Tidal Analysis

What You Need to Know I

A. The tidal forces are **periodic**, with **known frequencies** (really, really well-known).

B. The **amplitudes** of the tidal forces are also very well known.

C. The response of the Earth and ocean is such that, for frequencies close together, the relative amplitudes of the observed tides are close to those of the tidal forces: and the same for the phases.

$$\text{force} \quad F = A_1 \cos(2\pi f_1 t + \phi_1) + A_2 \cos(2\pi f_2 t + \phi_2)$$

$$\text{observed} \quad F = B_1 \cos(2\pi f_1 t + \theta_1) + B_2 \cos(2\pi f_2 t + \theta_2)$$

$$\text{if} \quad |f_1 - f_2| \ll f_1 \quad \text{then}$$

$$\frac{B_1}{B_2} \approx \frac{A_1}{A_2} \quad \text{and} \quad \theta_1 - \theta_2 \approx \phi_1 - \phi_2$$

This last result is referred to as the **smoothness of the admittance** (or the smoothness of the frequency response).

What You Need to Know II

D. The tidal frequencies separate into bands around 0, 1, 2 (and 3) cycles/day. The relative amplitudes of the bands depend on the location and type of measurement.

E. Each band separates into groups, spaced 1 cycle/month apart. Their relative amplitudes are invariant.

F. The groups are made up of individual **constituents**, spaced 1 cycle/year, or less, apart.

G. There is a system for identifying each constituent, but in practice we talk about the larger ones using their **Darwin symbols**. These are totally unmemorable—bad luck!

