



# SAR Image Formation

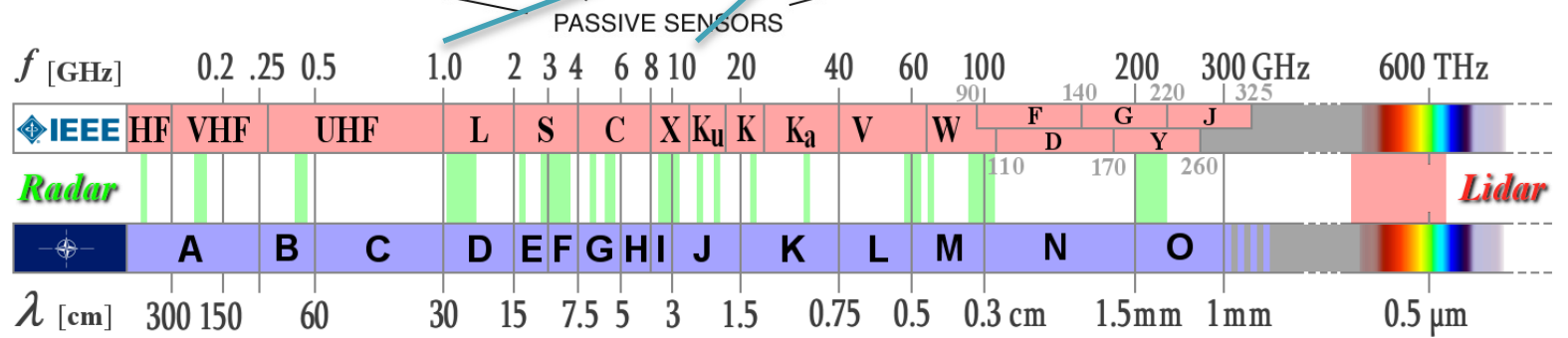
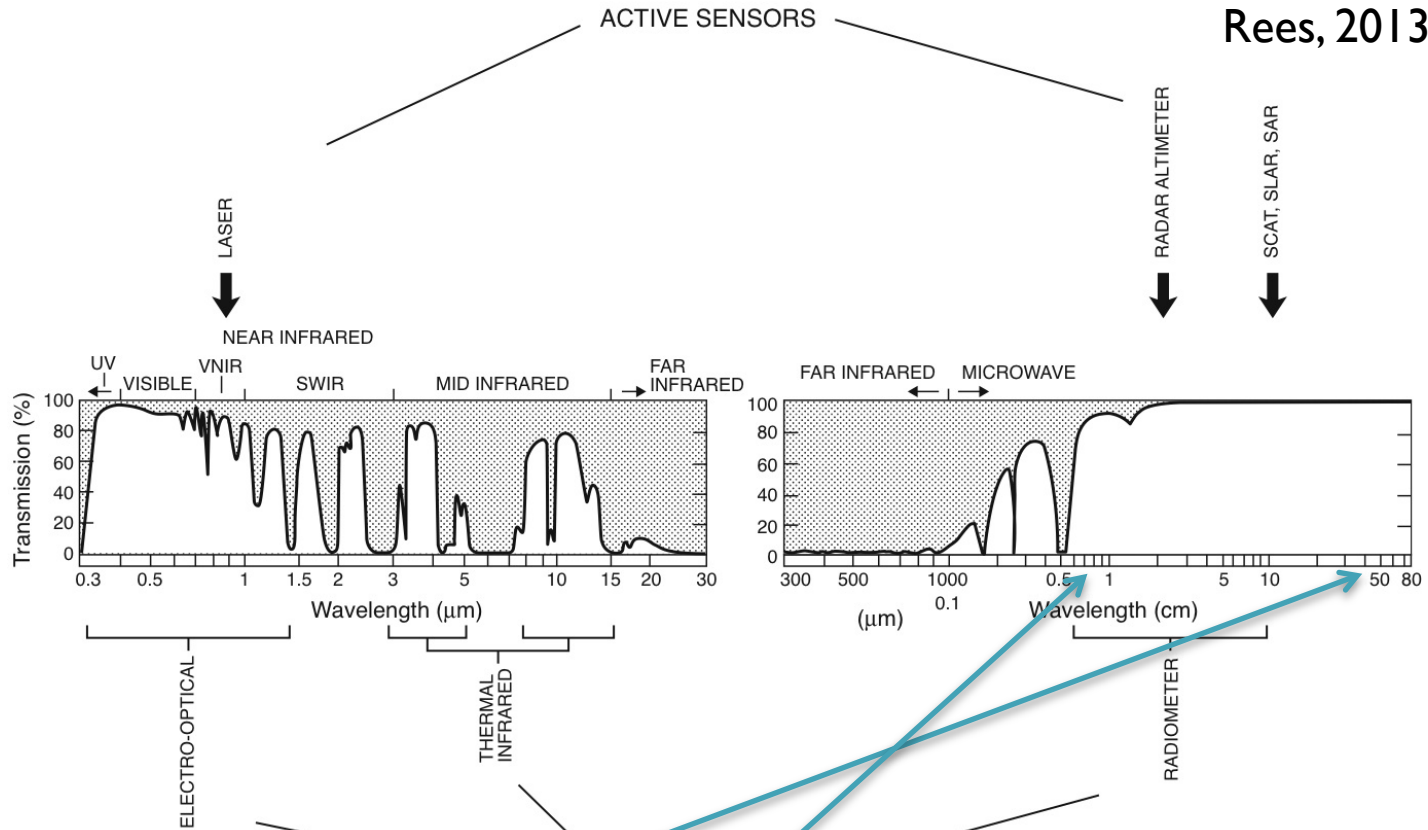
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David SANDWELL*

