

# **SPOTL: Some Programs for Ocean-Tide Loading**

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## **1. Introduction**

While the importance of ocean-load tides has increased with the increasing precision of geodetic measurements, to the point that this is no longer a topic solely of interest to the earth-tide community, the computation of such loading effects has remained a rather specialized activity. The purpose of this collection of programs is to make the ability to compute load tides generally available in a form that will (I hope) be straightforward to use, while at the same time allowing flexibility in combining different tidal and loading models. There are now a great many tidal models available, especially of the global ocean tide; while only a few are provided here, they have been chosen to be the ones most useful (or traditional) for loading computations. The package also includes programs to allow the computed loads to be converted into harmonic constants for any type of tide (including the ocean tide), and to compute the tide in the time domain from these constants. For completeness a program for direct computation of the body tides has also been included; while its accuracy is not as high as that of some recent versions (for example, Merriam, 1992), it should be more than adequate for comparison with all tidal data except perhaps for the best gravity-tide measurements.

Most of the details of how the programs should be run, with simple examples, are given in the manual pages that accompany this document. Section 2 provides some examples of how the programs may be combined to carry out more complex tasks. For many users, the rest of this manual should be necessary only for occasional reference. Section 3 describes the file formats; the parts of interest to most users will be Section 3.1 (on “polygon files”) and Section 3.2, which describes the Green functions available. Section 4 provides details about how these Green functions were computed. Sections 5 and 6 provide reference material on the global and local tidal models provided with the package; much of this has been taken directly from material written by the model developers.

### **1.1. Program Development and History**

This distribution is labelled as Verson 3.2 to acknowledge that it is a minor modification of the original version 3.0, which was distributed in June 1996. Version 3.1 (distributed in 1999) added of the induced potential to the quantities computed. The changes from 3.1 to 3.2 are the (A) inclusion of two new global models (GOT00.2 and TPXO6.2), (B) the revision of the local models for Canadian waters using an improved land-sea database, and (C) an improved Antarctic coastline.

The version number is 3 because there have been two earlier versions of this package, though neither was widely distributed. Version 1 was developed in 1981, and was based very loosely on the integrated Green-function load program of Goad (1980). Many of the program structures were devised in order to be able to run the program within the limited memory available on a PDP-11/34. Since this program was developed for research purposes, it was made as flexible as possible; for example, although most computations used the Schwiderski ocean models, it was capable of including other models, both global and local. Partly because of this flexibility, and also because of the memory restrictions, the implementation was complicated, requiring three programs to be run just to compute the loads. This version was therefore not distributed. Version 2 was developed for the National Geodetic Survey in 1987, and took advantage of a larger computer to combine these three programs into one, also hardwiring the choices available in the earlier version so that the only input required was the location of the place of interest.

With the appearance of the new, Topex/Poseidon-based, ocean tide models, it was clear that the programs needed to be updated. In this version I have attempted to retain the flexibility that proved useful in the first version while making the programs easy to use in an “automatic” way if that is appropriate (as indeed it is for most locations). I have also added ancillary routines to make it easier to convert the output of the loading program to harmonic constants for a specific type of tide.

Two of the programs included (for computing the time-domain tides) were not strictly part of the development described above. The body-tide program, `ertid`, has a history (and includes some code) going back to the work of Munk and Cartwright (1966); the program comments summarize the later developments. The program for computing tides from harmonic constants (including spline interpolation of small constituents) I developed in 1983; essentially the present version of it was distributed with Version 2 of the loading program.

## 1.2. Portability and Installation

Insofar as possible, the programs are written in standard Fortran 77. All the files are read with Fortran reads and writes, either binary (for the ocean-model and land-sea files) or ASCII (for the others). One C routine is used to do bitwise AND's for reading the bitmapped part of the land-sea database (described in more detail in Section 3.4).

The only required subroutine or function calls not standard to Fortran are to the routines `iargc` and `getarg` for reading arguments off the command line.<sup>1</sup> In addition, the function `fdate` is used in subroutine `lodout` to provide a time-stamp for the output of the loading program; this may be omitted if such a routine is not available. All these routines are available in most Unix implementations based on Berkeley Unix, and are available on HP-Unix if the appropriate flag (+U77) is used with the Fortran compiler.

Installation of the programs requires the following (many of which are of course specific to Unix systems).

1. Read the `tar` file into which all the files and directories are packed.
2. Modify the file `Makefile` in the `src` directory to have the appropriate flags for the compiler you are running.

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<sup>1</sup>The statement `narg = iargc()` puts the number of command-line arguments in `narg`, while `call getarg(n, string)` places the characters of the `n`-th argument in character variable `string`.

3. Run the `install` script in the main directory. This will:
  - A. Compile all the routines and load them into the `bin` directory.
  - B. Use `modcon` (through the script `Tobinary`) to convert the ocean models from compressed ASCII to binary.
  - C. Use `mapcon` to convert the land-sea database from compressed ASCII to binary (two files).
  - D. Link the files into the `/working` directory. This includes the Green-function files in directory `green`, the polygon files in directory `polys`, the ocean-model files in directory `tidmod`, and the land-sea database in directory `lndsea`.

This package was developed with support from the University of California, the US National Science Foundation, and the National Aeronautics and Space Administration. It may be copied and used without charge. It may not be modified in a way which hides its origin or removes this message or any copyright messages. It may not be resold for more than the cost of reproduction and mailing. Scientific ethics, courtesy, and completeness require the program to be referenced in any publications that use results from it; an adequate reference would be to Agnew (1996), which is this document, or Agnew (1997), which is more generally available.

Similar restrictions apply to the various ocean-tide models, in varying degrees (though the Schwiderski models, having been produced by a US federal agency, are free from US copyright). See the individual subsections in Section 5 for information on further restrictions, and on what references should be cited for particular models.

## 2. Examples

These examples are intended to show how the programs can be combined into simple scripts to do different tasks; consult the individual manual pages for the programs for an explanation of the command-line arguments. These scripts, and the files produced by them, are in subdirectory `working/Examp1` in the distribution; if these are rerun in directory `working`, the results they produce can be checked against these earlier ones.