M_2 largest semidiurnal

S_2 large semidiurnal, but at 2.000 cpd, so usually useless

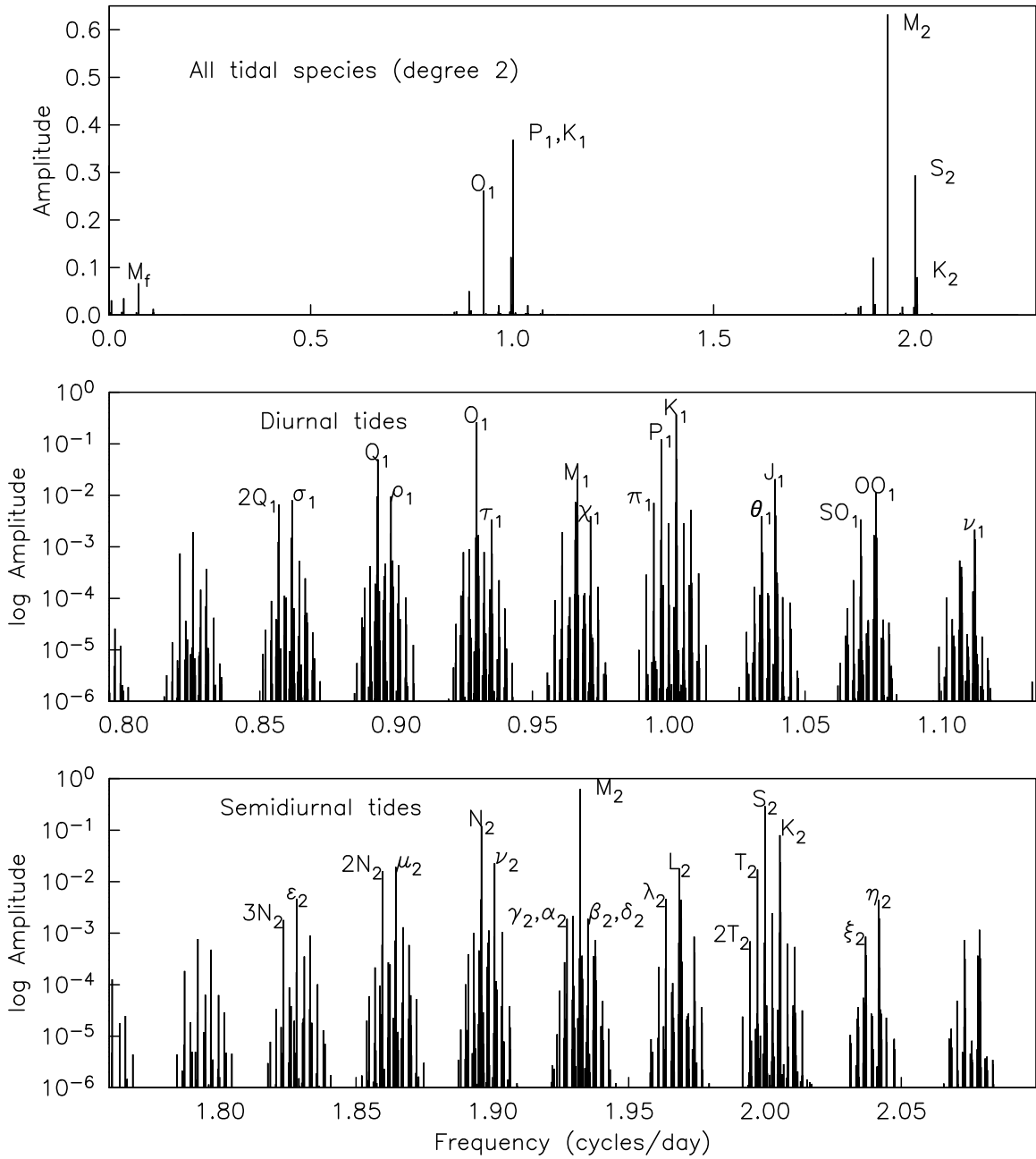N_2 smaller semidiurnal

K_2 smaller semidiurnal, close to S_2, but less contaminated

K_1 largest diurnal tide, but close to 1 cpd, so usually contaminated. Also affected by core-mantle boundary (core resonance).

O_1 next largest diurnal tide, far from 1 cpd

P_1 diurnal tide, close to 1 cpd

Tidal Potential: Amplitude Spectra

What Do We Get?

For "each tide", considered as $A\cos(2\pi f t + \phi)$ we get

- An amplitude, A, in counts (or whatever).

- A phase, ϕ, in degrees or radians (or, ϕ/f, in time).

There are two sources of confusion about the phase:

1. The sign. As we have written it, if $\phi = 0$, the tide is largest at $t = 0$. If ϕ were negative, the tide would be largest for t positive: that is, negative phase is the same as a lag. This is **not** completely standard.

2. What is the phase relative to?

   - One convention is the **local phase**, meaning that $\phi = 0$ when the tidal force for "that tide" maximum at the location of the data.

   - Another convention is the **Greenwich phase**, where $\phi = 0$ when the tidal force for "that tide" is maximum at the Greenwich meridian.

Plotting the Tidal Results

Given $A \cos(2\pi ft + \phi)$, how do we express this?

- Amplitude and phase is one way—it has the advantage that time shifts and scale factors are decoupled.
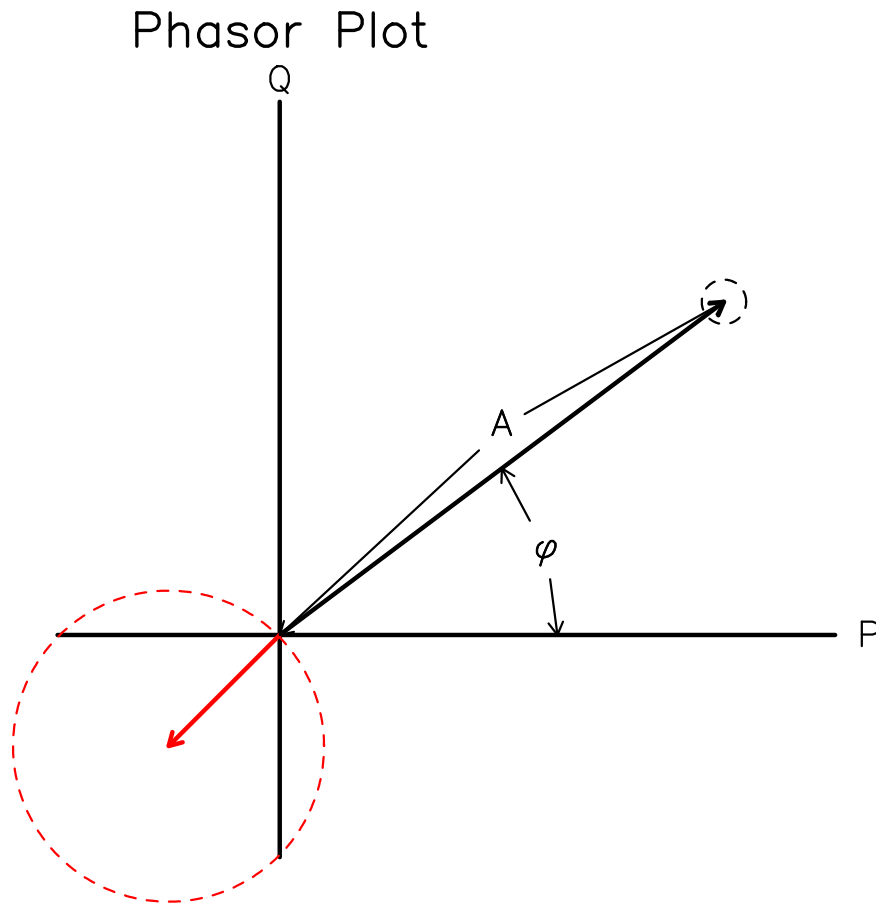
Phasor Plot

