Strain

1. Introduction

We begin by discussing, in outline form, the quantity that strainmeters measure. We explore some special points, relevant to understanding recorded strain, in more detail.

We begin with a discussion of what strain is, following the standard treatment in continuum mechanics (e.g., Malvern 1969) Strain is a consequence of the **deformation** of the Earth: that is, its change in shape. A material that does not deform is said to behave as a **rigid body**; the mathematical definition of this is that for any two particles $\mathbf{x}_1$ and $\mathbf{x}_2$, the distance $|\mathbf{r}^L(\mathbf{x}_1,t)-\mathbf{r}^L(\mathbf{x}_2,t)|$ does not change. Such motion can be decomposed into a **rigid-body translation** and a **rigid-body rotation**; if one point in the body does not translate then the most general motion is rotation (Euler's theorem used in plate tectonics). In geodesy these motions are used to describe the mapping from one reference frame to another, and are called Helmert transformations: 3-parameter for pure translation, and 6-parameter for translation and rotation.

For deformation, what matters is the relative motion of nearby particles. We therefore consider motions of the continuum relative to some particular (though arbitrary) particle, called the *reference particle*. Label this reference particle as $\mathbf{x}_R$, and consider the change in relative position between it and some other particle (labeled $\mathbf{x}$). This change is given by:

$$\mathbf{u}(\mathbf{x},t)=[\mathbf{r}^L(\mathbf{x},t)-\mathbf{r}^L(\mathbf{x}_R,t)]-[\mathbf{r}^L(\mathbf{x},0)-\mathbf{r}^L(\mathbf{x}_R,0)]$$

If $\mathbf{u}(\mathbf{x},t)=0$ for all $\mathbf{x}$ and all t, we are back to a pure translation, since any line between two particles has an unchanging length and direction.

We therefore want to develop descriptions for more general forms of $\mathbf{u}$. This can become quite complicated, so we shall focus on a situation with simple mathematical structure: small deformation.

2. Small Deformation

Define $\xi=\mathbf{x}-\mathbf{x}_R$ to be the vector from the reference particle to another, arbitrary, one. Our first assumption is to consider deformation in a small region only, so that ξ is infinitesimal, which we indicate by writing it as $d\xi$. Also, we look at $\mathbf{u}$ and related quantities for a fixed (but nonzero) value of t—that is, we consider the material in two configurations: one at $t=0$ and one at some other time. In this case, $\mathbf{x}$ describes the locations in the initial state, and $\mathbf{r}$ in the final (second) state—the material and spatial descriptions thus become the undeformed and deformed states respectively. (Though it is in a way arbitrary which state we call "undeformed").

By definition, when $d\xi=0$ (that is $\mathbf{x}=\mathbf{x}_R$), the vector $\mathbf{u}$ is always zero: the reference particle has to always be itself. We next assume that the deformation is smooth enough that we can write $\mathbf{u}$ as a Taylor series in $d\xi$. Remembering that $d\xi$ is a vector between particles (that is, it depends on particle labels $\mathbf{x}$), this means that we can write, for Cartesian coordinates,

$$u_i(d\xi) = \frac{\partial u_i}{\partial x_j}\, d\xi_j \quad + \quad \text{higher} - \text{order terms}$$

where we are using, as we shall in general, the Einstein convention that repeated indices denote summation over all components.

Our third assumption is that all we need from the Taylor series is the first term; this amounts to requiring that the gradients be much less than one. And our fourth assumption is that the motions are small enough that the axes for **r** and **x** will locally coincide: this is a separate requirement from the one for small gradients). Remember that for $t = 0$, the reference state, these vectors do coincide. This last assumption means that we may take **u** to be a function of **r** rather than of **x**. The last two assumptions, combined, allow us to write the displacement **u** as

$$u_i = \frac{\partial u_i}{\partial r_j}\, d\xi_j \tag{1}$$

$$= \tfrac{1}{2}\left[\frac{\partial u_i}{\partial r_j} + \frac{\partial u_j}{\partial r_i}\right] d\xi_j + \tfrac{1}{2}\left[\frac{\partial u_i}{\partial r_j} - \frac{\partial u_j}{\partial r_i}\right] d\xi_j$$