# Outline

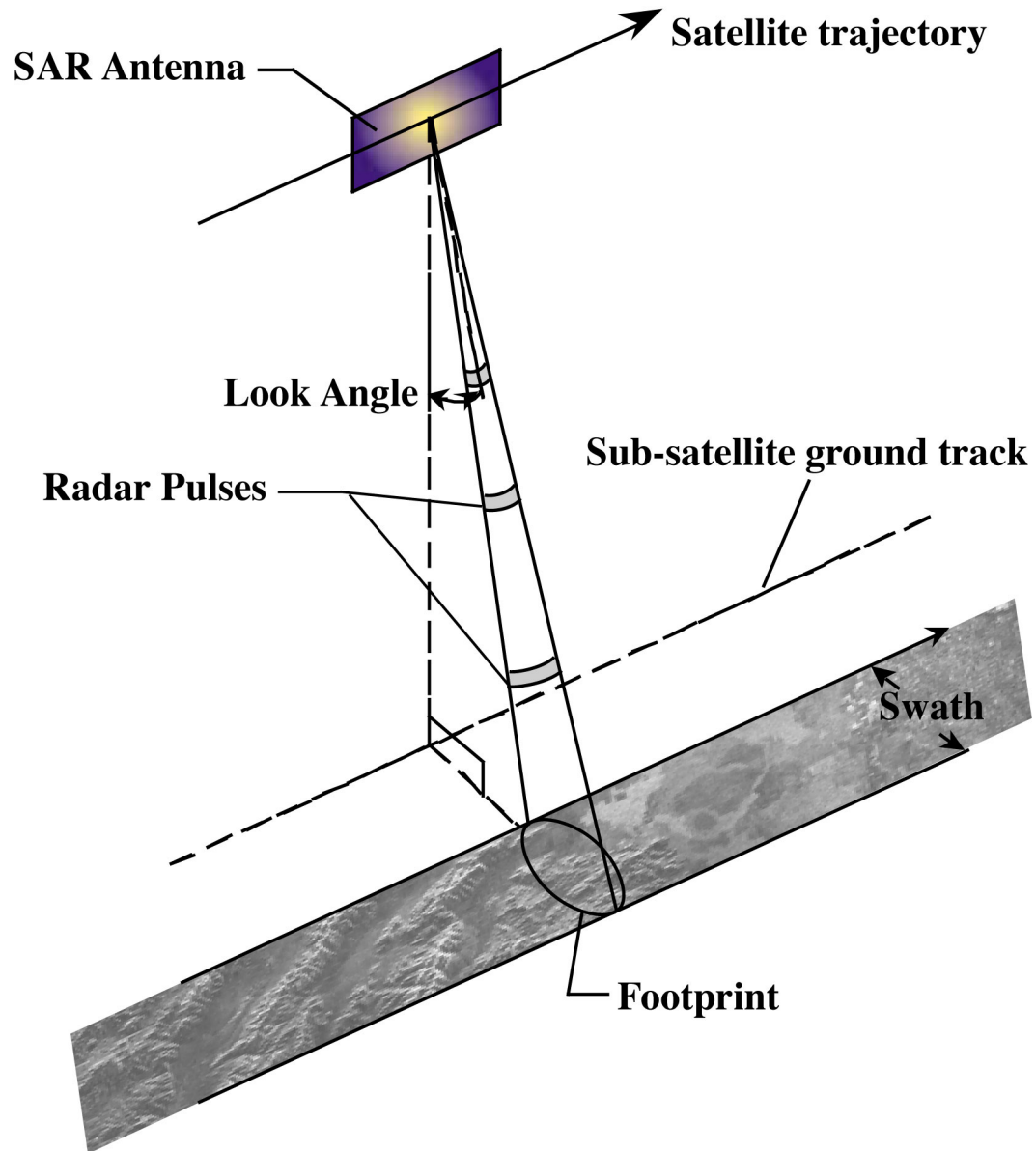
- Radar Bands
- SAR Image Formation
  - Range Compression
  - Range Migration
  - Azimuth Compression
- Length of Synthetic Aperture
- Precise orbit