- Another approach is

$$A \cos \phi \cdot \cos(2\pi ft) - A \sin \phi \cdot \sin(2\pi ft) =$$

$$P \cos(2\pi ft) + Q \sin(2\pi ft)$$

where P and Q are the **in-phase** and **quadrature** parts of the tide.

- If we took $A\cos(2\pi ft + \phi) = \mathrm{Re}[Be^{2\pi ift}]$, then P and Q are the real and imaginary parts of B.

We can plot P and Q on a **phasor plot** (same as the Argand diagram for complex numbers):

Why use P and Q?

- Because the errors in these are, generally, uncorrelated and equal, independent of the size of the sine wave.

- If the errors in P and Q are comparable to A, the pdf for ϕ and A is no longer Gaussian, and the "standard errors" are not meaningful. (See the red example in the plot above).

- If you add sine waves of the same frequency, the phasors add like vectors. So, this kind of plot is useful for comparisons.

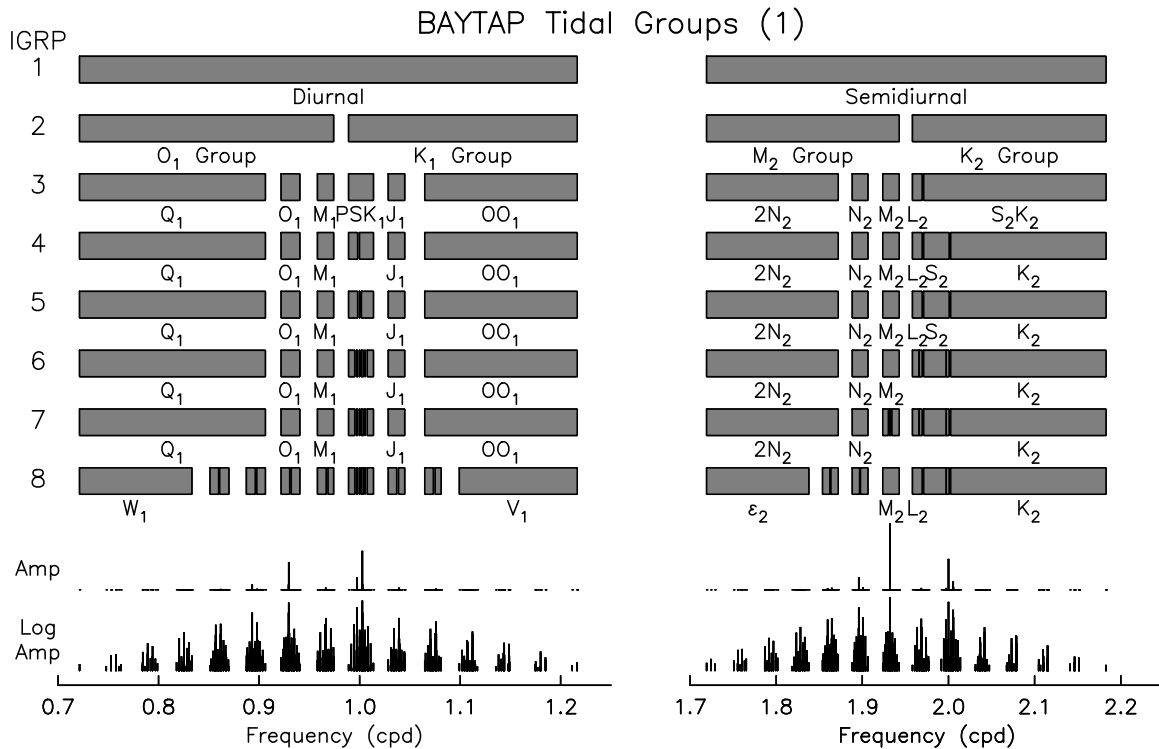# Baytap08

Derived from BAYTAP-G, to be easier to use.