$$=_{\text{def}}\ E_{ij}\, d\xi_j + \Omega_{ij}\, d\xi_j$$

The first expression gives the displacement in terms of the **displacement gradient**, which in coordinate-free terms is the dyad $\nabla\mathbf{u}$; on the next line, we add, subtract, and regroup terms to get particular combinations of these gradients. In dyadic form equation (1) becomes

$$\mathbf{u} = (\nabla\mathbf{u})d\xi = (\tfrac{1}{2}(\nabla\mathbf{u} + \mathbf{u}\nabla)d\xi + \tfrac{1}{2}(\nabla\mathbf{u} - \mathbf{u}\nabla)d\xi \ =_{\text{def}}\ \mathbf{E}d\xi + \mathbf{\Omega}d\xi$$

where we have defined new quantities $\mathbf{E} = \frac{1}{2}(\nabla\mathbf{u} + \mathbf{u}\nabla)$ and $\mathbf{\Omega} = \frac{1}{2}(\nabla\mathbf{u} - \mathbf{u}\nabla)$. From (1) it is clear that **E** is symmetrical and $\mathbf{\Omega}$ antisymmetrical in Cartesian coordinates.

2.1. Units and Signs

Strain, being a displacement divided by a distance, is always dimensionless, something that we express when we say that a strain is (for example) 10^{-9}. It is however convenient to have a named unit to talk about, so often we use the name "strain" and the symbol ε, so we can talk about 1 nanostrain, or label a plot as being in nε.

The sign for strain is given by the expressions above, which makes extension positive. This has the consequence in elasticity that positive stress corresponds to tension. In rock mechanics, which likes to make compressive stress positive, contractional strains are positive.

3. Strain in Two Dimensions

What do **E** and $\mathbf{\Omega}$ represent? To make this clearer, we consider the expressions in two dimensions. We then have, in Cartesian components,

$$\mathbf{E} = \begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix} = \begin{pmatrix} \partial_1 u_1 & \frac{1}{2}(\partial_1 u_2 + \partial_2 u_1) \\ \frac{1}{2}(\partial_2 u_1 + \partial_1 u_2) & \partial_2 u_2 \end{pmatrix}$$

$$= \begin{pmatrix} u_{1,1} & \frac{1}{2}(u_{1,2} + u_{2,1}) \\ \frac{1}{2}(u_{2,1} + u_{1,2}) & u_{2,2} \end{pmatrix}$$

where we have used the convenient contractions

$$u_{i,j} =_{\text{def}} \partial_j u_i =_{\text{def}} \frac{\partial u_i}{\partial r_j}$$

Now suppose the only nonzero term is E_{11}; then

$$u_1 = E_{11} d\xi_1$$

describes the displacement field: there is only displacement in the u_1 direction. If $E_{11} > 0$, this displacement increases as we move away from $d\xi_1 = 0$. This deformation is called a **uniaxial extension**. If $E_{11} < 0$, we have displacement, also increasing away from $d\xi_1 = 0$, but towards the 2-axis: this is **uniaxial contraction**. Together these are termed **uniaxial strain**. Of course, E_{22} gives the same kind of deformation field in the orthogonal direction.

Next consider E_{12} nonzero; then

$$u_1 = E_{12} d\xi_2 \quad u_2 = E_{12} d\xi_1$$

This is called a **pure shear**. Note that the axes (originally at right angles) will move to make an angle of $\pi/2 - 2E_{12}$. This suggests that shear may be specified by an angle change $\gamma_{12} = 2E_{12}$; γ is called **engineering shear** (to be distinguished from the **tensor shear** component E_{12}).

