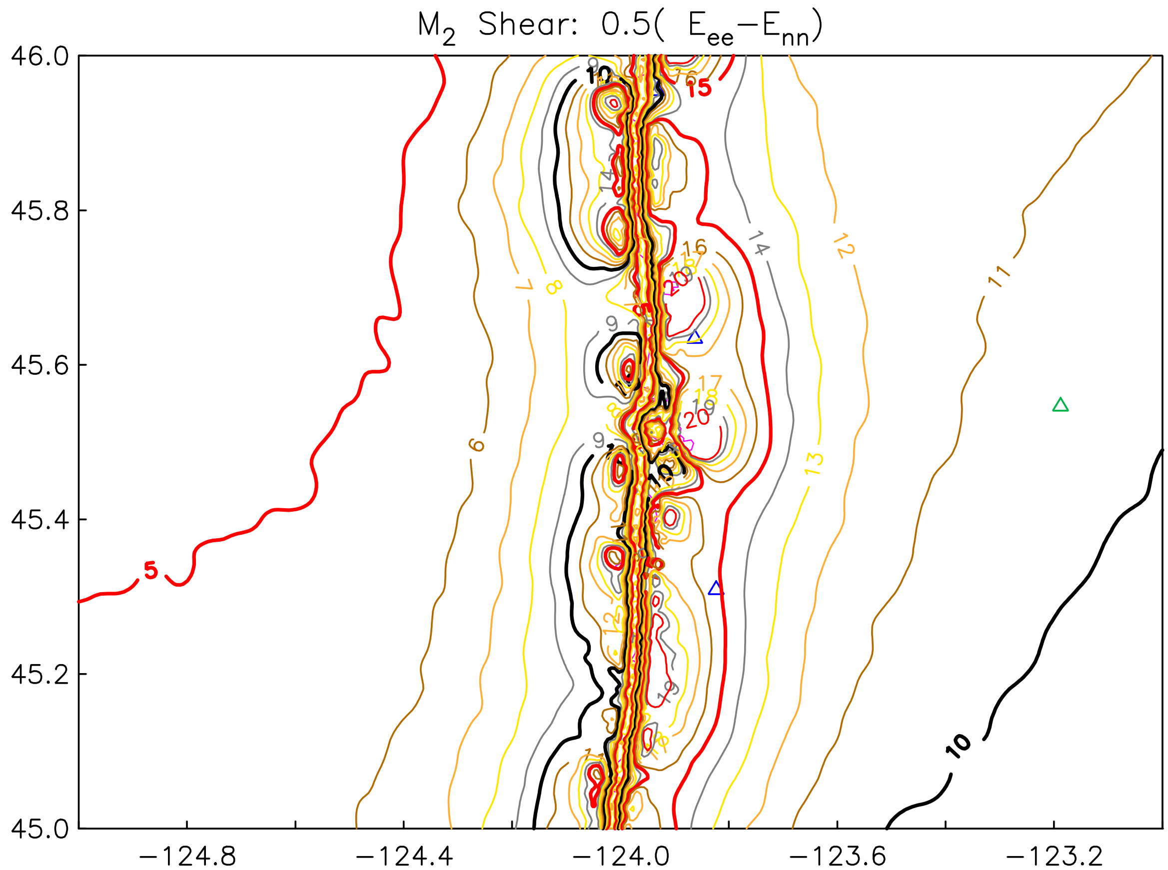




M$_2$ Areal Strain: 0.5(E$_{ee}$+E$_{nn}$)
46.0
45.8
45.6
45.4
45.2
45.0
−124.8
−124.4
−124.0
−123.6
−123.2
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2