# Radar Bands

Rees, 2013



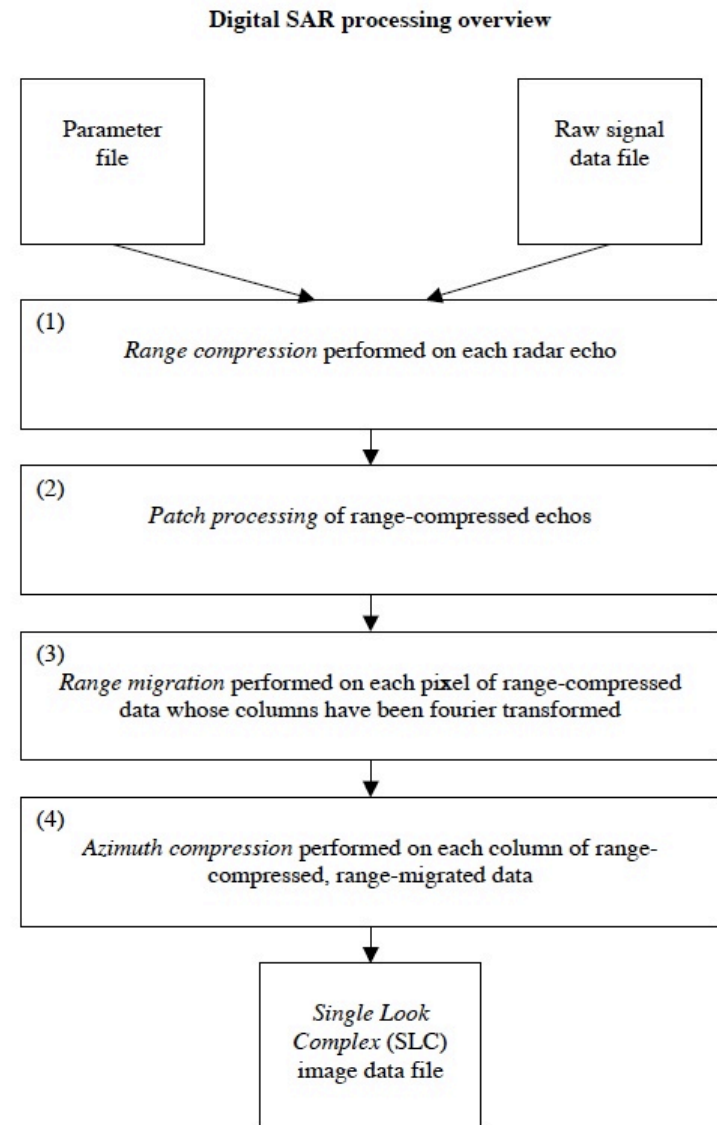
# How is the radar image focused?



# How is the radar image focused?

- Demodulate and Digitize
- Range Compression
  - Deconvolve a known chirp function
- Range Migration
  - Shift known Doppler in frequency domain and shift a known distance in range. (Orbit)
- Azimuth Compression
  - Deconvolve a known illumination chirp function

