Calibration of Three-Component Borehole Strainmeters

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1. Introduction

Multi-component borehole strainmeters are calibrated by comparing their output with "known" reference values of tidally-induced strains in the crust. More specifically, the amplitudes and phases of the M2 and O1 earth tide constituents are estimated from the data records of each strainmeter gauge, yielding 4 quantities for each gauge direction. The amplitudes and phases of the M2 and O1 earth tide constituents are calculated, including an elastic model for the earth, and a model for the loading applied by oceanic tides. Then a matrix is determined that, when multiplied times the observed tides, yields the calculated tides. It is generally assumed that this matrix provides an appropriate conversion from strainmeter output to strain over the entire useable frequency range of the strainmeter.

Objectives of this section are to:
1) describe conceptually how strainmeter output is coupled to regional strain
2) describe simplified forms of the coupling matrix
3) work through an example of 3-component strainmeter calibration
4) present the formal equations of calibration in their most general form and describe how PBO plans to implement them
5)discuss practical considerations such as stability of observed borehole strainmeter tides and appropriateness of simple coupling assumptions.

2. How is strainmeter output related to regional strain?

A strainmeter embedded in concrete inside a borehole constitutes an approximately cylindrical inclusion having different elastic properties from the surrounding rock. Strainmeter outputs are voltages proportional to the elongations of transducers (gauges) that cross the inclusion at several azimuths. The calibration procedure defines how to compute strain in the earth's crust from the strainmeter output.

2.1 Instrument, Local, and Regional strain tensors

Two-dimensional horizontal strain tensors in the earth are generally expressed in an east-north Cartesian coordinate system. We use the subscripts E and N to denote the coordinate directions, so that the components of a two-dimensional strain tensor, ε, are ε_{EE}, ε_{NN}, and ε_{EN}. The relationship between these quantities and the strain quantities often derived from 3-component strainmeter data (e.g., Hart et al., 1996) are:

$$\begin{aligned} e_a &= e_{EE} + e_{NN} \\ \gamma_1 &= e_{EE} - e_{NN} \\ \gamma_2 &= 2e_{EN} \end{aligned} \qquad (1)$$

e_a is an invariant of the strain tensor, so it is independent of which coordinate system is chosen, but γ_1 and γ_2 are not. The γ_1 and γ_2 notation will be avoided in these notes because similar symbols are used in other literature to denote the same quantities divided by 2.

In general, three separate strain fields are conceptually useful in describing the relationship of strainmeter output to strain in the crust (Figure 1).

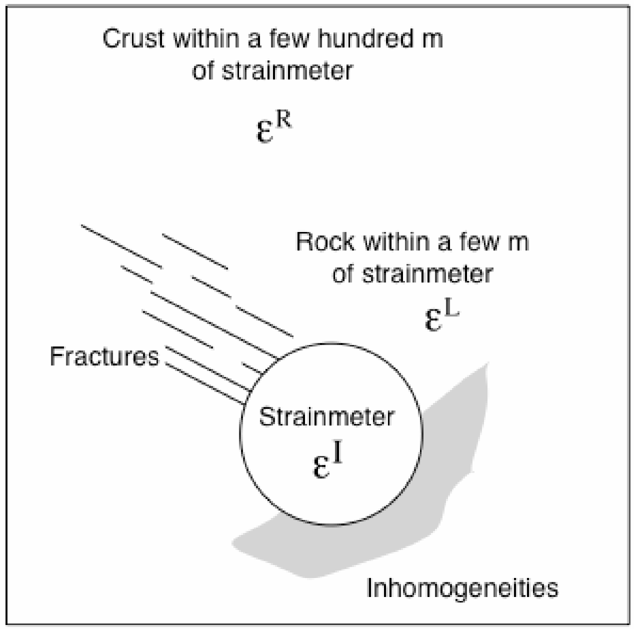

Figure 1. Cartoon showing approximate spatial scales for instrument, local, and regional strain fields .

The "instrument strain tensor", ε^{I}, is the tensor obtained by converting the gauge elongations to strain, using

$$e_\theta / R = \frac{\varepsilon^{I}_{EE} + \varepsilon^{I}_{NN}}{2} + \frac{\varepsilon^{I}_{EE} - \varepsilon^{I}_{NN}}{2} \cos 2\theta + \varepsilon^{I}_{EN} \sin 2\theta, \qquad (2)$$

in which e_θ is the elongation of a gauge of length R oriented at an angle θ measured counterclockwise from East. For a strainmeter with n gauges, equation (2) can be expressed in matrix form as