BAYTAP-G performs a least-squares fit for:

- The tides

- A smooth "trend" or "drift"

    Parts of the data that are correlated with other series.

The program also adjusts the smoothness of the trend for "minimum Akaike Bayesian Information Criterion".

Tidal analysis done by "groups" to avoid instability from trying to fit sinusoids with very nearby frequencies.

BAYTAP Tidal Groups (1)
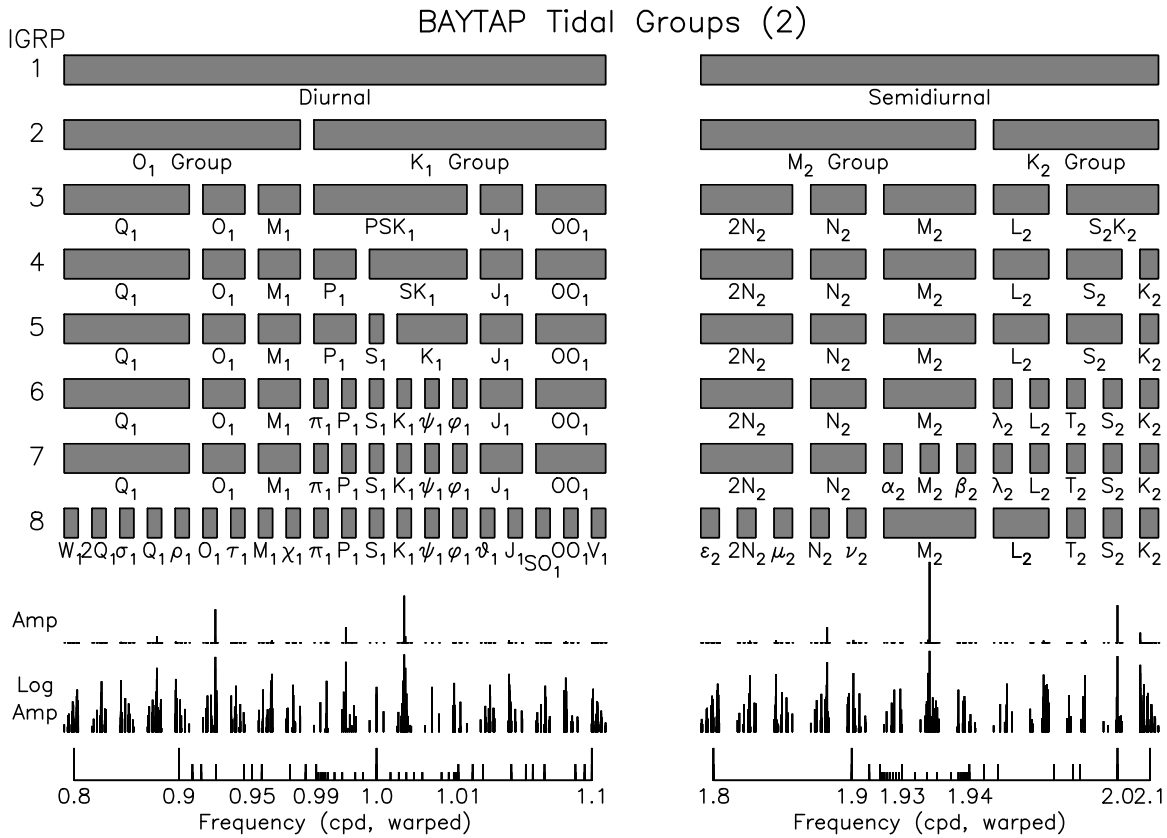
Baytap08 takes as input:

- The **data file**: various formats, simplest is a title line and 1 value/line

- A **control file**: instructions to the program

- Optional, **auxiliary files**: barometric pressure etc.

Baytap08 produces:

- A **results file**: results of the analysis, especially tidal results and residual spectrum

- A **series file**: the trend, tides, correlated part, and so on, for plotting.

Syntax:

```
cat data | baytap08 control results [aux[aux[aux]]] [ >
output]
```