SAR processor →



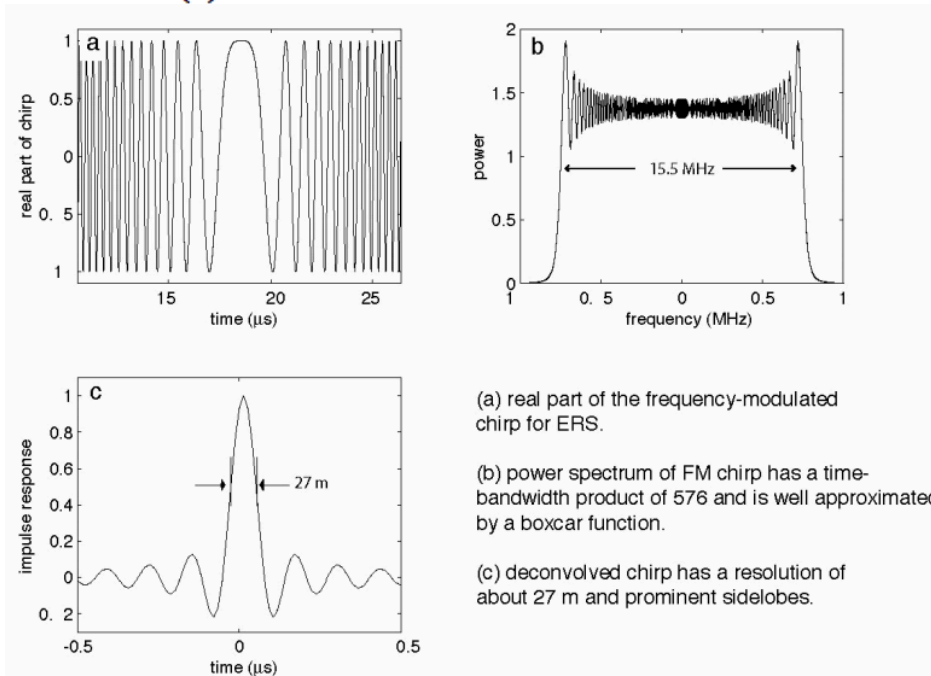
# Range Compression

- Ideal detection radar pulse (Delta Function),  $E = P(A) \cdot dt$ .
- Questions: Why not use a real delta function?
- Convolution between a pulse and a chirp function.  
Pulse duration  $\rightarrow \sim 10$  km
- Range Compression: De-convolve the range chirp from each row of data.  
(Focusing in Range)

$$s(t) = e^{i\pi k t^2}$$

- $k$  - chirp slope ( $4.17788 \times 10^{11} \text{ s}^{-2}$ )  
 $\tau_p$  - pulse duration ( $3.712 \times 10^{-5} \text{ s}$  or  $\sim 11$  km long)  
 $f_s$  - range sampling rate ( $1.89625 \times 10^7 \text{ s}^{-1}$ ).

An example of a portion of the chirp for the ERS- radar



(a) real part of the frequency-modulated chirp for ERS.

(b) power spectrum of FM chirp has a time-bandwidth product of 576 and is well approximated by a boxcar function.

(c) deconvolved chirp has a resolution of about 27 m and prominent sidelobes.