Figure 1 shows some of these simple two-dimensional deformations; of course for clarity we have to make them finite rather than infinitesimal, but since we have made them homogeneous they remain accurate. The tails of the arrows form a regular grid in the undeformed material; their heads show the positions of these particles after the deformation, so the arrows themselves show the displacement field $\mathbf{u}$. In addition to the two strain types already described, we also show a **pure rotation**, in which $\mathbf{E}$ is zero and $\mathbf{\Omega}$ is not; this is a rigid-body motion. If we add this rotation to the pure shear, we get the type of deformation known as **simple shear** (shear parallel to one axis): so this includes both strain and rotation. Simple shear has a special place in geodynamics, as being the kind of deformation that takes place across diffuse plate boundaries when the motion is parallel to the boundary: crustal deformation in California is one place where this is a good first approximation. Finally, Figure 1 shows the case in which $E_{11} = E_{22}$: this is often called **dilatation**; in the Earth, this kind of deformation is most typically found in volcanic areas. Note, in all these drawings, that there is nothing special about the point in the center; if we took displacements relative to some location on the edge, we would get the same sort of picture.

In two dimensions, the antisymmetric part $\mathbf{\Omega}$ is

$$\mathbf{\Omega} = \begin{pmatrix} 0 & -\Omega \\ \Omega & 0 \end{pmatrix}$$

so there is only one component, Ω, which describes just a rigid-body rotation—

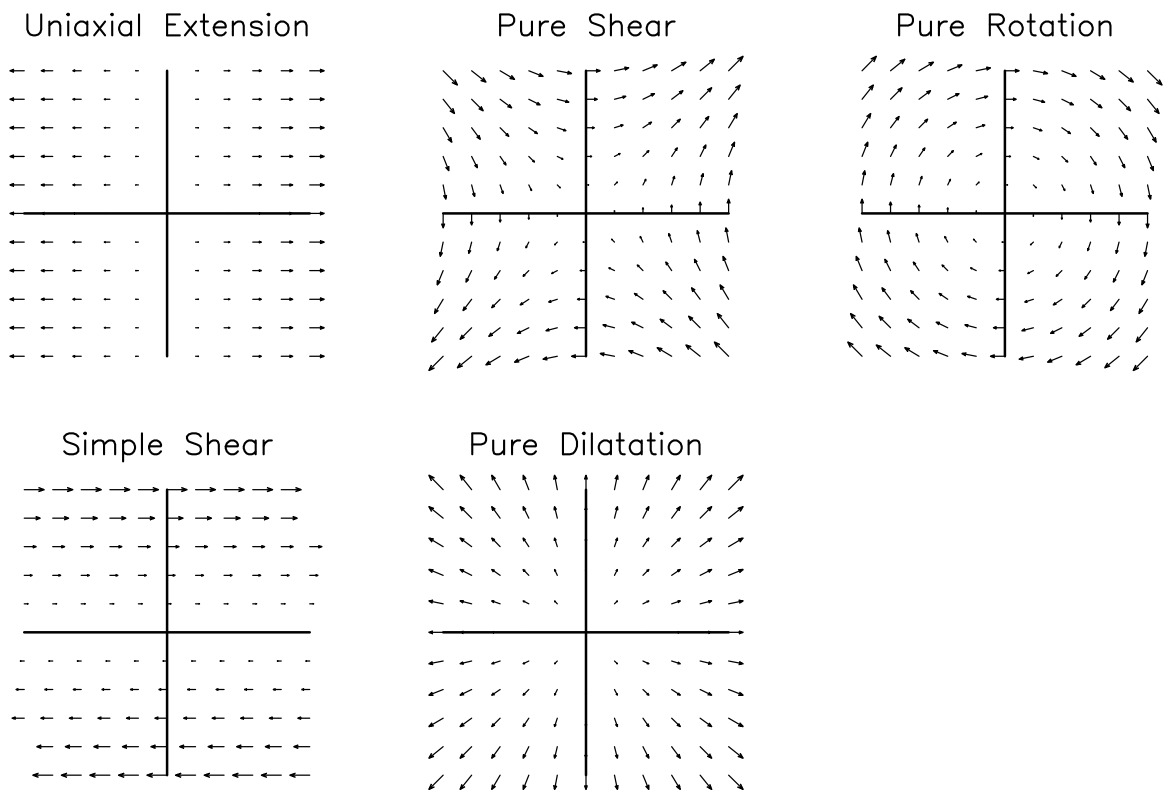

Figure 1

though this rotation has to be small for this description to work.[1]

3.1. Transformation of Two-Dimensional Strains

It is useful to state the rules for how the Cartesian components of **E** and **Ω** change in two dimensions as we change the direction of the coordinate axes—although this could be expressed simply by saying that **E** is a tensor.