$$\begin{bmatrix} e_1/R_1 \\ e_2/R_2 \\ \vdots \\ e_n/R_n \end{bmatrix}_{nx1} = \begin{bmatrix} 1/2 & 1/2\cos 2\theta_1 & 1/2\sin 2\theta_1 \\ 1/2 & 1/2\cos 2\theta_2 & 1/2\sin 2\theta_2 \\ \vdots & \vdots & \vdots \\ 1/2 & 1/2\cos 2\theta_n & 1/2\sin 2\theta_n \end{bmatrix}_{nx3} \begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^{I}_{3x1} = [\mathbf{O}] \begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^{I}_{3x1} \qquad (3a)$$

which is essentially equations (8) and (9) of Hart et al. (1996). Equivalently,

$$\begin{bmatrix} e_1/R_1 \\ e_2/R_2 \\ \vdots \\ e_n/R_n \end{bmatrix}_{nx1} = \begin{bmatrix} \cos^2\theta_1 & \sin^2\theta_1 & 2\sin\theta_1\cos\theta_1 \\ \cos^2\theta_2 & \sin^2\theta_2 & 2\sin\theta_2\cos\theta_2 \\ \vdots & \vdots & \vdots \\ \cos^2\theta_n & \sin^2\theta_n & 2\sin\theta_n\cos\theta_n \end{bmatrix}_{nx3} \begin{bmatrix} \varepsilon_{EE} \\ \varepsilon_{NN} \\ \varepsilon_{EN} \end{bmatrix}^I_{3x1} \tag{3b}$$

As will be seen below, the instrument strain tensor is generally not equal to what we will call the "local" strain tensor, ε^L, which represents strain in the rock near (within perhaps a few meters of) the borehole. Moreover, there may also be a significant difference between the local strain tensor and the "regional" (or "remote") strain tensor, ε^R. The regional strain tensor is a tensor that is most appropriate for comparison with calculated strains based on simplified earth models, such as coseismic strain fields of earthquakes or ocean-load corrected theoretical tides. It may be appropriate to think of the regional strain tensor as a spatial average of local strain tensors over an area large compared with fractures or heterogeneities that might affect the output of an individual borehole strainmeter.

Returning to equation (3a) or (3b), we rewrite the fractional elongations on the LHS as $e_j/R_j = g_j u_j$, where u_j represents the numbers in the strainmeter data file from the *j*-th gauge (voltages, counts, or whatever) and g_j is a "gauge factor" (or "transducer factor") that scales the strainmeter data to actual fractional elongations. For PBO GTSM's, the gauge factors are adjusted in the laboratory to 1 nanostrain/count. For previously installed USGS 3-component borehole strainmeters, the laboratory gauge factors were not always measured, so we will retain the gauge factors in the calibration equations. Equation (3a) becomes

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}_{nx1} = \begin{bmatrix} 1/g_1 & 0 & \cdots & 0 \\ 0 & 1/g_2 & 0 & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 1/g_n \end{bmatrix}_{nxn} [\mathbf{O}]_{nx3} \begin{bmatrix} \varepsilon_{EE}+\varepsilon_{NN} \\ \varepsilon_{EE}-\varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^I \tag{4}$$

The relationship in equation (4) between the gauge outputs and the instrument strain tensor depends only on features of the strainmeter - namely, the gauge orientations and weighting factors.

2.2 Coupling of Instrument Strain to Local Strain in the Rock

The first step in determining strain in the crust from the instrument strain is to determine a coupling matrix, **K**, relating the instrument strain to the local strain:

$$\begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^{I} = [\mathbf{K}]_{3x3} \begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^{L} \qquad (5)$$

If the borehole is uniform, and the instrument and surrounding material are axisymmetric, then **K** is a symmetric matrix with only two independent entries (e.g., Agnew, 1986, section 3.4.3), and equation (5) can be written

$$\begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^{I} = \begin{bmatrix} C & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{bmatrix}_{3x3} \begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^{L} \qquad (6)$$

where C and D are areal and shear coupling coefficients, respectively. This situation will be referred to as "isotropic" coupling. C and D are dimensionless quantities derived from the ratios of the rock elastic moduli to the effective elastic moduli of the instrument. The ratio D/C is theoretically expected to be in the range 1 to 3 and can be calculated for various axisymmetric models of concentric rock, borehole, grout, and strainmeter (Gladwin and Hart, 1985). The need for distinct coefficients for shear and areal coupling arises from the expectation that the borehole/instrument inclusion is more compliant in response to shear strain than to areal strain.