### Example 1

```
../bin/nloadf PFO 33.609 -116.455 1280 m2.csr3tr green.gbavap.std 1 poly.cortez - > ex1.f1
../bin/nloadf PFO 33.609 -116.455 1280 m2.cortez green.gbavap.std 1 poly.cortez + > ex1.f2
cat ex1.f1 ex1.f2 | ../bin/loadcomb c > ex1.f3
```

This script computes the  $M_2$  loads at station PFO, in southern California. The first line computes the load from a global model (CSR3), with the Gulf of California excluded using the polygon file `poly.cortez`. (The Sea of Cortez is another name for the Gulf of California). The second line computes the load from a separate model for this Gulf, with only the inside of the polygon included; this eliminates any overlap. Figure 1 illustrates this process: 1a shows the global model around the Gulf and in it, while 1b shows the local model and the polygon. Figures 1c and 1d then show the grid of cells used in the loading computation: 1c for the global model (top line of the example), and 1d for the local model (second line). The removal of overlap is clear.<sup>2</sup> The Green function (see

<sup>2</sup> In this case, the coverage of the Gulf by the global model is adequate for the load computation; the reason for using a local model is that the global one fails to represent the tides adequately, because of a resonance near

Section 3.2) is for the Gutenberg-Bullen earth model. The third line adds up the loads put into the two files to give the total load, and also copies all the header lines from each of the loading computations.

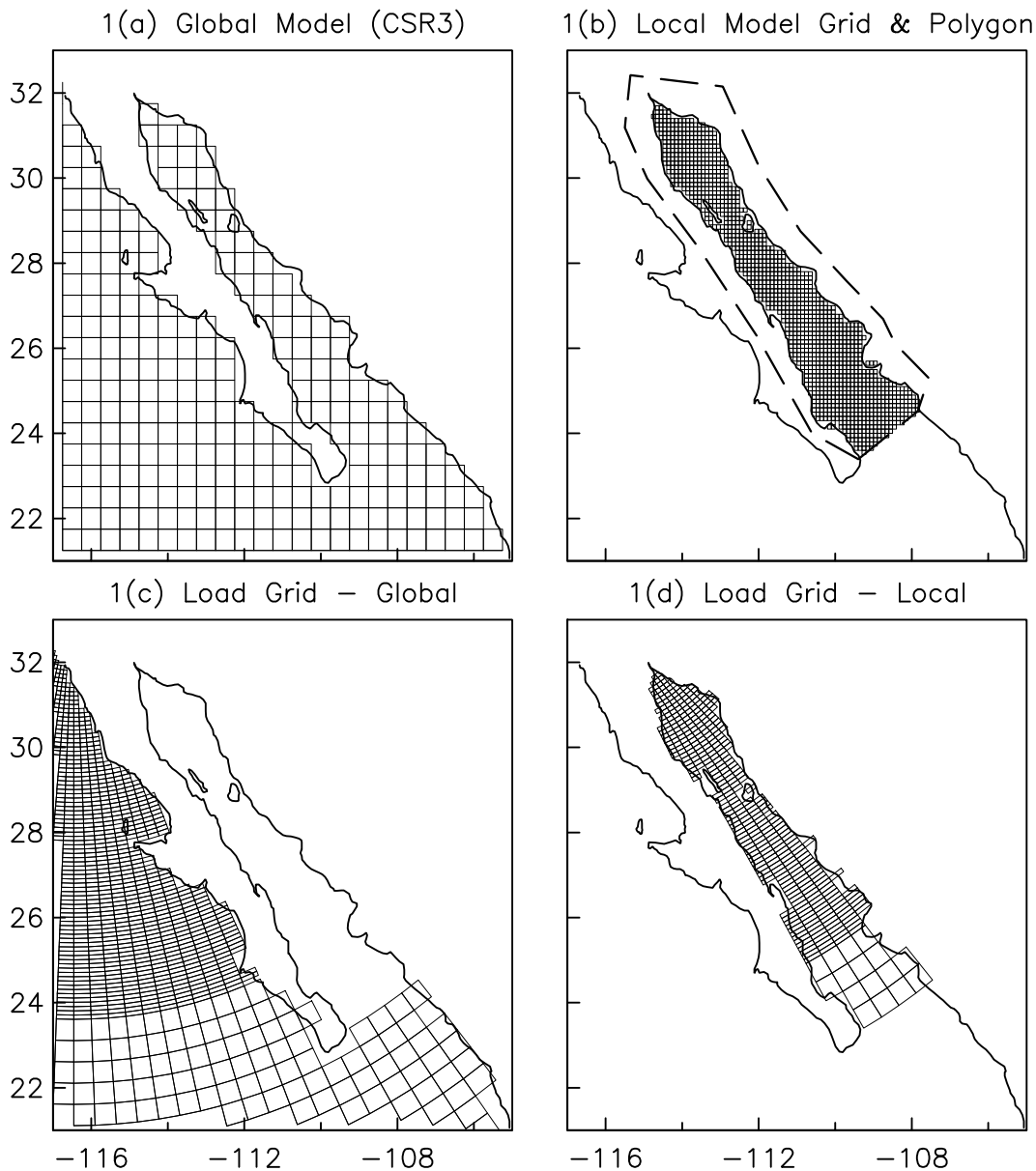


Figure 1

## Example 2

```
../bin/nloadf BFO 48.33 8.32 582 m2.tpxo62 green.gbavap.std l poly.tplim + > ex2.f1
../bin/nloadf BFO 48.33 8.32 582 m2.csr3tr green.gbavap.std l poly.tplim - > ex2.f2
cat ex2.f1 ex2.f2 | ../bin/loadcomb c > ex2.f3
```

This example is similar to the first one, but illustrates how the method could be used to combine two global models, one of which (TPXO6.2) is incomplete at high latitudes (a the frequency of this constituent.

common difficulty with models based on satellite altimetry).<sup>3</sup> In this case the polygon file (`poly.tplim`) includes all latitudes between 66°N and south of 66°S, and is used to limit the loading to this range in the first line, while in the second line it is overridden to include only the Arctic and Antarctic in the convolution. This script computes the  $M_2$  loads at station BFO, the Black Forest Observatory in Schiltach, Germany.

### Example 3

```

../bin/oclook q1.schw 44.6682 -130.3622 o > ex3.f1
../bin/oclook o1.schw 44.6682 -130.3622 o >> ex3.f1
../bin/oclook p1.schw 44.6682 -130.3622 o >> ex3.f1
../bin/oclook k1.schw 44.6682 -130.3622 o >> ex3.f1
../bin/oclook n2.schw 44.6682 -130.3622 o >> ex3.f1
../bin/oclook m2.schw 44.6682 -130.3622 o >> ex3.f1
../bin/oclook s2.schw 44.6682 -130.3622 o >> ex3.f1
../bin/oclook k2.schw 44.6682 -130.3622 o >> ex3.f1
cat ex3.f1 | ../bin/harprp o > ex3.f2
cat ex3.f2 | ../bin/hartid 1995 246 0 0 0 145 1800 >> ex3.f3

```

The first 8 lines extract the amplitude of the ocean tide at the specified location, for all the constituents of the Schwiderski ocean model. This file is then piped through `harprp`, with the option set to extract the ocean-tide amplitude and turn it into a file of constituent amplitudes and phases, which in turn is sent to `hartid` to compute the actual tide at this location. The last two lines could of course be combined.