For extensions, the tranformation is

$$e = E_{11}\cos^2\theta + E_{22}\sin^2\theta + 2E_{12}\sin\theta\cos\theta \tag{4}$$

Where e is just the uniaxial strain at an angle θ to the original coordinate system—or to put it another way, this is an expression for $E_{11}(\theta)$, the 11 component of strain, but in a coordinate system rotated counterclockwise by θ from the original. For example, if in the original coordinates we have pure shear, with $E_{11} = E_{22} = 0$ and $E_{12} \neq 0$, then $E_{11}(\theta)$ has a four-lobed pattern, with two of the lobes being negative. This is apparent if we look at Figure 1 for pure shear: at 45° to the axes we see uniaxial extension and contraction.

We can also derive how the shear strain will change for a rotation of coordinate axes. The result is

[1] Large rotations (usually called **finite rotations**) cannot be described by this simple means, in part because they do not commute, as small rotations do. (Though in two dimensions even finite rotations commute).

$$E_{12}(\theta) = (E_{22} - E_{11}) \sin\theta \cos\theta + E_{12}(\cos^2\theta - \sin^2\theta)$$

which shows, among other things, that $E_{11} - E_{22}$ is just as much a shear as E_{12}.

It is very useful to write the Cartesian components of strain as a vector (not a physical vector but a methematical one), since then we can write the full transformation strains in two dimensions in matrix form:

$$\begin{pmatrix} E'_{11} \\ E'_{22} \\ E'_{12} \end{pmatrix} = \begin{pmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{pmatrix} \begin{pmatrix} E_{11} \\ E_{22} \\ E_{12} \end{pmatrix}$$

The above expressions for transformations need to be used with care because of the possibility of different sign conventions. Some possible traps are:

1 In spherical coordinates the usual strains are $E_{\lambda\lambda}$, $E_{\phi\phi}$, and $E_{\lambda\phi}$. Here λ is colatitude, measured positive going south. If we want to represent these (as is sensible) as local two-dimensional strains, we would have

$$E_{11} = E_{EE} = E_{\phi\phi}\$ \qquad E_{22} = E_{NN} = E_{\lambda\lambda}\$ \qquad E_{12} = E_{EN} = -E_{\lambda\phi}\$$$

2. The angle θ is measured counterclockwise from the 1-axis (East, locally); instrument azimuth is usually given counterclockwise from North.

3.2. Alternative Expressions for Two-Dimensional Strain

This transformation in turn can be used to find additional ways of looking at the strain components. While we shall continue to consider the two-dimensional case, most of these results carry over to three dimensions as well. If we consider $\Delta_A = E_{11} + E_{22}$, we can see from that this will not vary with θ at all—that is, this quantity is invariant, or more properly **an invariant of E**. (There are other invariants, but they involve powers of the components). Δ_A is called the **areal strain** and amounts (for small strains) to the ratio of areas in the deformed and undeformed states, minus one. In three dimensions, the equivalent ($\Delta_V = E_{11} + E_{22} + E_{33}$) is called the **dilatation**, or alternatively the **volume strain** since it is related to the change in volume. If we subtract the dilatation from the strain tensor to form $\mathbf{E}^D = \mathbf{E} - \Delta_A \mathbf{I}$, we have the **deviatoric strain $\mathbf{E}^D$**. In three dimensions we use Δ_V, or we can just write $\mathbf{E}^D = \mathbf{E} - \mathrm{tr}(\mathbf{E})\mathbf{I}$, where tr() is the trace of the tensor—for Cartesian components the sum of the diagonal terms.