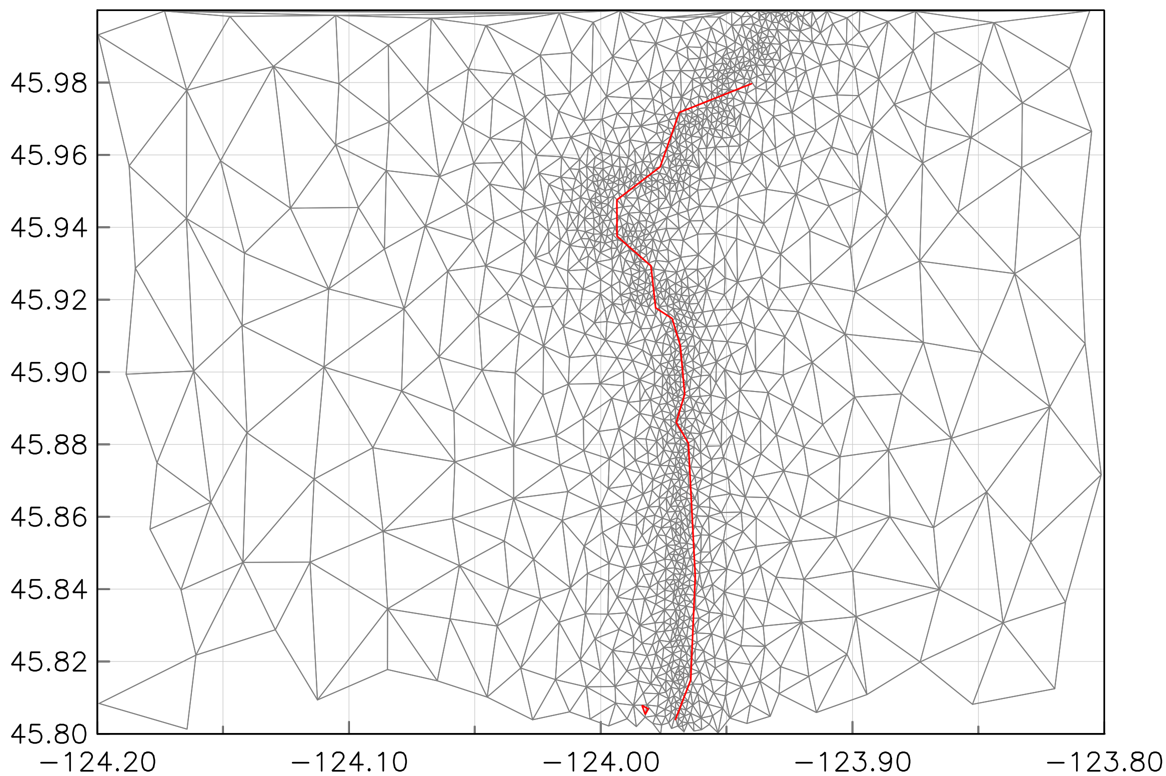


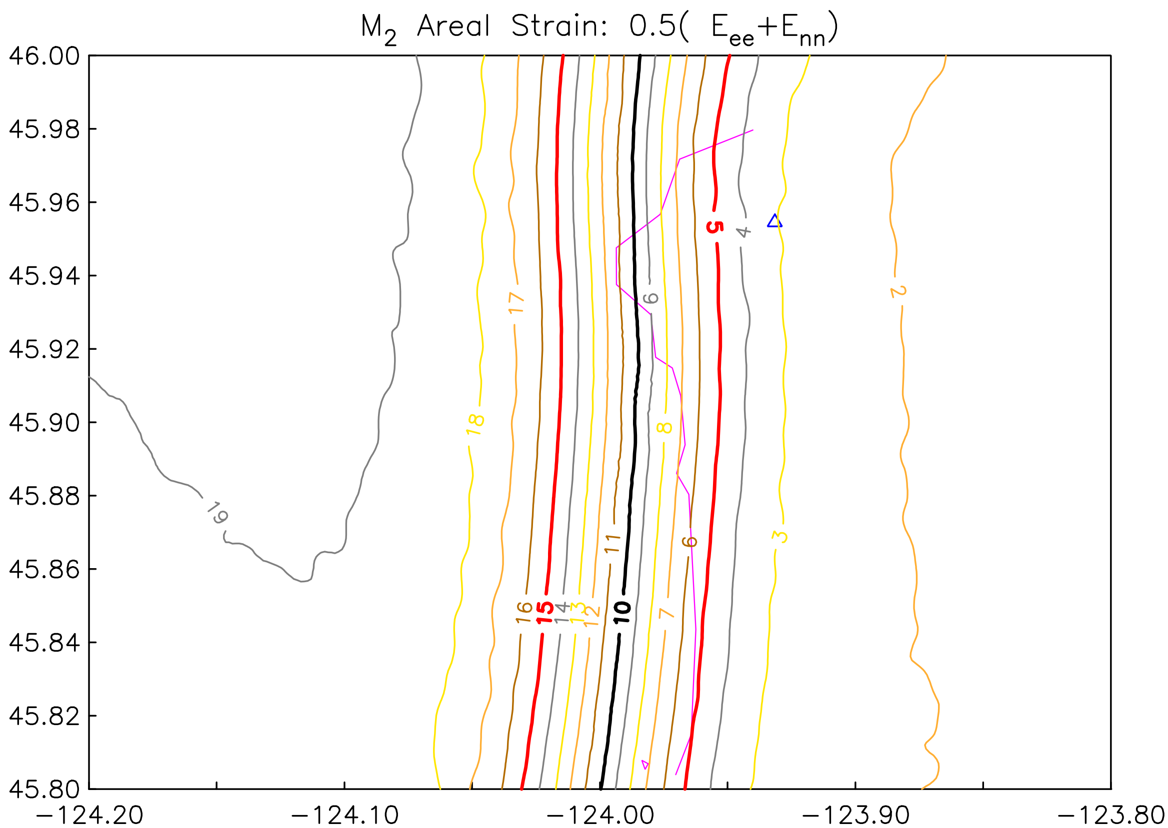
M2 Shear: EEN
46.0
45.8
45.6
45.4
45.2
45.0
−124.8
−124.4
−124.0