# Range Migration

- A range shift based on Doppler and Range

$$R_{rd}(f_\eta) \approx R_0 + \frac{\lambda^2 R_0}{8 V_r^2} f_\eta^2$$

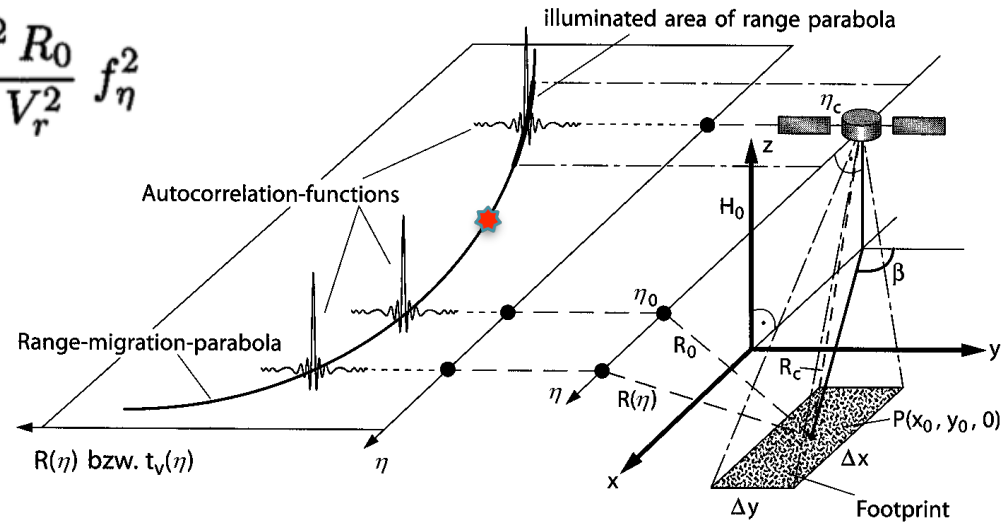


Image 3.11: Illustration of the range migration

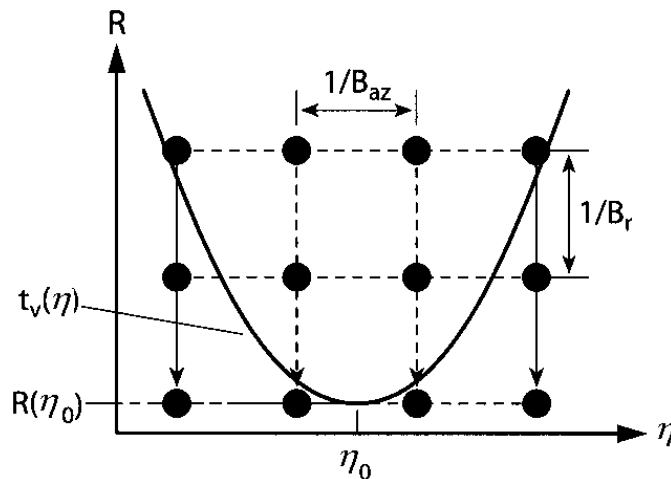


Image 3.12: Near-range parabola

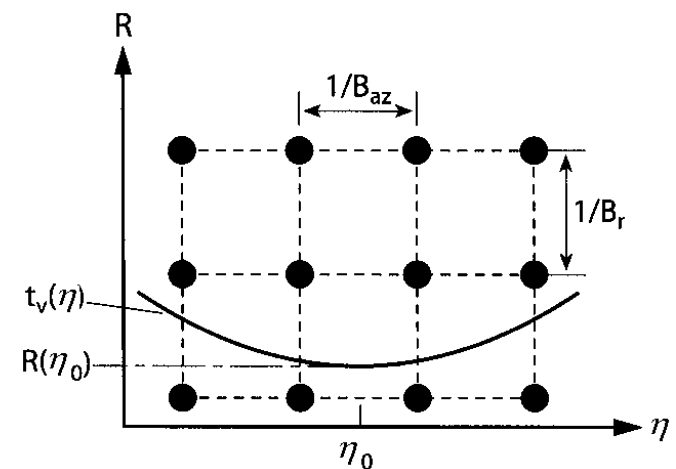


Image 3.13: Far-range parabola

# Azimuth Compression

## *Azimuth compression*

Azimuth compression or azimuth focusing involves coherent summation of echos at a constant range from the point reflector. The geometry of the strip-mode acquisition is shown in Figure B6

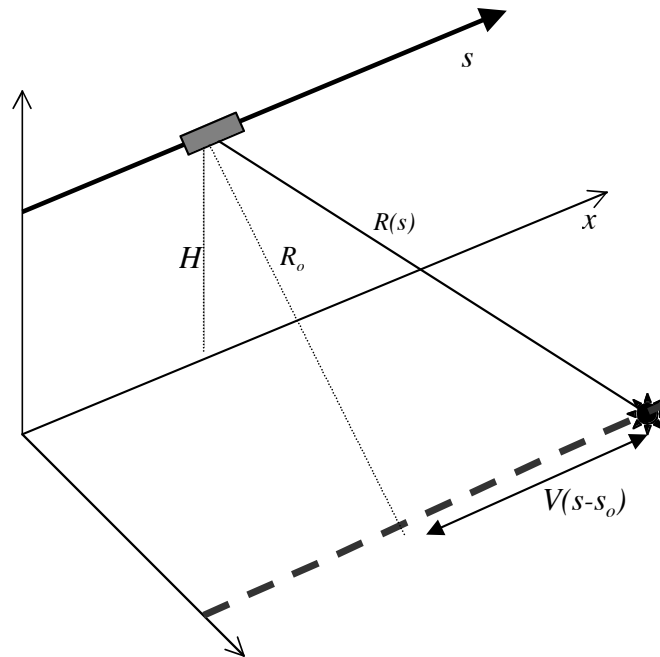
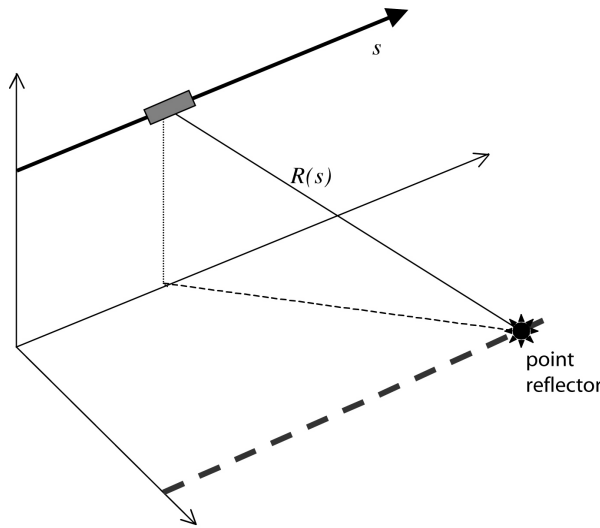