### Example 4

The script for this example is not given here to save space, but is included in the directory with the others. It takes the usage of example 1 to compute the load tide at Haystack Observatory, combining the FES95.2 model with a local model for the Bay of Fundy and Gulf of Maine. This is done (as in example 3) for several (seven) tidal constituents, and a final file is produced of the harmonic constants for vertical displacement.

## 3. File Formats and Information

This section summarizes the formats of the various files. Most of the files described do not require user modification (with the possible exception of the “polygon files” of Section 3.1); as noted above, Section 3.2 also provides a list of the Green-function files that are available.

### 3.1. Polygon Files

These files are designed to specify, relatively crudely, a particular region, or set of regions, which either is to be the only one used in the convolution, or is to be excluded from it. This is useful in merging local and global models: if we specify a region that includes the local model, and which has a boundary (in part) along the overlap between local and global models, then by including and excluding this region in using the local and global models we may ensure that the total convolution has no overlap. These files

<sup>3</sup> This is not actually true for this model, though it was for its predecessor, TPX02, which was used in this example in earlier versions.

are ASCII, and are formatted as follows:

Variable(s)	Format	Description
filnam	a80	Name appropriate to the whole file
npoly	i	Number of polygons in file (maximum of 10).
polynam	a80	Name of the polygon (not used, but in file for reference)
npoints	i	Number of points in the polygon
use	a1	This is + if the polygon defines the region to be included, – if it defines the region to be excluded. Note that it can be overridden by use of the appropriate symbol in the command line of <code>nloadf</code> .
xpoly, ypoly	*	The vertices of the polygon, given as longitude/latitude pairs (East and North positive). These coordinates may not cross a 360-degree discontinuity and must be in the range –180 to 180 or 0 to 360. It is only necessary to give each vertex once.

Of course, the last four elements (`polynam`, `npoints`, `use`, and the `xpoly`, `ypoly` arrays) are in the file `npoly` times.

Several choices are possible in deciding, should the file contain polygons with both excluded and included areas, what should be done. The choice implemented here is that if the point falls in any excluded area it is excluded; then if there are any included areas it must fall in one of them, though if there are not any included areas the point may fall anywhere outside the excluded areas.

The polygon files included in the distribution are `poly.bfund` (Bay of Fundy/Gulf of Maine), `poly.cortez` (Gulf of California), `poly.gefu` (Strait of Georgia/Juan de Fuca), `poly.tplim` (limits of most Topex/Poseidon models), `poly.sfbay` (San Francisco Bay), and `poly.sl` (Gulf of St. Lawrence). Except for the `.tplim` file (which simply includes everything between  $\pm 66^\circ$  latitude), and the `.cortez` file (shown in Figure 1a), these polygons are illustrated in Figure 3 with the appropriate local model.

### 3.2. Green-function files

The Green-function files contain the values of the integrated Green functions, specified over a given grid of radial distances. The integrated Green function is defined as:

$$G_i = a^2 \int_{\Delta_i}^{\Delta_{i+1}} G(\Delta) \sin \Delta d\Delta$$

where  $\Delta$  is the distance from the station and  $G$  is the mass-loading Green function for the quantity of interest, as defined by Farrell (1972). (For this definition, the effect of a constant load of material of thickness  $A$  and density  $\rho$  over this distance range is then [azimuthal effects aside]  $\rho A G_i$ .) In principle we could choose the  $\Delta_i$ 's arbitrarily; in this implementation they have been chosen to be spaced at equal intervals within different distance ranges. Suppose there are  $M$  such ranges, the  $j$ -th one of which has  $N_j$  intervals, each with width (radially from the station) of  $\delta_j = \Delta_i - \Delta_{i-1}$ . Further define (omitting the subscript on  $N_j$ )  $\Delta_L = \Delta_1 + \delta_j/2 = \frac{1}{2}(\Delta_1 + \Delta_2)$  and  $\Delta_H = \Delta_{N+1} - \delta_j/2 = \frac{1}{2}(\Delta_N + \Delta_{N+1})$ ; the total distance coverage is thus from  $\Delta_L$  through  $\Delta_{N+1}$ , with the centers of the intervals running from  $\Delta_L$  though  $\Delta_H$ . The format of the Green function file can then be described as

Variable(s)	Format	Description
title	a70	Identification of Green function, usually by Earth model
$n_{gr}, j, M, N_j, \Delta_L, \Delta_H, \delta, fine$	i1,3x,2i4,3f10.4,5x,a1	The variables are as defined above, except for <i>fine</i> , a character variable. If this is F, the detailed land-sea database will be used in determining if a point is on land or not; if it is C, the ocean-model grid will be used instead. Also, if it is F, the value of the tide at the center of the loading cell will be found using bilinear interpolation of the ocean model cells (Section 3.3); if C, the value will simply be taken to be that of the ocean-model cell which the center of the load cell is inside of. In the files used with Version 3.1 $n_{gr}$ has been added; this gives the number of Green functions.
$G_i(j)$	7e13.6	The integrated Green function (see below for the normalization), for the six load types defined in Farrell (1972): vertical and radial displacement, gravitational acceleration, radial tilt, and the strains $e_{\theta\theta}$ and $e_{\lambda\lambda}$ ; and, in Version 3.1, the induced potential height. <b>Note</b> that the induced potential height is relative to the (moving) surface of the Earth, as defined by Farrell (1973), and not relative to the geocenter, as defined by Francis and Mazzega (1990). There are $N_j$ lines of this type, followed by another line (of the type given above) for the next range, and so on.

There are several Green-function files distributed with the programs; this table summarizes their characteristics. Use of Green functions with a wider range of Earth models (and especially of crustal structure) would be desirable, although indications are that newer models produce very similar results to the old, except perhaps at near distances for strain and tilt. Unfortunately, many computations for new earth models do not include the all the quantities of interest. The value of  $\delta$  has been chosen to be comparable to the size of the land-sea grid for very close distances, and to that of most ocean models for the farther ones.