−123.6
−123.2



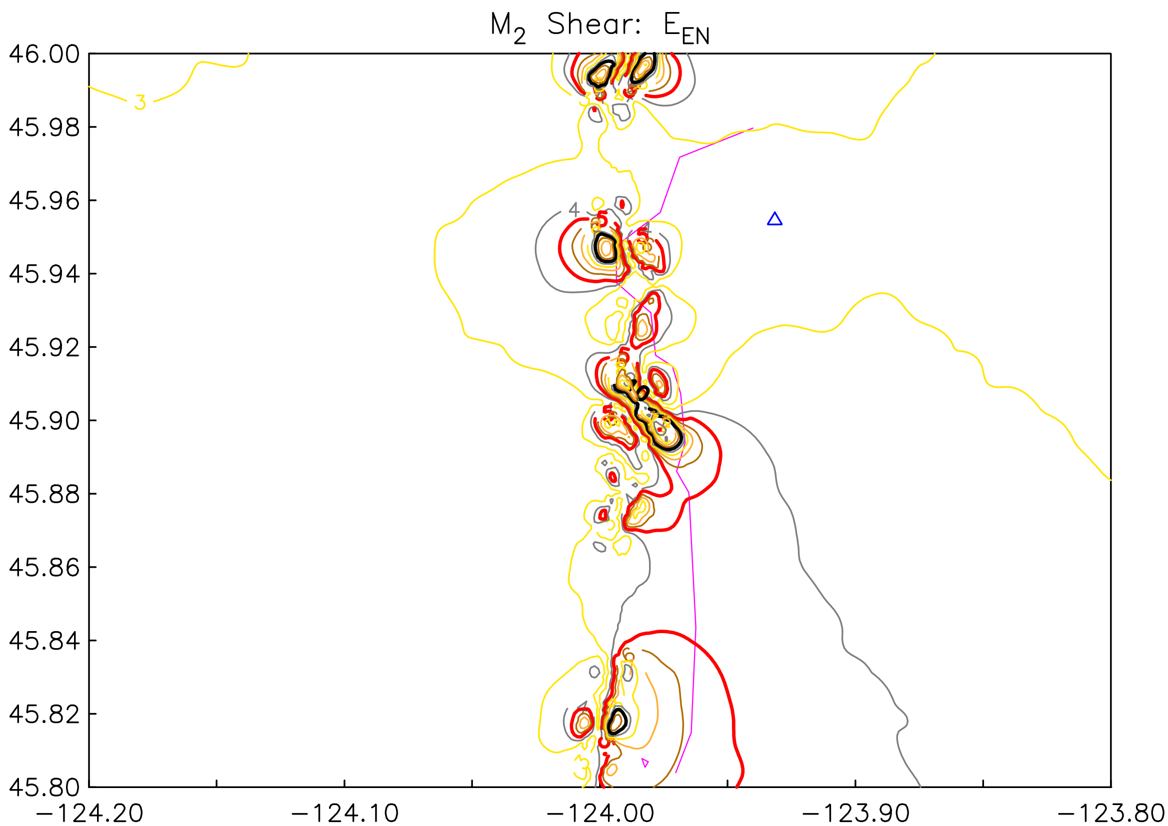
M_2 Shear: 0.5($E_{ee}-E_{nn}$)
46.0
45.8
45.6
45.4
45.2
45.0
−124.8
−124.4
−124.0
−123.6
−123.2