From our transformation expression, we can see that at 45° to our original (and arbitrary) choice of axes the shear strain would be $\frac{1}{2}(E_{22} - E_{11})$: as noted above, this is thus just as much a shear as E_{12}. We thus can express the strain as:

$$[\Delta_A, \tfrac{1}{2}(E_{22} - E_{11}), E_{12}] \equiv [\Delta_A, \tfrac{1}{2}\gamma_1, \tfrac{1}{2}\gamma_2)$$

where γ_1 and γ_2 are the **engineering shear strains**. This is often a useful representation because the material may respond differently to shear than to change in area: for example, in rocks shear leads to failure, whereas dilatation does not.

Alternatively, we can see that there is an orientation of coordinate axes that will make $E_{12} = 0$. Let the angle of the axes (relative to the original set) be $\theta = \theta_p$;

then our expression shows that to make the shear zero we have to have

$$\tfrac{1}{2}(E_{22} - E_{11})\sin 2\theta_p + E_{12}\cos 2\theta_p = 0$$

which means that the angle is given by

$$\theta_p = \tfrac{1}{2}\arctan\left[\frac{2E_{12}}{E_{11} - E_{22}}\right]$$

For axes that make this angle to the original axes, only E_{11} and E_{22} are nonzero, so yet another way to express strain is as

$$[\,\theta_p,\ E_{11}(\theta_p),\ E_{22}(\theta_p)]$$

These are termed the **principal axis strains**: the principal axes are those for which $E_{12} = 0$, which is to say, the axes for which $\mathbf{E}$ is diagonal (in Cartesian coordinates). While this set of numbers does not directly transform to other coordinates, can be useful to look at strains in this way. This result also extends to three dimensions, and follows from the fact that symmetric matrices can be decomposed into the product of two matrices

$$\mathbf{E} = R_p^{-1} P R_p$$

where R_p is an orthogonal matrix and P is diagonal. An orthogonal matrix is one in which the transpose is the inverse, and it transforms the Cartesian components of a tensor as a finite rotation would, so this says that there is some set of axes in which the components of $\mathbf{E}$ are

$$\begin{pmatrix} E_I & 0 & 0 \\ 0 & E_{II} & 0 \\ 0 & 0 & E_{III} \end{pmatrix}$$

The axes in this coordinate system are the **principal axes** of the strain tensor; in this particular coordinate system there are no shears, only extensions (or contractions), something we have already seen for the two-dimensional case.

4. Strain Near the Surface of an Elastic Halfspace

So far we have simply asserted that we can safely deal with two-dimensional strain only. We now justify this for the special case of strains near the surface of an elastic halfspace. Note that up to this point we have not assumed anything about the nature of the material we are measuring strain in. For now we assume it is a perfectly elastic solid; we will see, later on, at least one way in which this is probably an inadequate model.

The equations that apply to an isotropic elastic medium are the displacement/strain relationship for small strains

$$\mathbf{E} = \tfrac{1}{2}(\nabla\mathbf{u} + \mathbf{u}\nabla)$$

the conservation of linear momentum (the equation of motion)

$$\rho\ddot{\mathbf{u}} = \mathbf{f} + \nabla\cdot\mathbf{T}$$

and the constitutive (stress-strain) relationship for an elastic material

$$\mathbf{T} = \lambda_e \mathrm{Tr}(\mathbf{E})\mathbf{I} + 2\mu_e \mathbf{E}$$

which can be inverted to give strain in terms of stress:

$$\mathbf{E} = \frac{\mathbf{T}}{2\mu_e} - \frac{\lambda_e}{2\mu_e(3\lambda_e + 2\mu_e)} \mathrm{Tr}(\mathbf{T})\mathbf{I}$$