File name	$M$	$j$	$N_j$	$\delta$	$\Delta_L$	$\Delta_H$	fine
green.gbavap.std	3	1	98	0.01	0.025	0.995	F
		2	90	0.10	1.05	9.95	F
		3	160	0.50	10.25	89.75	C
		4	90	1.00	90.50	179.50	C

Gutenberg-Bullen Model A average Earth, Green functions computed by W. E. Farrell 10/19/1971 (and taken from the original card deck produced when these were computed). (The Green functions for the induced potential, in this and all other models, are for the Harkrider model (see below), as described by Farrell (1973), and taken from a listing provided by him. Note that this is not the same function as tabulated by Francis and Mazzega (1990).)

green.contap.std	3	1	98	0.01	0.025	0.995	F
------------------	---	---	----	------	-------	-------	---

2	90	0.10	1.05	9.95	F
3	160	0.50	10.25	89.75	C
4	90	1.00	90.50	179.50	C

Gutenberg-Bullen average Earth, top 1000 km replaced by the continental shield crust and mantle structure of Harkrider (1970). Computed from the Green functions computed by Farrell (1972) (as preserved on cards).

green.ocenap.std	3	1	98	0.01	0.025	0.995	F
		2	90	0.10	1.05	9.95	F
		3	160	0.50	10.25	89.75	C
		4	90	1.00	90.50	179.50	C

Gutenberg-Bullen average Earth, top 1000 km replaced by the oceanic crust and mantle structure of Harkrider (1970). Computed from the Green functions tabulated by Farrell (1972).

### 3.3. Ocean-model files

The ocean models are all specified on an array of cells, bounded by parallels of latitude and meridians of longitude, and all of equal size in degrees. The files as stored for use are written in binary, using Fortran sequential direct-access. For distribution the files are given in ASCII (compressed), being converted to binary through program `modcon`. The format of the ASCII file is as follows:

Variable(s)	Format	Description
<code>dsym</code>	a4	Darwin symbol of constituent
<code>icte(6)</code>	6i3	Doodson number of constituent, in Cartwright-Tayler form
<code>latt1, latt2</code>	2i8	Integer part (degrees) and fractional part (0.001 degrees) of the latitude of the top edge of the grid
<code>latb1, latb2</code>	2i8	Integer part (degrees) and fractional part (0.001 degrees) of the latitude of the bottom edge of the grid
<code>lonr1, lonr2</code>	2i8	Integer part (degrees) and fractional part (0.001 degrees) of the longitude of the right (east) edge of the grid
<code>lonl1, lonl2</code>	2i8	Integer part (degrees) and fractional part (0.001 degrees) of the longitude of the left (west) edge of the grid
<code>latc, longc</code>	2i8	The number of cells in latitude and longitude
<code>modname</code>	a50	A name for the model used
<code>ireal()</code>	10i7	The real part of the tidal height, in integer mm, the phase being Greenwich phase with lags positive.
<code>imagi()</code>	10i7	The imaginary part of the tidal height, in the same conventions.

For example, here is the ASCII version of the first lines of a file for one of the Schwiderski models:

```
M2
  2  0  0  0  0  0
    90      0
   -78      0
   360      0
     0      0
   168    360
```

Schwiderski 1980

Note that the ordering of the cells is first from west to east, then north to south: for



example, in the case given, the first cell is centered at 89.5°N, 0.5°E, the second one at 89.5°N, 1.5°E, number 361 at 88.5°N, 0.5°E, and so on.

In interpolating to an arbitrary coordinate, the values for the tides are assumed to apply to the center of each cell. The interpolated value is found by bilinear interpolation from the four cell centers closest to the point of interest. If some number of these cells have a zero value, then the values at these points are set (for the purpose of interpolation) so that bilinear interpolation is equivalent to interpolation along a plane surface (as usual, for the complex-valued amplitude).

### 3.4. Land-sea Database

The land-sea database is a representation of where there is land and ocean, for the whole world, at a resolution of 1/64 of a degree (1.7 km at the Equator). This is based on the World Vector Shoreline data, as converted to land-sea polygons for version 3.0 of the GMT (Global Mapping Tools) package (Wessel and Smith 1996). The original form of these data includes the Antarctic ice shelves as land. In order to improve the representation of the Antarctic coast, coastal and grounding-line data were obtained from the Antarctic Digital Database (ADD), which is maintained as a public database for the Scientific Committee on Antarctic Research. This database has been digitized from the best available maps and photographic coverage, and covers all points south of 60°S. The coastal and grounding-line program were converted from the ADD ArcInfo format to geographic coordinates. For the Antarctic ice shelves the determination of the true grounding-line is a difficult task with conventional coverage. The ADD grounding lines for these regions have been updated from recent determinations using local deformation measurements from InSAR: for the Amery Ice Shelf from H. Fricker (pers. commun.) and for the Ross Ice shelf (Siple coast) from I. Joughin (pers. commun.).

If stored as a single bit-mapped array this database would require 33.1 Mb; so save on memory needs this is instead stored as two arrays (each stored in one disk file, and read into memory at run time). Array 1 (stored on disk as file `landsea.ind`) covers the world at a coarse spacing (0.5°), and each element contains one of three values:

- 2 ocean (meaning that the cell is all ocean)
- 1 land (meaning that the cell is all land)
- ≥0 the index of the cell in the second array, meaning also that the cell has both land and ocean (“mixed”).

The second array contains only the “mixed” cells, at full resolution, stored in the order the first array indexes them in. These store the land-sea information as bits. The cell size in the coarse array was chosen to (roughly) minimize the overall storage needed. As it turns out, only 16,423 (6% of the total of 259,200) of the cells are “mixed”; their distribution is plotted in Figure 2. The two files needed are stored in directory `landsea`. They are generated from a file (`landsea.ascii.Z`) which contains both in compressed ASCII, using the program `mapcon`.

## 4. Integrated Green Function Computation

This section gives the details of how the integrated Green functions are computed. For the “Newtonian” (direct attraction) part this is done within the loading program; the analytical expressions needed are given later in this section. I first discuss the method by

which the files of integrated Green functions (for the “elastic” part) are computed before distribution.