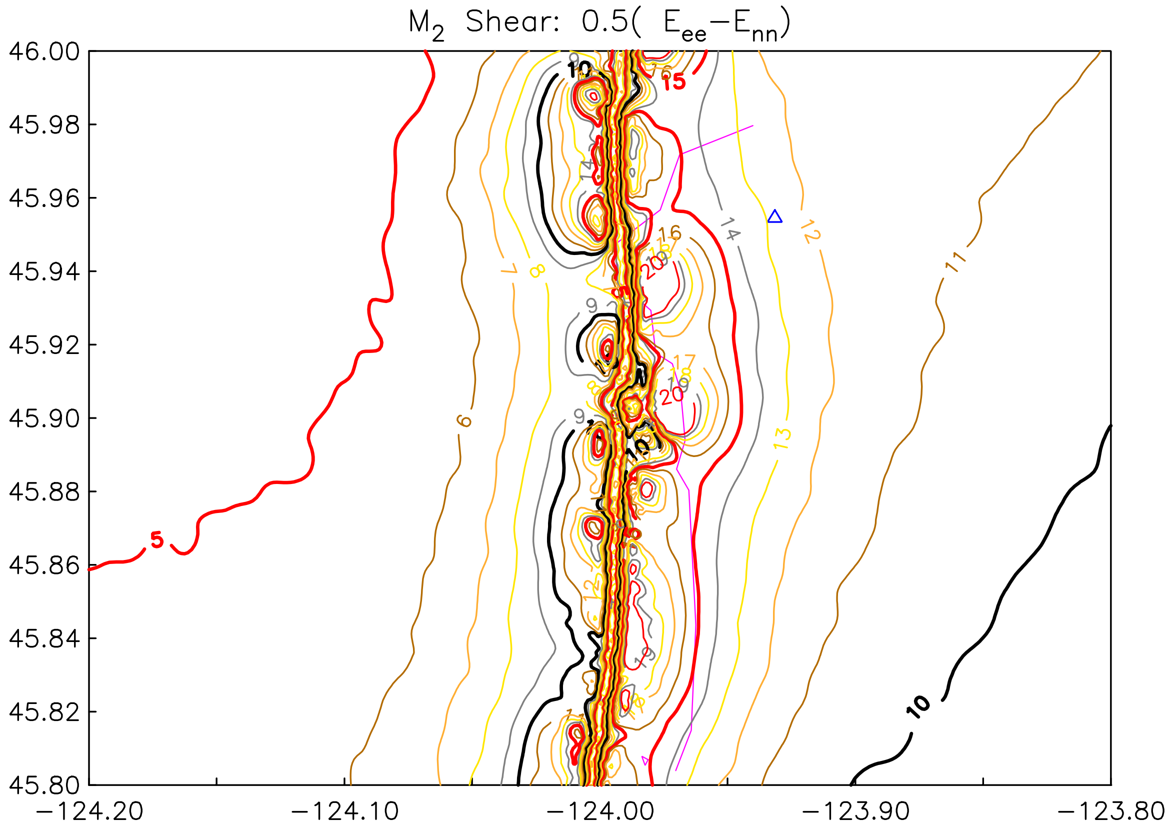
Even finer spacing (100 m near the coast); this shows the approximate nature of the global land-sea model.