We digress briefly to note that Gladwin and Hart (1985) and Sakata and Sato (1986) discuss coupling in the context of principal strains. These are useful papers and are for the most part consistent with the development here. However, working with principal strains can be problematic in the analysis of real-world borehole strainmeter data, which are generally sums of strain tensors having various time-dependent orientations. In particular, equations (16) and (17) of Gladwin and Hart (1985), and the corresponding equations in section 3.2 of Sakata and Sato (1986), which are nonlinear formulae for extracting the maximum shear strain and the direction of the principal axes from the gauge outputs, yield meaningless results when applied to tidal variations, which have time-varying principal strains.

Combining equations (4) and (6) leads to a commonly-used, although not fully general, form of the calibration equation:

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}_{nx1} = \begin{bmatrix} 1/g_1 & 0 & \cdots & 0 \\ 0 & 1/g_2 & 0 & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 1/g_n \end{bmatrix}_{nxn} [\mathbf{O}]_{nx3} \begin{bmatrix} C & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{bmatrix}_{3x3} \begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^L \tag{7}$$

In equation (7), the vector of strains on the RHS represents the local strain tensor. Note that to obtain a meaningful strain in the rock it is necessary to estimate the coupling coefficients C and D; the instrument strain tensor formed without these quantities will differ significantly from the local strain tensor.

We will for now ignore the difference between the local and regional strain tensors, and proceed with an example, before discussing more general coupling and possible differences between the local and regional strain tensors.

3. Tidal Calibration Examples

We can use equation (7) to determine the relationships between tidal variations in the strainmeter output and calculated or independently measured tides, for the special case of three equally-spaced gauges. In preparation for the calibration, we assume that the amplitudes and phases of the M2 and O1 tidal constituents have been determined for each of the strainmeter gauges, and that there are “known” M2 and O1 tidal amplitudes and phases that have been calculated numerically, and/or measured with an instrument such as a LSM.

The tidal amplitudes and phases will be expressed in complex "phasor" notation. As an example, the East-West M2 tidal strain component, ε_{EE}^{M2}, has a sinusoidal time dependence given by

$$\varepsilon_{EE}^{M2} = A_{EE}^{M2}\cos(\omega_{M2}t + \phi_{EE}^{M2}) \tag{8}$$

in which t denotes time, ω_{M2} is the (known) frequency of the M2 tide, A_{EE}^{M2} is the amplitude, usually given in nanostrain, and ϕ_{EE}^{M2} is the phase. The phase is often given in degrees relative to the phase of the M2 tidal potential at the same site. Because equation (8) is a nonlinear function of the phase, it is more convenient to consider ε_{EE}^{M2} to be the real part of a complex function $\tilde{\varepsilon}_{EE}^{M2}$, such that:

$$\varepsilon_{EE}^{M2} = \mathrm{Re}[\tilde{\varepsilon}_{EE}^{M2}] = \mathrm{Re}[A_{EE}^{M2}\exp(i\omega_{M2}t)\exp(i\theta_{EE}^{M2})] \tag{9}$$

where $i = \sqrt{-1}$. We will only be using these M2-related quantities in equations that will contain a factor of $\exp(i\omega_{M2}t)$ in every term, so these factors are dropped in anticipation of their cancellation. With this "phasor" notation, $\tilde{\varepsilon}_{EE}^{M2}$ is written as $A_{EE}^{M2}\exp(i\theta_{EE}^{M2}) = A_{EE}^{M2}[\cos(\theta_{EE}^{M2}) + i\sin(\theta_{EE}^{M2})]$, and the factor of $\exp(i\omega_{M2}t)$ is implied. Or, $\tilde{\varepsilon}_{EE}^{M2}$ can be written as $A_{EE}^{M2}\cos(\theta_{EE}^{M2}) + iA_{EE}^{M2}\sin(\theta_{EE}^{M2}) = \mathrm{Re}[\tilde{\varepsilon}_{EE}^{M2}] + i\,\mathrm{Im}[\tilde{\varepsilon}_{EE}^{M2}]$. Similar notation is used for the O1 constituent and for the other strain tensor components.

Determining both the real and imaginary parts of the tide phasor is equivalent to determining the amplitude and phase. In our example, we will work with amplitudes and phases. In practical calibrations, however, linear least-squares estimation is used, so real and imaginary parts are used to avoid needing to solve simultaneous transcendental equations.