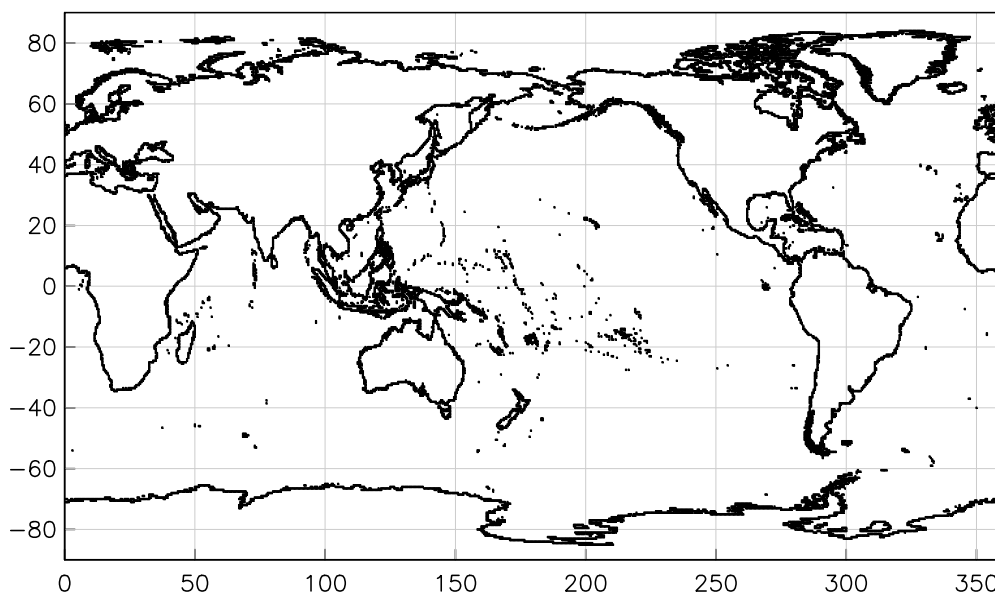


Figure 2

#### 4.1. Elastic Green Functions

Goad (1980) showed how to find the integrated Green functions for gravity and displacement by forming suitable sums of Love numbers. I have instead chosen to assume that such a sum has already been done so that the Green functions are available in tabular form, in particular in the form given by Farrell (1972). The tables to that paper give the Green functions for displacement, gravity, tilt, and radial strain ( $e_{\theta\theta}$ ), normalized in the following way:

$$G_t(\Delta) = Ka\Delta G(\Delta) \quad G_t(\Delta) = Ka^2\Delta^2 G(\Delta) \quad (1)$$

where the first equation applies to displacement, gravity, and induced potential, and the second one to tilt and strain;  $K$  is  $10^{12}$  (SI units;  $10^{18}$  for gravity) and  $a$  is the mean radius of the Earth,  $6.371 \times 10^6$  m. The quantity we wish to compute is

$$a^2 \int_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} G(x) \sin x dx$$

We do this by renormalizing the tabulated Green functions,  $G_t$ , in the following way (again, the left is for displacement and gravity, the right for strain and tilt).

$$G'_t(\Delta) = a^2 G_t(\Delta) [2 \sin(\Delta/2)/\Delta] / Ka \quad G'_t(\Delta) = a^2 G_t(\Delta) [2 \sin(\Delta/2)/\Delta]^2 / Ka^2 \quad (2)$$

which for small values of  $\Delta$  retains the feature of the earlier normalization of removing the singularities in  $G$ .

For displacement, gravity, or the potential, the integral is then

$$\int_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} G'_t(x) \frac{\sin(x)}{2 \sin(x/2)} dx = G'_t(\Delta_c) \int_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} \cos(x/2) dx =$$

$$4G'_t(\Delta_c) \cos(\Delta_c/2) \sin(\delta/4)$$

where in the first equation we have assumed that  $G'_t(x)$  is constant over the interval of integration. In practice the intervals  $\delta$  have been chosen sufficiently small that halving them produces essentially equivalent results; the value of  $G'_t$  at  $\Delta_c$  is evaluated from the tabulated values using Lagrange interpolation.<sup>4</sup>

In the case of the strains and tilt, the integral is

$$\int_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} G'_t(x) \frac{\sin(x)}{4 \sin^2(x/2)} dx = \frac{1}{2} G'_t(\Delta_c) \int_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} \cot(x/2) dx =$$

$$G'_t(\Delta_c) \ln \left[ \frac{\sin(\Delta_c/2) \cos(\delta/4) + \cos(\Delta_c/2) \sin(\delta/4)}{\sin(\Delta_c/2) \cos(\delta/4) - \cos(\Delta_c/2) \sin(\delta/4)} \right]$$

## 4.2. Newtonian Green Functions

For a point at elevation  $h$  above sea level, the vertical gravitational attraction from a mass  $m$  an angular distance  $\Delta$  away is

$$\frac{-G_N m}{a^2} \frac{\varepsilon + 2 \sin^2(\Delta/2)}{[4(1 + \varepsilon) \sin^2(\Delta/2) + \varepsilon^2]^{1.5}}$$

where  $G_N$  is the Newtonian constant of gravitation;  $\varepsilon$  is  $h/a$ ; and we reckon positive acceleration upwards. From this expression, the Green function is

$$a^2 G = -G_N \frac{\varepsilon + 2 \sin^2(\Delta/2)}{[4(1 + \varepsilon) \sin^2(\Delta/2) + \varepsilon^2]^{1.5}} \quad (3)$$

This may be put into the form given by Farrell (1972) if we set  $\varepsilon = 0$  and realize that the gravitational acceleration on a spherical Earth,  $g$ , is given by  $G_N m_E / a^2$ , which makes the Green function

$$\frac{g}{m_E} \frac{-1}{4 \sin(\Delta/2)}$$

Note that for  $\Delta$  small, it is possible for the expression (3) to be much larger (for reasonable values of  $\varepsilon$ ) than the expression with  $\varepsilon$  zero. While the expression in (3) is exact it is not easily integrated; to compute the integral, we resort to two approximations.

For essentially all land-based observations, we have  $\Delta \gg \varepsilon$ , and so can ignore the  $\varepsilon^2$  term in (3). If we do this, the integrated Green function is

<sup>4</sup> In the actual program used to compute the integrated Green functions, the Green functions are read from the un-normalized form used by Farrell in his programs, and immediately normalized to the form (equation (1)) given in Farrell (1972), using a subroutine provided by him, before renormalizing (equation (2)) for subsequent interpolation. In this initial subroutine all three strain Green functions are also computed, using the relations in Farrell (1972).

$$\begin{aligned}
& -G_N \int_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} \frac{\varepsilon \cos(x/2)}{4(1 + 1.5\varepsilon) \sin^2(x/2)} + \frac{\cos(x/2)}{2(1 + 1.5\varepsilon)} dx \\
& = \frac{-2G_N}{1 + 1.5\varepsilon} \cos(\Delta_c/2) \sin(\delta/4) \left[ 1 + \frac{\varepsilon}{\cos(\delta/2) - \cos(\Delta_c)} \right] \tag{4}
\end{aligned}$$

However, if we have measurements made on a cliff, or on the bottom of the ocean, it is no longer true that  $\Delta \gg \varepsilon$ , and we need another approximation. This is suggested by the fact that  $\Delta$  is small in such cases, and is to take  $\sin(\Delta) = \Delta$ . The integrated Green function is then

$$G_N \int_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} \frac{\varepsilon + \frac{1}{2} x^2}{[(1 + \varepsilon)x^2 + x^2]^{1.5}} dx = -G_N \left[ \frac{(1 + \varepsilon)x^2 - 2\varepsilon}{2(1 + \varepsilon)^2 \sqrt{\varepsilon^2 + (1 + \varepsilon)x^2}} \right]_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} \tag{5}$$

Numerical comparisons of these two versions for a reasonable range of elevations (–6 to 4 km) show that it is appropriate to use (4) for  $\Delta$  greater than  $3^\circ$ , and (5) for smaller distances, the maximum error (for very large depths or elevations) being less than 1%.