In terms of Young's modulus E_y and Poisson's ratio ν this is

$$\mathbf{E} = \left(\frac{1+\nu}{E_y}\right)\mathbf{T} - \frac{\nu}{E_y} \mathrm{Tr}(\mathbf{T})\,\mathbf{I}$$

where we have used

$$\frac{\nu}{1+\nu} = \frac{\lambda_e}{3\lambda_e + 2\mu_e}$$

4.1. No Load: Plane Stress

We first consider **plane stress**. Because of the continuity of the tractions across a boundary, normal stress must be continuous across a boundary; the other stress components need not be. A particularly important example is a free surface: a boundary with nothing on the other side. The normal stress in a material at a free surface must be zero: for example, at the Earth's surface the vertical stress is zero (actually, the atmospheric pressure, but for now we idealize this to zero, dealing with changes in it later). So the stress is exactly two-dimensional. For a halfspace with the 3-axis perpendicular to a free surface, the condition that the traction be zero across this surface means that stresses associated with that axis are zero: $T_{33} = T_{13} = T_{23} = 0$. Given the plane stress conditions, we may write the strain-stress relationshop as four equations

$$\text{(a):}\;\; E_{11} = \frac{T_{11} - \nu T_{22}}{E_y} \quad \text{(b):}\;\; E_{22} = \frac{T_{22} - \nu T_{11}}{E_y}$$

$$\text{(c):}\;\; E_{33} = \frac{-\nu}{E_y}(T_{11} + T_{22}) \quad \text{(d):}\;\; E_{12} = \frac{T_{12}}{2\mu_e}$$

Combining the first two of these gives

$$T_{11} = \frac{E_y}{1-\nu^2}(E_{11} + \nu E_{22}) = \frac{2\mu_e}{1-\nu}(E_{11} + \nu E_{22})$$

from which, together with the parallel equation for T_{22} and the equation (c), gives

$$E_{33} = \frac{-\nu}{1-\nu}(E_{11} + E_{22})$$

so that, under plane stress, the vertical strain (in the commonest geophysical case) is proportional to the areal strain. Hence, under plane stress the strains are effectively two-dimensional, in that E_{11}, E_{22} and E_{12} are the only independent strains.

Strictly speaking, plane stress only applies at a free surface; close to such a surface it may or may not be valid. (And in assuming this to be the stress state in geophysics, we omit the increase of T_{33} with depth because of overburden pressure). Generally plane stress is a good approximation if the horizontal wavelength of the stress is greater than the depth. For borehole strainmeters (depth of burial at 100 m or so) this is a good approximation for the tides, and for seismic waves with wavelengths longer than the depth of burial.

4.2. Uniform Loading

The next case we consider is different in that we assume T_{33} to be uniform, and nonzero: this applies to the case of changes in atmospheric pressure, which at most periods can be assumed uniform over an area much greater than the depth of burial. We assume that T_{11}, T_{22} and T_{12} are zero, and still have part of the restriction on the stress near a free surface, which makes $T_{13} = T_{23} = 0$. Given all this, we find that the stress-strain relation is

$$E_{33} = \frac{Ts33}{E_y} \qquad E_{11} = E_{22} = -\nu E_{33}$$

just as for uniaxial compression of an elastic material—which this essentially is.

It should be noted that this is *not* the solution usually given in the hydrological literature. The standard solution assumes that the horizontal deformation must be zero, by symmetry. But this is incorrect: the horizontal strains are finite. While this does, for the halfspace, produce infinite displacements at infinity, this does not invalidate the solution, since both stress and strain remain everywhere finite. If we consider a geometrically limited problem (compression of a uniform sphere by external pressure) it is easy to see that the surface strains are finite, not zero: as the sphere shrinks in radius, it also decreases in area (McGarr 1988).

4.3. Seismic Waves

If we have a plane wave for which the displacement is (in one dimension)

$$u(t, x) = A\cos(\omega t - kx)$$

then the strain is just $\frac{\partial u}{\partial x} = Aku$, and the particle velocity is $\dot{u} = A\omega u$. Taking the ratio of these gives that

$$\frac{\varepsilon}{\dot{u}} = \frac{k}{\omega} = v_{phase}$$

which is to say, the strain and particle velocity are related by the phase velocity of the wave (the apparent phase velocity for a non-surface wave). While there have been attempts to use this for phase velocity meaurement (Mikumo and Aki 1964; Sacks *et al* 1976) and for discrimination of wave types (Fix and Sherwin 1970) the relationship appears in practice too complicated to be useful (Gomberg and Agnew 1996).