3.1 Isotropic coupling matrix, three equally-spaced gauges with one East-West

Equation (7) becomes quite simple for the case of three gauges with equal azimuthal spacings, one of which is oriented East-West. (These assumptions are not very restrictive, since most currently operating GTSM's do have 3 equally-spaced gauges, and because reference tides can always be rotated to a coordinate system whose *x*-axis is parallel to one of the gauges.) It may be necessary to work with the gauges in an order other than the one in which they are numbered, so in the equations below we use subscripts a,b,c instead of integers to denote the three gauges. Assume without loss of generality that the East-West gauge is Gauge b, that Gauge a is oriented 120° CCW of Gauge b, and Gauge c is oriented 120° CW of Gauge b. Equation (7) reduces to:

$$\begin{bmatrix} g_a u_a \\ g_b u_b \\ g_c u_c \end{bmatrix}_{3x1} = \frac{1}{2}\begin{bmatrix} 1 & -1/2 & \sqrt{3}/2 \\ 1 & 1 & 0 \\ 1 & 1/2 & -\sqrt{3}/2 \end{bmatrix}\begin{bmatrix} C(\varepsilon_{EE}+\varepsilon_{NN}) \\ D(\varepsilon_{EE}-\varepsilon_{NN}) \\ D(2\varepsilon_{EN}) \end{bmatrix}^L \tag{10}$$

Summing the three equations represented by equation (10) yields:

$$(g_a u_a + g_b u_b + g_c u_c)/3 = C(\varepsilon_{EE}^L + \varepsilon_{NN}^L)/2 \tag{11a}$$

which says that for isotropic coupling, the areal strain is proportional to the weighted average of the gauge outputs. (Note that equation (11a) does not depend on the number or the orientation of the gauges; this relation holds for any number of gauges at equal azimuthal spacings.) It can also be shown from (10) that:

$$(2g_b u_b - g_a u_a - g_c u_c)/3 = D(\varepsilon_{EE}^L - \varepsilon_{NN}^L)/2 \tag{11b}$$
$$(g_c u_c - g_a u_a)/\sqrt{3} = D\varepsilon_{EN}^L \tag{11c}$$

Equations (11b) and (11c) show that the shear strains are proportional to differences of the signals from the three gauges, again under the assumption of isotropic coupling.

Equations (11a), (11b), and (11c) contain 5 real, scalar unknown parameters (three gauge factors and the areal and shear coupling coefficients). Actually only 4 of these unknowns are independent, since one of the gauge factors can be assumed without loss of generality to be 1. When used for tidal calibration, the gauge outputs and strains in equations (11a), (11b), and (11c) can be viewed as complex numbers, so that each equation represents two real equations obtained by taking the amplitudes and phases of both sides. Moreover, each of these equations may be written for both M2 and O1 phases.

Several useful facts can be deduced by inspecting equations (11a)through (11c):

a) The areal and shear coupling factors, C and D, are real scalars, so their values cannot affect the phase equations. It follows that only by adjusting the relative gage weightings can mismatches between observed and calculated tide phases be reconciled. It also follows that if the two sides of any of these equations have different phases, then a complex coupling factor would be needed, which is physically unreasonable.

b) If g_b is the gauge factor chosen to be unity, then the equations obtained by taking the phase of both sides of (11a), (11b), and (11c) are a system of 3 equations per tide constituent in the two remaining gauge factors. If there is a pair of gauge factors that satisfies all of these equations to a close enough approximation, then this could be viewed as evidence that the tidal strains are well modeled, the rock surrounding the strainmeter is approximately isotropic and uniform, and the strainmeter is well coupled to the formation in the tidal band.

c) Once the gauge factors are determined, there will always be an areal coupling factor, C, that solves the amplitude equation corresponding to (11a), if only M2 is considered. If O1 is also used in the calibration, then there may or may not be a single value of C that works for both constituents.

d) Once the gauge factors are determined, there may or may not be a single value of the shear coupling factor, D, that satisfies the amplitude equations corresponding to both (11b) and (11c), even if only M2 is considered. The existence of a single value that works satisfactorily would indicate tidal strains are well modeled, the rock is isotropic and uniform, and coupling is good in the tidal band.

3.2 Example: Calibration of the Piñon Flat GTSM assuming isotropic coupling

We implement the approach above in an Excel spreadsheet to get an idea of how well the isotropic coupling matrix works in reconciling the tides of the Piñon Flat GTSM with those of the Piñon Flat Laser Strainmeter (LSM). Working through the equations “by hand” illustrates the roles of the gauge factors and coupling factors in reconciling the

GTSM tides with theoretical or, in this case, independently measured, tides. The Piñon Flat example is chosen because it will lead into a brief review of the paper by Hart et al. (1996), who carried out a fully cross-coupled calibration of the GTSM against the LSM tides.