Figure B6. Geometry of radar passing over a point reflector where

- $V$  – the effective speed which is about equal to the ground track speed
- $s$  – slow time along the satellite track
- $s_o$  – time when the center of the radar echo passes over the point reflector
- $R_o = R_{near} + n * (C / fs)$  minimum range from the spacecraft to the target
- $R_{near}$  – near range to first data sample in the swath



# Azimuth Compression



phase history of point reflector

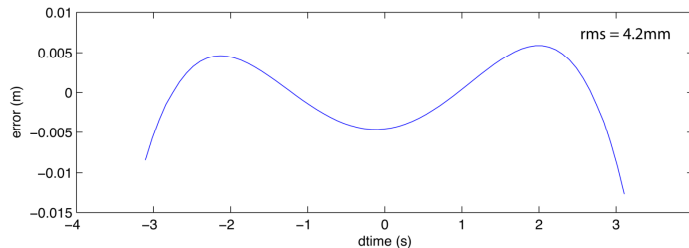
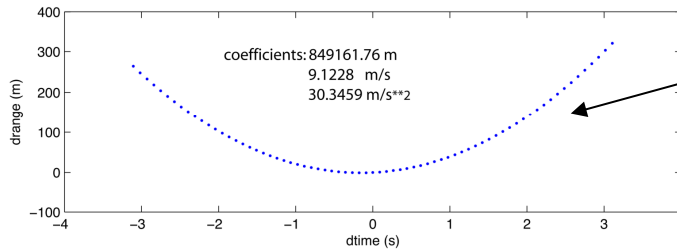
$$C(s) = \exp \left\{ i \frac{4\pi}{\lambda} [R(s)] \right\}$$

parabolic approximation to range history

$$R(s) = R_o + \dot{R}_o (s - s_o) + \frac{\ddot{R}_o}{2} (s - s_o)^2 + \dots$$

$$f_{DC} = \frac{-2\dot{R}}{\lambda}$$

$$f_R = \frac{2\ddot{R}}{\lambda}$$



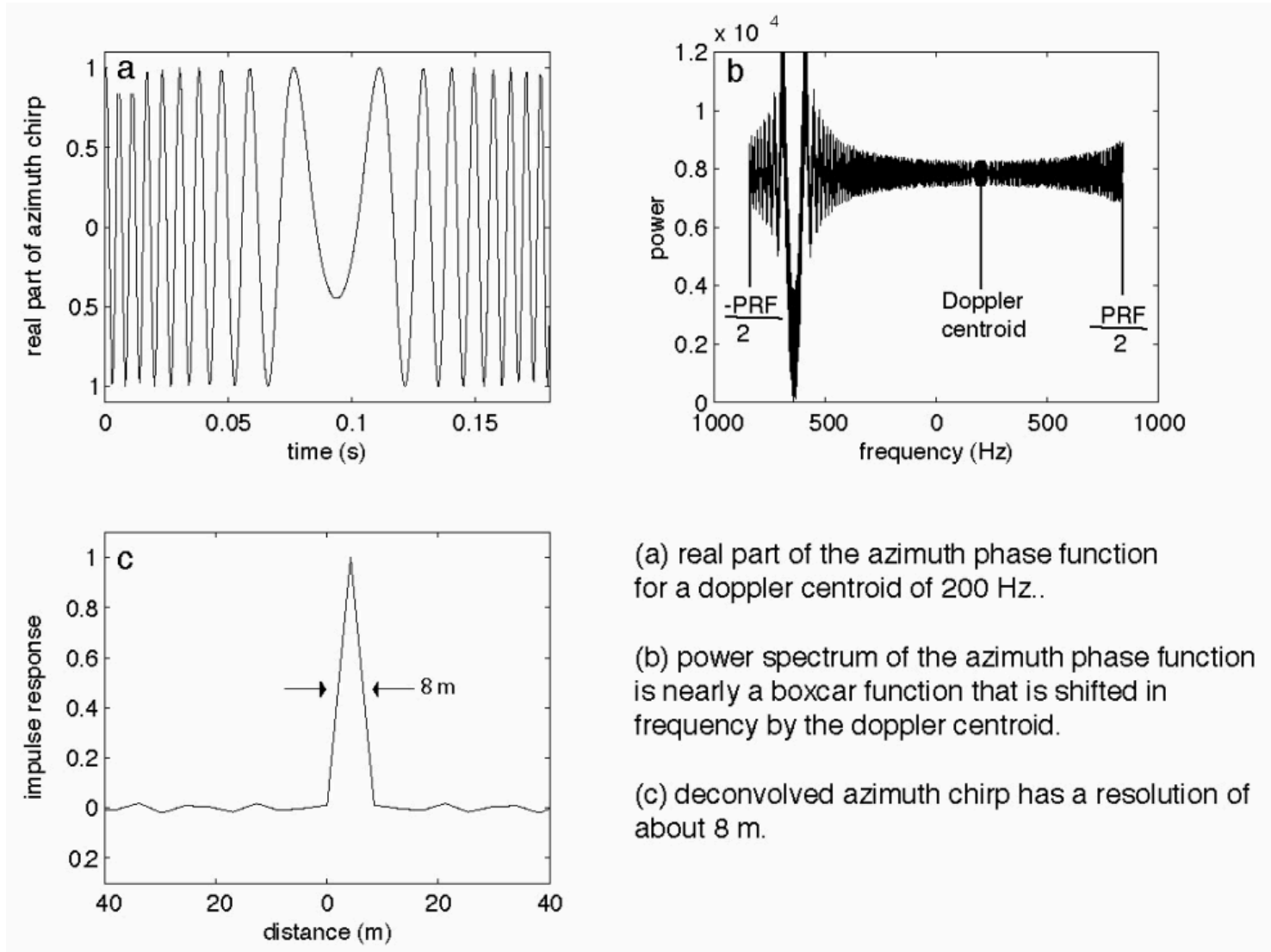
Least-squares fit of range history for each point in DEM provides both the accurate position in range azimuth  $[R_o, s_o]$  space and the Doppler centroid and rate parameters needed to focus the image. This analysis only needs to be applied to the master image.