5. Strain-Strain Coupling

As we move away from these idealized models, we need to introduce another concept, to relate the strains we measure to these models. The problem is that topography, geology, and the instrument installation itself can distort the deformation field. This is most important for instruments installed in tunnels or caves; this problem was first realized by King and Bilham (1973). For this specific case it is generally called, following Harrison (1976), the **cavity effect**. However, the problem is more general, and allows us to discuss other departures of actual from ideal strain.

We divide the displacement field $\mathbf{u}$ into two parts (notionally at least):

$$\mathbf{u} = \mathbf{u}_0 + \delta\mathbf{u}$$

where $\mathbf{u}_0$ is the displacement that would occur in an idealized system, and $\mathbf{u}$ the actual displacement. We can then write the strains as

$$\mathbf{E} = \mathbf{E}_0 + \delta\mathbf{E}$$

In general, the additional deformation $\delta\mathbf{E}$ will depend on the various departures of the model from reality, and on $\mathbf{E}_0$. If we suppose $\mathbf{E}_0$ to be uniform, then for a particular situation, $\delta\mathbf{E}$ at any place is a linear functions of $\mathbf{E}_0$:

$$\delta\mathbf{E} = \mathbf{C_E}\mathbf{E}_0 \tag{15}$$

where $\mathbf{C_E}$ is the fourth-order **strain-strain coupling tensor**. In Cartesian coordinates the symmetry of $\mathbf{E}_0$ and $\mathbf{E}$ means that $\mathbf{C_E}$ has 36 independent components. For uniform strain near the surface of the earth there are only three independent components of $\mathbf{E}_0$ (e_{11}, e_{12} and e_{22}, if the 3-axis is vertical), halving the number of components in $\mathbf{C_E}$ (King et al. 1976). It is usually convenient to express the strain-strain coupling tensor as $\mathbf{C_E} + \mathbf{I}$; this transforms $\mathbf{E}_0$ to $\mathbf{E}$. For near-surface conditions this may be written as a 3×3 matrix; adding a fourth row gives e_{33} in terms of the independent components of $\mathbf{E}_0$. Note that these effects do not add noise; they merely mean that the instrument does not measure what it is supposed to.

Boreholes cause cavity effects, but because of different geometry different distortions take place. Because there is no vertical shear strain near a horizontal free surface, there is no coupling between strain and vertical tilt (measured along the borehole), although a tiltmeter attached to the bottom of the hole will measure strain-coupled tilts. Vertical tilts are also not distorted by any casing. Strains are distorted by the borehole and by any material placed in it. If we assume that the borehole is uniform and that any inserts are axisymmetric, then the coupling tensor is isotropic for horizontal strains, and may be written as

$$\begin{bmatrix} e_{11} \\ e_{22} \\ e_{12} \\ e_{33} \end{bmatrix} = \begin{bmatrix} (H_A + H_S)/2 & (H_A - H_S)/2 & 0 & 0 \\ (H_A - H_S)/2 & (H_A + H_S)/2 & 0 & 0 \\ 0 & 0 & H_S & 0 \\ 0 & 0 & 0 & H_V \end{bmatrix} \begin{bmatrix} e^0_{11} \\ e^0_{22} \\ e^0_{12} \\ e^0_{33} \end{bmatrix}$$