The Green function for tilt is derived from the the exact expression for the horizontal attraction, which is

$$\frac{G_N m}{a^2} \frac{\sin(\Delta/2) \cos(\Delta/2)}{[4(1 + \varepsilon) \sin^2(\Delta/2) + \varepsilon^2]^{1.5}}$$

from which the Green function (scaling by  $a^2/mg$ , where  $g$  is now the local gravitational acceleration) is

$$\frac{2G_N}{g} \frac{\sin(\Delta/2) \cos(\Delta/2)}{[4(1 + \varepsilon) \sin^2(\Delta/2) + \varepsilon^2]^{1.5}}$$

The primary difference between this and the usual expression with  $\varepsilon$  zero is that this expression goes to zero for  $\Delta < \varepsilon$ ; for  $\Delta > \varepsilon$  we may take  $\varepsilon = 0$  with little error. In that case, the integrated Green function is

$$\begin{aligned}
& \frac{G_N}{g} \int_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} \frac{\cos^2(x/2)}{\sin(x/2)} dx \\
& = \frac{G_N}{g} \left[ -2 \sin(\Delta_c/2) \sin(\delta/4) + \ln \left( \frac{\tan(0.25(\Delta_c + \delta/2))}{\tan(0.25(\Delta_c - \delta/2))} \right) \right] \tag{6}
\end{aligned}$$

For small distances we may again make the approximation that  $\sin(\Delta) = \Delta$ , in which case the integrated Green function is

$$\frac{G_N}{g} \int_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} \frac{x^2}{(\varepsilon^2 + (1 + \varepsilon)x^2)^{1.5}} dx$$

which we may integrate exactly, although it is sufficient to set  $1 + \varepsilon = 1$ , in which case the integrated Green function is

$$\frac{G_N}{g} \left[ \frac{-x}{(x^2 + \varepsilon^2)^{0.5}} + \ln(x + \sqrt{x^2 + \varepsilon^2}) \right]_{\Delta_c - \delta/2}^{\Delta_c + \delta/2} \quad (7)$$

As with the gravity Green function, we use (6) for  $\Delta > 3^\circ$ , and (7) for smaller distances.

For the induced potential (expressed as height by dividing by  $g$ ), the Newtonian Green function (for height zero) is

$$\frac{a}{m_E} \frac{1}{2 \sin(\Delta/2)}$$

which makes the integrated Green function

$$\frac{4a^3}{m_E} \cos(\Delta/2) \sin(\delta/4)$$

## 5. Ocean-model Information: Global Models

All of the global models are presented with the same grid spacing as originally provided, and have simply been reformatted to present the data in the form described in Section 3.3 above; when needed, the routines provided with the data files have been used to produce values for individual tidal constituents, rather than as “orthoweights”. Only two adjustments have been made to the data, and only in some cases. For the Schwiderski models, a uniform sheet has been subtracted to force the models to conserve mass. For several of the altimetric models, a number of cells that are nonzero include cells that are mostly or all on land. While this is convenient for processing altimetric data, it is clearly undesirable for loading computations, so that these models have been “trimmed” of such cells by setting to zero the values in all cells that are more than 50% land, as determined from the Shum *et al.* (1987) database. The table below summarizes the characteristics of the models available:

Suffix	Model	Cell Size	Latitude Limits	Constituents
.schw	Schwiderski	1°	-78°—90°	$M_f, O_1, P_1, K_1, Q_1, N_2, M_2, S_2, K_2$
.csr3tr	CSR 3.0	0.5°	-78°—90°	$O_1, P_1, K_1, Q_1, N_2, M_2, S_2, K_2$
.fes952	FES 95.2	0.5°	-86°—90°	$O_1, K_1, Q_1, N_2, M_2, S_2, K_2$
.got00	GOT00.2	0.5°	-90°—90°	$O_1, P_1, K_1, Q_1, N_2, M_2, S_2, K_2$
.tpxo62	TPXO6.2	0.25°	-90°—90°	$M_f, M_m, O_1, P_1, K_1, Q_1, N_2, M_2, S_2, K_2$

(Cell size, if two dimensions are given, is NS by EW).

It should be noted that some of the global models either do not include, or do not well match, a number of areas which may have large local tides that will be important for the loads nearby. To take two examples that I am aware of, neither the FES95.2 or the CSR3.0 models include the Bay of Fundy; neither accurately captures the semidiurnal resonance in the northern Gulf of California. Local models have been provided for both these cases, but there may be others.

### 5.1. Schwiderski Models

These models are taken from a data tape provided by E. W. Schwiderski and L. T. Szeto in 1981, and are believed to embody essentially the form of these models summarized by Schwiderski in his NSWC reports. They have been included not because they represent the best available model, but as a “reference standard” against which to compare other loading computations, as these models have been widely used.

### 5.2. CSR 3.0

The CSR 3.0 model (with land cells set to zero) was provided by Richard Eanes (University of Texas Center for Space Research). The following description is from the README file distributed with the model (December 1995):

The third generation CSR ocean tide model (CSR 3.0) was obtained from analysis of TOPEX/POSEIDON altimeter data using the response method (Eanes and Bettadpur, 1995). “The third generation model is based upon 89 cycles (2.4 years) of T/P altimetry. First diurnal orthoweights were fit to the Q1,O1,P1 and K1 constituents of the Grenoble hydrodynamical model (FES94, Le Provost *et al.*, 1994), and semidiurnal orthoweights were fit to the N2,M2,S2 and K2 constituents of Andersen’s adjusted Grenoble model (Ole B. Andersen, private communication, Dec 1994). Tides in the Mediterranean from Canceil were used in both tidal bands as they appeared in the Andersen adjusted Grenoble model. Radial ocean loads from the CSR 2.0 model were added to the Grenoble ocean tides to convert them to geocentric tides.”

“Then T/P altimetry was used to solve for corrections to these orthoweights in 3x3 degree spatial bins. The orthoweight corrections so obtained were then smoothed by convolution with a 2-d gaussian for which the full-width-half-maximum (FWHM) was 7.0 degrees. The smoothed orthoweight corrections were output on the 0.5 x 0.5 degree grid of the Grenoble model and then added to the Grenoble values to obtain the new model.”