The PF GTSM does not have a gauge oriented exactly East-West, so in order to use equations like (11a) through (11c), the LSM tides must be rotated to a reference frame with x'-axis parallel to one of the gauges. This is done using the following equations, which give the 2-dimensional strain tensor in an x', y' coordinate system rotated by an angle ϕ counterclockwise from an x, y coordinate system:

$$\varepsilon_{x'x'} = \left[\frac{\varepsilon_{xx}+\varepsilon_{yy}}{2}\right] + \left[\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\right]\cos(2\phi) + \varepsilon_{xy}\sin(2\phi) \qquad (12a)$$

$$\varepsilon_{y'y'} = \left[\frac{\varepsilon_{xx}+\varepsilon_{yy}}{2}\right] - \left[\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\right]\cos(2\phi) - \varepsilon_{xy}\sin(2\phi) \qquad (12b)$$

$$\varepsilon_{x'y'} = -\left[\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\right]\sin(2\phi) + \varepsilon_{xy}\cos(2\phi) \qquad (12c)$$

The gauges need to be considered in a different order (see Figure 2). Gauges 1,2 and 3 of the PF GTSM are oriented counterclockwise from East by 24.5°, 84.5°, and 144.5°; ie., their azimuths are 65.5°, 5.5°, and -54.5°. Gauge 1 is the closest to East-West, so we identify Gauge 1 with Gauge b (this choice is totally arbitrary). Gauge 2 can be viewed as having either the azimuth 5.5° or 185.5°; this is 120° CW from Gauge b, so Gauge 2 is Gauge c. Gauge 3 has azimuth -54.5°, so it is Gauge a, 120° CCW from Gauge 2. Gauge-b-parallel coordinates are rotated 24.5° CCW from E-W. The original and rotated LSM tides are shown in Table 1. Notice that the rotation does not affect the amplitude or phase of the areal strain, which is an invariant of the 2-D strain tensor.

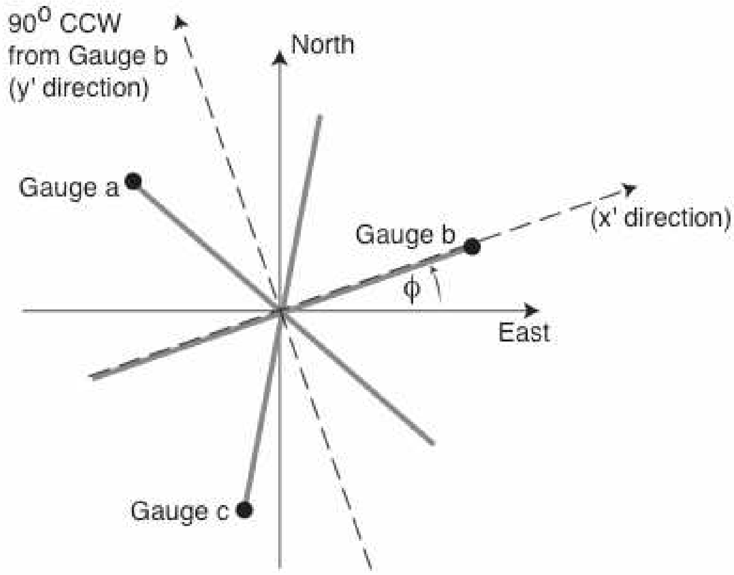

Figure 2. Diagram of strainmeter gauges and coordinate system rotated so that x'-axis is parallel to gauge b.

Table 1. Amplitudes and phases of Pinon Flat LSM tidal strains. Strains in east-north coordinates are from Table 4 of Hart et al. (1996). Strains in x'y' coordinates have been computed using equations (12a) thru (12c) for an x' axis rotated 24.5 degrees CCW from East.

	|M2| (nanostrain)	M2 phase (degrees)	|O1| (nanostrain)	O1 phase (degrees)
Eee+Enn	17.39	5.5	8.66	-1.7
Eee-Enn	7.56	164.4	2.02	-43.3
2Ene	8.09	168.6	1.74	109.8
Eee	5.34	20.2	5.13	-9.2
Enn	12.29	-0.9	3.64	9.0
Ene	4.04	168.6	0.87	109.8
Ex'x'	3.89	32.7	4.59	0.5
Ey'y'	14.04	-1.8	4.08	-4.0
Ex'y'	0.28	-59.0	1.30	125.2
(Ex'x'+Ey'y')	17.39	5.5	8.66	-1.7
(Ex'x'-Ey'y')	11.05	166.7	0.61	31.6
Ex'y'	0.28	-59.0	1.30	125.2

Table 2. Average Piñon Flat GTSM gauge tides, from Table 1 of Hart et al. (1996)

	|M2| (nanostrain)	M2 phase (degrees)	|O1| (nanostrain)	O1 phase (degrees)
Gauge a (Gauge 3)	16.78	5.3	8.72	−17.4
Gauge b (Gauge 1)	5.29	71.1	8.13	7.4
Gauge c > (Gauge 2)	26.53	−2.7	5.96	17.2