H_A relates internal and external areal strain, H_S shear strain, and H_V vertical strain. All three depend on the elastic constants of the surrounding material and of the contents of the borehole. Gladwin and Hart (1985) compute values of H_A and H_S for realistic parameters. For an empty borehole, $H_V = 1$, $H_S = 4(1-\nu)$, and $H_A = 2/(1-\nu)$, so that instead of (17) the transformation matrix for near-surface strain is

$$\frac{1}{1-\nu}\begin{bmatrix} 3-4\nu+2\nu^2 & 4\nu-1-2\nu^2 & 0 \\ 4\nu-1-2\nu^2 & 3-4\nu+2\nu^2 & 0 \\ 0 & 0 & 4(1-\nu)^2 \\ -\nu & -\nu & 0 \end{bmatrix}$$

All horizontal strains are thus substantially magnified. For $\nu = 1/4$ the areal strain $e_{11} + e_{22}$ is amplified by 2.7 and the volume strain by 3. An insert (casing or instrument) will add stiffness to the hole and thus reduce these values; ideally, a borehole strainmeter would be designed to have the same average elastic moduli as the material around it, but in practice this condition can only be met approximately. Just as in tunnels, fractures near the borehole can amplify the strain (Beavan *et al.* 1979) and should be avoided.

But it is important to realize that there can be strain-strain coupling from other factors as well. One unavoidable source is the irregular topography of the Earth, which causes variable strains on its surface, even if the strain at depth would be uniform. Another one, generally very difficult to model, comes from the elastic inhomogeneities associated with different geology. In general, the only approach to these is through finite-elements modelling, although some simple analytical methods have been developed for restricted classes of topography (Berger and Beaumont 1976; Emter and Zurn 1985; Mctigue and Segall 1988). Modelling these effects remains a frontier in understanding geophysical strain measurement.

References

J. Beavan, R. Bilham, D. Emter, and G. King, "Observations of strain enhancement across a fissure," *Veroeff. Deutsch. Geodaet. Komm., Reihe B,* 231, p. 47–58 (1979).

J. Berger and C. Beaumont, "An analysis of tidal strains from the United States of America, II, The inhomogeneous tide," *Bull. Seismol. Soc. Am.,* 66, p. 1821–1846 (1976).

D. Emter and W. Żurn, "Observations of local elastic effects on earth tide tilts and strains" in *Earth Tides,* ed. J. C. Harrison, p. 309–327, Van Nostrand Reinhold, New York (1985).

J. E. Fix and J. R. Sherwin, "A high-sensitivity strain-inertial seismograph installation," *Bull. Seismol. Soc. Am.,* 60, p. 1803–1822 (1970).

J. Gomberg and D. C. Agnew, "The accuracy of seismic estimates of dynamic strains: an evaluation using strainmeter and seismometer data from Piñon Flat Observatory, California," *Bull. Seismol. Soc. Am.,* 86, pp. 212-220 (1996).

J. C. Harrison, "Cavity and topographic effects in tilt and strain measurement," *J. Geophys. Res.,* 81, p. 319–328 (1976).

G. C. P. King and R. Bilham, "Tidal tilt measurement in Europe," *Nature,* 243, p. 74–75 (1973).

G. C. P. King, W. Zürn, R. Evans, and D. Emter, "Site correction for long-period seismometers, tiltmeters, and strainmeters," *Geophys. J. R. Astron. Soc.,* 44, p. 405–411 (1976).

L. E. Malvern, *Introduction to the Mechanics of a Continuous Medium,* p. 713, Prentice-Hall, Englewood Cliffs, N.J. (1969).

A. McGarr, "On the state of lithospheric stress in the absence of applied tectonic forces," *J. Geophys. Res.,* 93, pp. 13609-13617 (1988).

D. F. McTigue and P. Segall, "Displacements and tilts from dip-slip faults and magma chambers beneath irregular surface topography," *Geophys. Res. Lett.,* 15, pp. 601-604 (1988).

T. Mikumo and K. Aki, "Determination of local phase velocity by intercomparison of seismograms from strain and pendulum instruments," *J. Geophys. Res.,* 69, p. 721–731 (1964).

I. S. Sacks, J. A. Snoke, R. Evans, G. King, and J. Beavan, "Single-site phase velocity measurements," *Geophys. J. R. Astron. Soc.,* 46, p. 253–258 (1976).