“The T/P orbit used for this tide model development was computed at CSR and used JGM-3 and a dynamical ocean tide model based upon the first generation CSR model (CSR 1.6). Tests performed by the T/P precision orbit determination team have shown that this orbit is close to those expected when updated models are used at GSFC to produce the new GDR’s.”

### 5.3. TPXO6.2

This is the Oregon State University TOPEX/Poseidon solution, Version 6.2, from the OSU website.<sup>5</sup> To quote from the description there:

“TPXO.6 is a global model of ocean tides, which best-fits, in a least-squares sense, the Laplace Tidal Equations and along track averaged data from 324 TOPEX/Poseidon orbit cycles. The 69076 data sites include all cross-overs and 4 equally spaced sites between. All available along track data are averaged to the choosen data sites. The methods used to compute the model are described in detail by Egbert, Bennett, and Foreman (1994) and further by Egbert and Erofeeva (2002).”

“TPXO.6 reduced the RMS T/P data difference from 0.3471m (no tidal correction), to 0.1382 m. For comparison, the RMS difference for the other tidal corrections on the

<sup>5</sup> <http://www.coas.oregonstate.edu/research/po/research/tide>

TP GDR are 0.1407 m for TPXO.5.1, and 0.1432 m for FES99. TPXO.6 thus resulted in variance reductions of 1.8% and 3.5% relative to TPXO.5.1 and FES99, respectively. Note that all models were compared for the same data set, and that the quoted RMS values are unweighted. Smaller RMS values are obtained for all models when the comparison is restricted to open ocean (e.g., greater than 1000 m depth). For the 102 pelagic validation gauges proposed by Cartwright and Ray, the RMS misfits of TPXO.6 for the four constituents are: 1.43 cm ( $M_2$ ); 0.93 cm ( $S_2$ ); 0.79cm ( $O_1$ ); 0.99cm ( $K_1$ ). For comparison, the RMS misfits of TPXO.5 are 1.57 cm ( $M_2$ ); 0.99cm ( $S_2$ ); 0.84 cm ( $O_1$ ); 1.12 cm ( $K_1$ )."

#### 5.4. FES95.2

This is the third-generation version of the "Grenoble" tide models, provided by Christian Le Provost and Jean-Marc Molines (Laboratoire des Ecoulements Geophysiques et Industriels, Institut de Mecanique de Grenoble). They are based on the FES 94.1 hydrodynamic model, which was computed using a finite-element model with variable mesh size (Le Provost *et al*, 1994). This was the first global ocean-tide model to include the regions under the Ross, Filchner, and Ronne ice shelves. The FES95.2 models were developed by adjusting the FES94.1 models using the CSR2.0 solution (Eanes, 1994) that was derived from the first year of T/P data, using an improved orbit. Any use of these models should reference Le Provost *et al* (1994) and Le Provost *et al* (1995).

#### 5.5. GOT00.2

This is Version 00.2 (Year 2000) of the Goddard Ocean Tide Model, produced by Richard Ray based on 286 cycles of Topex/Poseidon data.

### 6. Ocean-model Information: Local Models

These local models were obtained from different sources; those not on grids of the type described above were converted to that form. This conversion was done by interpolating the real and imaginary parts of each model from the original irregular grid to the regular one, using spline interpolation, and then setting the land areas to zero, defining these using (in most cases) the Shum *et al*. (1987) land-sea database. The table below summarizes the characteristics of the models available; the region covered by each is best shown in Figure 3.

Suffix	Region	Cell Size	Constituents
.cortez	Gulf of California	0.1°	$K_1, M_2, S_2$
.sfbay	San Francisco Bay	0.01°	$O_1, K_1, M_2, S_2$
.bfund	Bay of Fundy	0.1°	$O_1, K_1, N_2, M_2, S_2$
.gefu	Strait of Georgia/Juan de Fuca	0.1°	$O_1, K_1, N_2, M_2, S_2$
.sl	St. Lawrence River/Gulf	0.1°	$O_1, K_1, N_2, M_2, S_2$

(Cell size, if two dimensions are given, is NS by EW).