First, we compare observed and calculated phases. Taking arguments of both sides, equation (11a) for the areal strain becomes

$$Arg(g_a u_a + g_b u_b + g_c u_c) = Arg(\varepsilon_{x'x'} + \varepsilon_{y'y'}) \quad (13a)$$

where scalar factors have been cancelled since they do not affect the phase. Similarly equations (11b) and (11c) become:

$$Arg(2g_b u_b - g_a u_a - g_c u_c) = Arg(\varepsilon_{x'x'} - \varepsilon_{y'y'}) \quad (13b)$$

$$Arg[g_c u_c - g_a u_a] = Arg[\varepsilon_{x'y'}] \quad (13c)$$

Table 3 compares the phases obtained from (13a) through (13c) under the assumption of equally weighted gauges. The phase disagreements for areal strain are only about 2° for both M2 and O1; but for some of the shear strains, the discrepancies are greater. Note that these phase comparisons can be made independently of determining C and D, and no choice of C or D can improve the phase matches. Neglecting the phase difference, the areal coupling coefficient is found to be 1.72 for M2 and 1.70 for O1; these values are very close to the "isotropic" estimates in Table 5 of Hart et al. (1996). The reason there is a small difference is that the value in Table 5 of Hart et al. (1996) is fit by least-squares to both the M2 and O1 information.

Table 3. Phase comparisons for equal gauge weights. Columns labeled LSM are the LSM-observed values, in x',y' coordinates rotated 24.5 degrees CCW from E. Columns labeled GTSM are calculated from equations (13a) through (13c), assuming all gauge weights are equal.

	M2, LSM	M2, GTSM	O1, LSM	O1, GTSM
Phase(Ex'x'+Ey'y')	5.5	6.7	-1.7	0.5
Phase(Ex'x'-Ey'y')	166.7	166.3	31.6	54.3
Phase(Ex'y')	-59.0	-15.9	125.2	121.1

We consider whether unequal gauge weighting can reduce the phase discrepancies for the shear strains. Experience shows that for many 3-component strainmeters, the areal strain phase is not that sensitive to the relative gauge weightings, so we solve the two shear strain equations (13b) and (13c) in succession for the two unknown gauge factors that will exactly match the shear strain phases:

$$g_a / g_c = \frac{\tan[Arg(\varepsilon_{x'y'})]\mathrm{Re}(u_c) - \mathrm{Im}(u_c)}{\tan[Arg(\varepsilon_{x'y'})]\mathrm{Re}(u_a) - \mathrm{Im}(u_a)} \tag{14a}$$

$$g_c = 2\frac{\tan[Arg(\varepsilon_{x'x'} - \varepsilon_{y'y'})]\mathrm{Re}(u_b) - \mathrm{Im}(u_b)}{\tan[Arg(\varepsilon_{x'x'} - \varepsilon_{y'y'})]\mathrm{Re}[(g_a/g_c)u_a + u_c)] - \mathrm{Im}[(g_a/g_c)u_a + u_c)]} \tag{14b}$$

A word of caution is needed with respect to (14a) and (14b), which can yield misleading answers in cases where the GTSM tides cannot be reconciled with the reference tides using an isotropic coupling matrix. More specifically, equations (14a) and (14b) will always provide values for g_a and g_c, but when these value are substituted into the left-hand-sides of equations (13b) and (13c), the result may be $Arg(\varepsilon_{x'y'})$+/-180°, rather than just $Arg(\varepsilon_{x'y'})$. For this reason, it is essential to verify that equations (13b) and (13c) are correctly satisfied. Also, it's possible for the values returned by equations (14a) and (14b) to be negative, or unreasonably different from unity. Such results indicate the isotropic coupling assumption is not valid, or possibly that some other problem, such as misorientation, exists. When the isotropic coupling assumption is reasonably appropriate, and reasonable gauge weightings exist, equations (14a) and (14b) will yield correct solutions.

For M2, equations (14a) and (14b) yield the gauge factors g_a=1.211, g_b=1.0 (assumed), and g_c=0.8291. The phase of the M2 areal strain computed using these gauge factors is 7.95°, only 2° different from the value with equally weighted gauges, although a little futher from the LSM value. Using the isotropic coupling matrix, there are no further parameters to adjust that would bring the observed and calculated areal strain phases into closer agreement. Ignoring the small phase difference, equating the amplitudes of the observed and calculated areal strains leads to the areal coupling factor, C, having the value 1.70, not very different from the value obtained with equal gauge weights (Table 4).