$$C(s) = \exp \left\{ -i \frac{4\pi R_o}{\lambda} \right\} \exp \left\{ i 2\pi \left[ f_{DC} (s - s_o) + f_R (s - s_o)^2 / 2 \right] \right\}$$

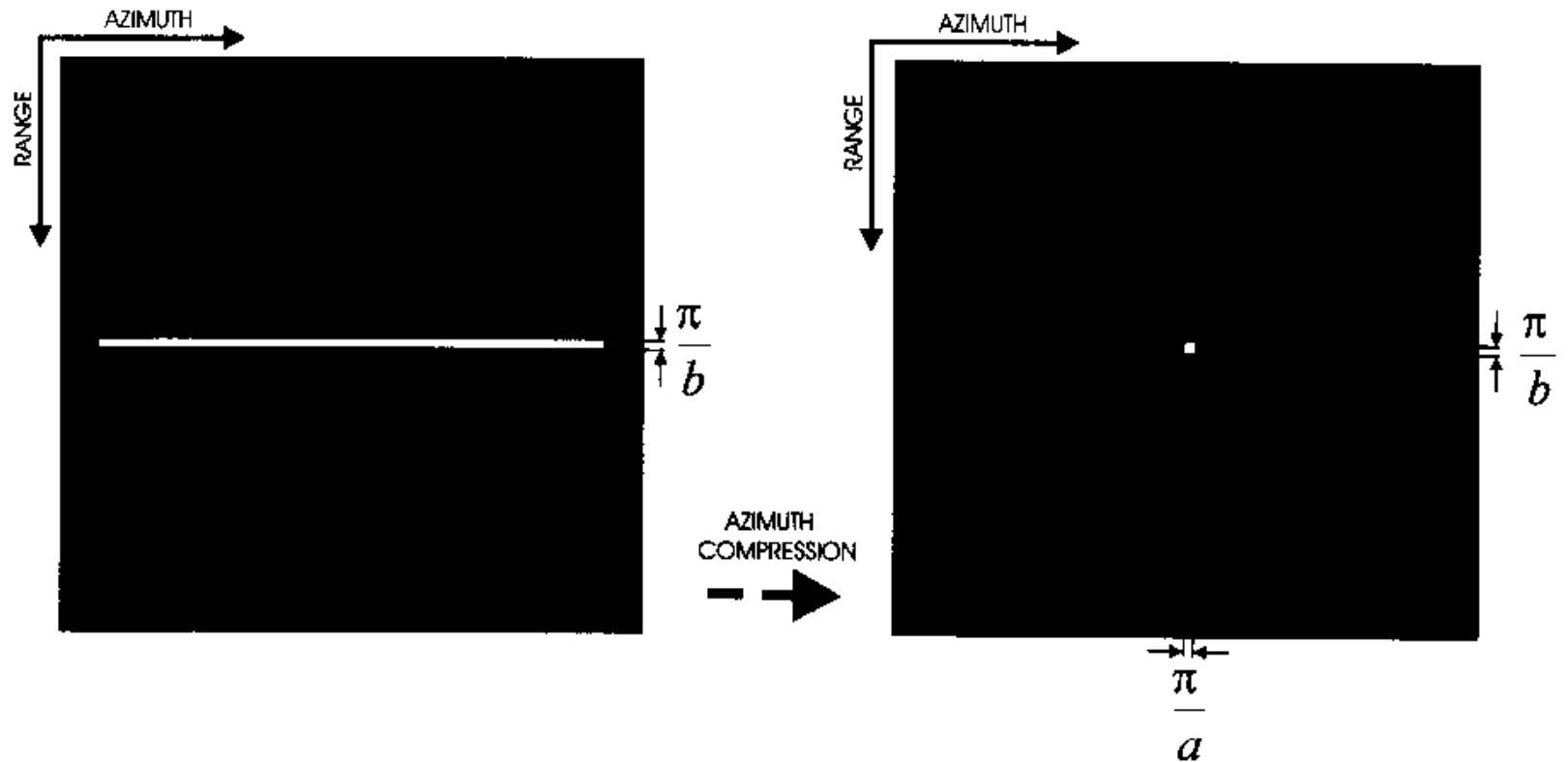
Orbit!!!