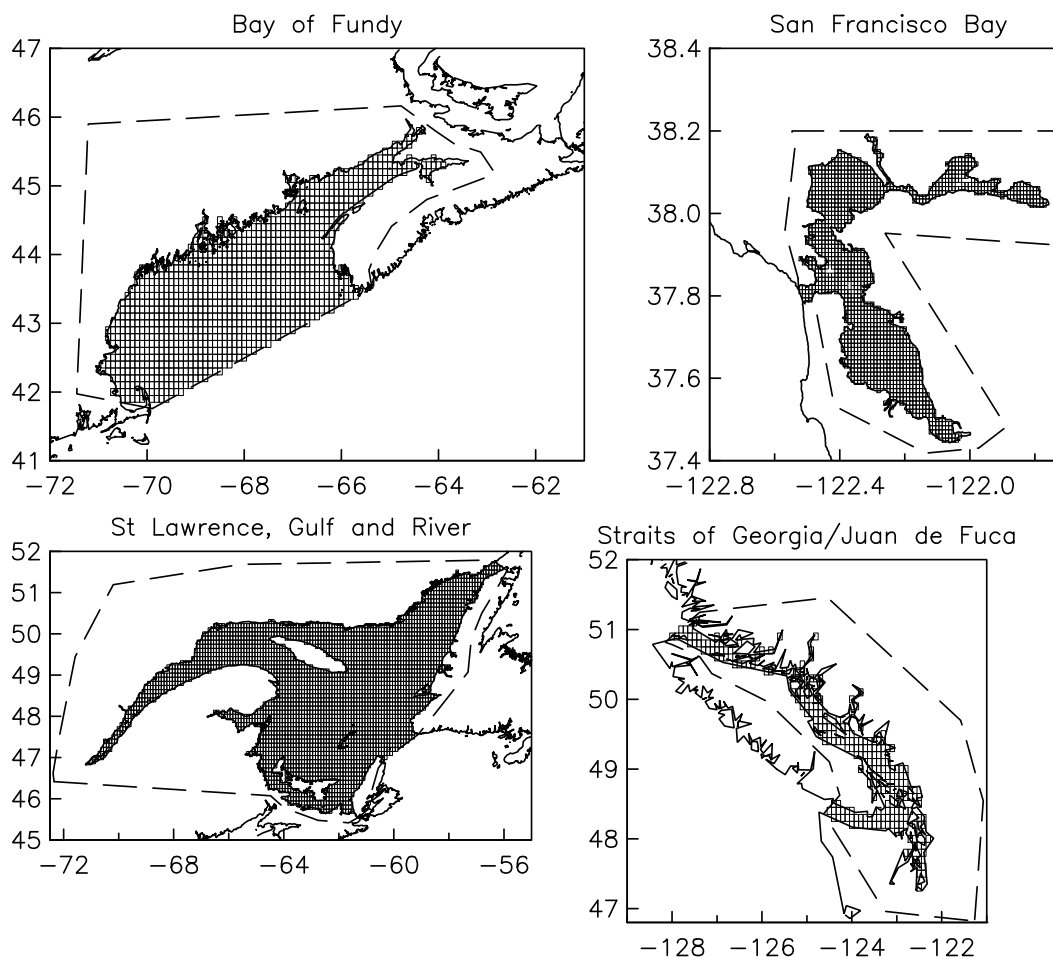


Figure 3

### 6.1. Gulf of California (Sea of Cortez)

This model was re-interpolated from a three-dimensional hydrodynamic model developed by Stock (1976), which appears to still be the only one for this area. (Stock's grid was aligned along the axis of the Gulf of California). These models are available only for a few constituents, and have been included because even the detailed global models do not adequately capture the  $M_2$  resonance in this gulf. The FES.95.2 and CSR 3.0 global models appear to be adequate for the diurnal tides.

### 6.2. San Francisco Bay

These tides were interpolated between observed values around the Bay, taken from Cheng and Gartner (1984); the boundaries of the Bay were defined using a detailed coastline file. A comparison of the amplitude with the model results in Cheng *et al.* (1993) shows that the  $M_2$  amplitude is within 5 cm almost everywhere, and often better: 10% error would probably be conservative. (Unfortunately, these model results were not available). One known effect not included is that there is a  $20^\circ$  phase shift in  $M_2$  between the mouth of San Francisco Bay and the tides several tens of km offshore (as reflected by the global models and also by the tides observed on the Farallon Islands).



### 6.3. Canadian Waters

A large number of local models for Canadian coastal waters were made available by Dr. A. Lambert of the Seismology and Geomagnetism Branch of the Geological Survey of Canada. Those for areas not well-covered by one of the global models were converted to standard form, re-interpolated to uniform grids as described above; most were on nearly uniform grids to start with, and this original grid spacing was retained. The bases for these models were described in two reports by Andrew Billyard of the University of Waterloo; excerpts from these are given in the following sections. A summary is given by Lambert *et al.* (1991), and by Lambert *et al.* (1998); the latter paper, as the most easily found, should be referenced if these models are used.

#### 6.3.1. St. Lawrence River

“The cotidal models here were from Godin (1980). The tidal representation continued to 71°W, since beyond that point, the river narrowed too much to have a noticeable effect on gravity measurements.”

#### 6.3.2. Gulf of St. Lawrence

“The data here were from Godin (1980). Few minor alterations were made to make the model consistent with the Scotian Shelf model and the Newfoundland coast model, by Lank and de Margerie (1986). A major correction was the relocation of the amphidrome in K1. Most other corrections were slight repositionings of co-amplitude or co-phase lines to fit the shelf models.”

#### 6.3.3. Bay of Fundy - Scotian Shelf Area

“There are three models in this area that have been combined together: Bay of Fundy, Gulf of Maine and Scotian Shelf. There were places where the separate models met and slightly disagreed. In these areas, the models were smoothed together, and uncertainties were increased.”

“The Bay of Fundy is unique in that the tide there is a nearly resonant tidal system which mostly resembles the partial tide M2. Thus, there are extensive models and research papers only on M2 for the Bay. Greenberg’s full numerical model (Greenberg 1979) of M2 was chosen over Godin’s empirical interpolation (Godin, 1980) for M2, S2 and N2. However, Godin’s model was used for K1 and O1.”

“Since Greenberg’s study concerned M2 only, amplitude ratios S2/M2 and N2/M2, as well as S2–M2 and N2–M2 phase lag differences were derived using the files from MEDS to create the S2 and N2 cotidal charts. These factors were compared to the factors extrapolated from Godin’s model, and close agreements were found, namely a difference of 1.2% for amplitude ratios and 10.5% for phase lag differences.”

“The K1 and O1 cotidal lines were slightly altered around Grand Manan Island, to fit the models for the Gulf of Maine and Scotian Shelf. For K1, the 180° co-phase line originated from the Gulf and ended at Grand Manan Island. It was altered to end at Long Island, Nova Scotia, to compensate for the large phase lag values found in the Gulf. In O1, the 170° co-phase line between Grand Manan Island and Maine, U.S.A., was removed, and a 175° was added. Only a few minor shifts in position of some co-amplitude lines were necessary.”

“Models of Gulf of Maine, like the Bay of Fundy, were mostly limited to M2. Here, Greenberg’s model was used but conversion factors for S2 and N2 were not derived from the MEDS’s files, as in the Bay of Fundy. The files did not contain tidal constituent data for American waters, and therefore, conversion factors and their standard deviations . . . were derived from Schwiderski’s model of the Gulf for S2 and N2.”

“A model by Beaumont and Boutilier (1978) was used for O1. The inaccuracy in the ocean tide model discussed by them was thought to be only in the region east of Nova Scotia; the Gulf of Maine cotidal data was thought to be accurate, though. The K1/O1 (amp) and K1–O1 (phase) values for the Gulf of Maine were derived from Schwiderski’s model there.”

“The Scotian Shelf model was taken from Lank and de Margerie (1986). Because this model was the most recent of all, the adjacent Gulf of St. Lawrence and the Gulf of Maine tidal data were made to fit it. The Lank and de Margerie model gave cotidal charts for M2, K1 and O1 as well as S2/M2 (amp), N2/M2 (amp), S2–M2 (phase) and N2–M2 (phase) values, and their standard deviations.”

#### 6.3.4. Georgia - Fuca Area

“This area encompasses the Juan de Fuca strait and the Strait of Georgia, east of Vancouver Island. The cotidal data is from Crean, Murty and Stronach (1988). Discrepancies between observed and calculated amplitude and phase lag values for M2 and K1 were quoted to be less than 6 cm and 6 deg, respectively. Only the M2 and K1 models were given, so S2, N2 and O1 had to be derived from amplitude ratios and phase lag differences (table 3). These ratios and differences, as well as their standard deviations, were calculated from tables given by Crean, Murty and Stronach.”

“Each area within this system (i.e. San Juan Island passages, Strait of Georgia, northern passages, etc.) has different amplitude ratios and phase lag differences. As a result, for S2, N2 and O1, the different areas of model had to be smoothed together, raising uncertainties along "borders" between these areas.”

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