For O1, the same method yields somewhat different gauge factors (g_a=0.838 and g_c=0.783). These values and the M2 gauge factors agree well with the "tranducer factor" calibration values determined by Hart et al. (1996). The O1 areal strain phase changes by only 0.2° if these factors are used. The areal coupling factor, C, based on O1, has the value 1.50, slightly lower than the value with equal gauge weights. The two shear coupling factors determined from the equations for $\varepsilon_{x'y'}$ and $\varepsilon_{x'x'} - \varepsilon_{y'y'}$ are quite different from each other for both M2 and O1 (Table 4).

Table 4. Areal and shear coupling factors from M2 and O1 for the Piñon Flat GTSM with gauges individually weighted.

	from M2	from O1
D from Ex'y':	8.32	2.48
D from Ex'x'-Ey'y'	2.40	5.99
C from Ex'x'+Ey'y'	1.70	1.50

3.3 LSM tides compared with theoretical tides

For Piñon Flat, the LSM provides a direct measure of in situ tides, but such information is not available for the great majority of GTSM sites. The Piñon Flat theoretical tidal amplitudes are within 6% of the LSM values for the areal strain and for ε_{EN}, but the amplitudes differ by 35% for $\varepsilon_{EE} - \varepsilon_{NN}$; phases are within 1° for areal strain, but differ by up to 40° for shear strain (Hart et al., 1996). The discrepancy is probably due to the ocean load calculation. It's instructive to see what happens if the theoretical tides (body + ocean loads) are used to determine gauge weights and coupling factors. (This can be done using the accompanying spreadsheet and the theoretical tides from Hart et al. (1996) Tables 6a, 6b, and 6c). In fact, equation (14a) does not provide a correct answer for O1 in this case: although the reasonable value of g_a / g_c=1.53 is obtained, back-substitution shows that this value corresponds to $Arg(\varepsilon_{x'y'})$-180°, and so is not really a solution of equation (13c). This situation indicates that the GTSM tides and theoretical tides as given in Hart et al. (1996) cannot be reconciled under the assumption of isotropic coupling.

4. More General Coupling

The procedure above reconciles the tides observed by the strainmeter with the reference tides by assuming that the calculated tides are realistic, but that the gauges are not equally weighted. The result for the Piñon Flat GTSM is that different gauge factors seem to be needed for M2 as opposed to O1, an indication that differing gauge weights are not the only additional parameters needed to reconcile the GTSM and LSM tides. Hart et al. (1996) consider a coupling more general than the isotropic matrix. One way such a matrix could arise is based on the idea that regional strains differ from the local strain tensor. More specifically, heterogeneities, anisotropy, or topographic variations within a few meters or tens of meters of the strainmeter borehole are postulated to modify the local strain field relative to the regional strain field, which is representative of an area hundreds of meters across. Hart et al. (1996) express this relationship using a perturbation matrix, $\mathbf{P}$, such that

$$\begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^L = [\mathbf{P}]_{3x3} \begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^R \qquad (15)$$

The matrix $\mathbf{P}$ is in general not symmetric. A more general form of equation (7) is thus

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}_{nx1} = \begin{bmatrix} 1/g_1 & 0 & \cdots & 0 \\ 0 & 1/g_2 & 0 & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & 1/g_n \end{bmatrix}_{nxn} [\mathbf{O}]_{nx3} \begin{bmatrix} C & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{bmatrix}_{3x3} [P]_{3x3} \begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^R \tag{16}$$

For a strainmeter with n gauges, equation (16) is generally solved numerically by finding a nx3 matrix, $\mathbf{C}$, such that

$$\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}_{nx1} = [\mathbf{C}]_{nx3} \begin{bmatrix} \varepsilon_{EE} + \varepsilon_{NN} \\ \varepsilon_{EE} - \varepsilon_{NN} \\ 2\varepsilon_{EN} \end{bmatrix}^R_{3x1} \tag{17}$$

For a tidal calibration, (17) expands to

$$\begin{bmatrix} \mathrm{Re}[u_1^{O1}] & \mathrm{Im}[u_1^{O1}] & \mathrm{Re}[u_1^{M2}] & \mathrm{Im}[u_1^{M2}] \\ \mathrm{Re}[u_2^{O1}] & \mathrm{Im}[u_2^{O1}] & \mathrm{Re}[u_2^{M2}] & \mathrm{Im}[u_2^{M2}] \\ \vdots & \vdots & \vdots & \vdots \\ \mathrm{Re}[u_n^{O1}] & \mathrm{Im}[u_n^{O1}] & \mathrm{Re}[u_n^{M2}] & \mathrm{Im}[u_n^{M2}] \end{bmatrix}_{4xn}$$