M$_2$ Areal Strain: 0.5(E$_{ee}$+E$_{nn}$)
46.00
45.98
45.96
45.94
45.92
45.90
45.88
45.86
45.84
45.82
45.80
−124.20
−124.10
−124.00
−123.90
−123.80
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2


M_2 Shear: E_{EN}
46.00
45.98
45.96
45.94
45.92
45.90
45.88
45.86
45.84
45.82
45.80
−124.20
−124.10
−124.00
−123.90
−123.80



The fine grids show that the singularity in strain is confined to the water's edge; if the strain were computed at the depth of the strainmeter, the response would no longer be singular.

Sample SPOTL Syntax

```
nloadf HOKO 48.202 -124.427 100. m2.gefu   green.contap.std l poly.gefu + > tmp1
nloadf HOKO 48.202 -124.427 100. m2.tpxo70 green.contap.std l poly.gefu - > tmp2
cat tmp1 tmp2 | loadcomb c > tmp3
```

Sample SPOTL Output

```
S  HOKO                                          48.2020 -124.4270      100.
O M2        2 0 0 0 0 0     Straits of Georgia and Juan de Fuca
G                GUTENBERG BULLEN GREENS FUNCTIONS  JOBO2Q 10/19/71
G    Rings from   0.03 to   1.00 with spacing 0.01 - detailed grid used
G    Rings from   1.05 to   9.95 with spacing 0.10 - detailed grid used
G    Rings from  10.25 to  89.75 with spacing 0.50 - ocean model grid used
G    Rings from  90.50 to 179.50 with spacing 1.00 - ocean model grid used
P Polygon to include the Straits of Georgia and Juan de Fuca
P   all polygon areas included
C   Version 3.2 of load program, run at Wed Jun 11 08:10:31 2008
C   closest nonzero load was   0.09 degrees away, at   48.28 -124.39
C       23 zero loads found where ocean present, range   0.78-   3.05 deg
L l          Phases are local, lags negative
O M2        2 0 0 0 0 0     OSU TPXO 7.0
G                GUTENBERG BULLEN GREENS FUNCTIONS  JOBO2Q 10/19/71
G    Rings from   0.03 to   1.00 with spacing 0.01 - detailed grid used
G    Rings from   1.05 to   9.95 with spacing 0.10 - detailed grid used
G    Rings from  10.25 to  89.75 with spacing 0.50 - ocean model grid used
G    Rings from  90.50 to 179.50 with spacing 1.00 - ocean model grid used
P Polygon to include the Straits of Georgia and Juan de Fuca
P   all polygon areas excluded
C   Version 3.2 of load program, run at Wed Jun 11 08:10:32 2008
C   closest nonzero load was   0.17 degrees away, at   48.21 -124.69
C       39 zero loads found where ocean present, range   0.83-   9.85 deg
L l          Phases are local, lags negative
X
g            5.6220  179.0940
p           17.1925   -7.5024
d            7.3408 -179.8589    2.1931 -102.5371   19.5748  176.3950
t          130.4428 -163.5393   25.0510  -77.1638
s           15.3493    1.0151    3.4055  136.6995    7.7535    5.3059
```

Last line is amp and local phase of strain: ε_{EE}, ε_{NN}, ε_{EN}.