$$= [\mathbf{C}]_{nx3} \begin{bmatrix} \mathrm{Re}[\tilde{\varepsilon}_{EE}^{O1}] & \mathrm{Im}[\tilde{\varepsilon}_{EE}^{O1}] & \mathrm{Re}[\tilde{\varepsilon}_{EE}^{M2}] & \mathrm{Im}[\tilde{\varepsilon}_{EE}^{M2}] \\ \mathrm{Re}[\tilde{\varepsilon}_{NN}^{O1}] & \mathrm{Im}[\tilde{\varepsilon}_{NN}^{O1}] & \mathrm{Re}[\tilde{\varepsilon}_{NN}^{M2}] & \mathrm{Im}[\tilde{\varepsilon}_{NN}^{M2}] \\ \mathrm{Re}[\tilde{\varepsilon}_{EN}^{O1}] & \mathrm{Im}[\tilde{\varepsilon}_{EN}^{O1}] & \mathrm{Re}[\tilde{\varepsilon}_{EN}^{M2}] & \mathrm{Im}[\tilde{\varepsilon}_{EN}^{M2}] \end{bmatrix}_{3x4} \tag{18}$$

where phasor notation is used for the tide components. It is assumed that the reference tides on the RHS of (18) represent the "regional" tides, ie., the tidal strain in an area large compared with the borehole and strainmeter. There are always 12 regional tidal quantities. For a strainmeter with three gauges, there are 9 unknown entries in the calibration matrix and 12 observables. For a strainmeter with 4 gauges, there are 12 unknowns and 16 observables. The system (4) is overdetermined for either number of gauges. After obtaining the calibration matrix by least squares, the orientation matrix can be factored out, and so can the gauge factors and areal and shear coupling coefficients, if they are known, leaving the perturbation matrix, $\mathbf{P}$. Hart et al. (1996) go through this process for the Piñon Flat GTSM in their equations (20) thru (23), obtaining the following perturbation matrix:

$$\mathbf{P} = \begin{bmatrix} 0.988 & -0.088 & 0.149 \\ -0.012 & 1.034 & 0.281 \\ 0.056 & -0.231 & 0.945 \end{bmatrix}$$

They note the off-diagonal entries as large as 28% are too large to be explained by the small topographic features in the area, but consider them consistent with geological inhomogeneities or fractures near the GTSM borehole. Equation (25) of Hart et al. (1996) gives the percent errors in each strain component resulting from using an isotropic coupling matrix rather than equation (17). These values range from 1% to 26%.

In principle, perturbation matrices may be calculated using finite-element models that incorporate topography and geologic heterogeneity, but this procedure has not been carried out for sites other than Piñon Flat.

5. Practical Experience and Considerations

Several features of existing USGS California GTSM deployments make it difficult to fully investigate the relationship between tides recorded by GTSM's and calculated tides. First, none of the GTSM's have been installed far enough from the coast to allow ocean loading to be neglected. Second, GTSM's have not been installed in close proximity to each other, so the degree of agreement to be expected among the shear components has not been explored. Third, information about laboratory-measured gauge weights has not been available. The PBO GTSM installations will supply information in all of these areas, so progress can be expected in understanding how to relate the GTSM-observed tides to reference tides. In the meantime, some experience with the existing instruments is described below.

5.1 Suitability of isotropic coupling for existing GTSM's

Experience with GTSM's in California, as well as with 3-component Sakata-type strainmeters used in the mini-PBO, shows that the tidal signal for some instruments agrees well with theoretical tides (ocean + body) using an isotropic coupling matrix, provided gauge factors are also solved for. In particular, areal strain phases generally agree well with calculated tides and nearby dilatometers. However, for most instruments there is at least one shear component that cannot be matched well, and lacking co-located LSM's, there is little information to diagnose the reason for the disagreement. A closer reconcilation of observed and reference tides will always be obtained if the assumption of isotropic coupling is relaxed. A general calibration matrix obtained numerically using equation (18) should always be decomposed as in equation (16) to assess the reasonableness of the perturbation matrix.

5.2 Stability of GTSM Tidal Parameters

Generally, tidal analyses of GTSM transducer data show variations of less than a few degrees over periods of years. However, Hart et al. (1996) note that the M2 phase of transducer 1 of the Piñon Flat GTSM decreased by about 1°/year between 1985 and 1991, clearly outside the measurement error. It was also found that the M2 phase for Gauge 1 of the Parkfield Frolich GTSM increased by 22° from 1988 to 2001. These changes are difficult to explain, because instrument drift should affect amplitude as well as phase, and timing errors should affect both M2 and O1.

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