# Azimuth Compression

- De-convolve an azimuth chirp (Focusing in Azimuth)



# Example



**FIGURE 1C** Same as Figure 1A. Azimuth compression; normalized azimuth resolution is given by  $\pi/a$ .

# More Examples

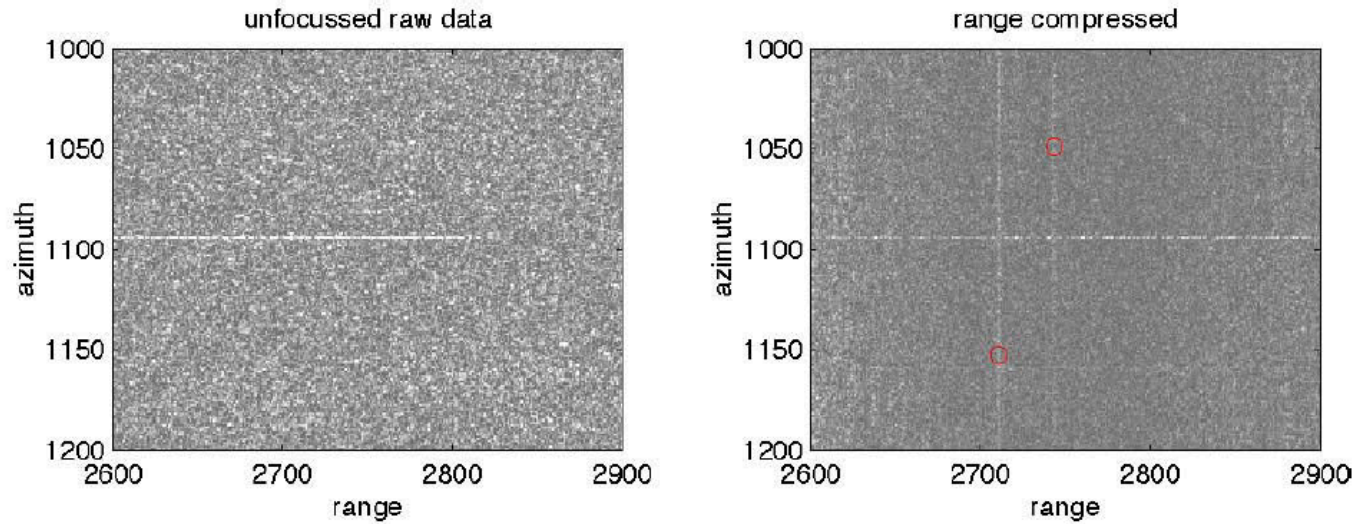


Figure B5. Raw ERS signal data (left) and range compressed data (right) for data over Pinon Flat California where two radar corner reflectors are installed (lower).

# More Examples

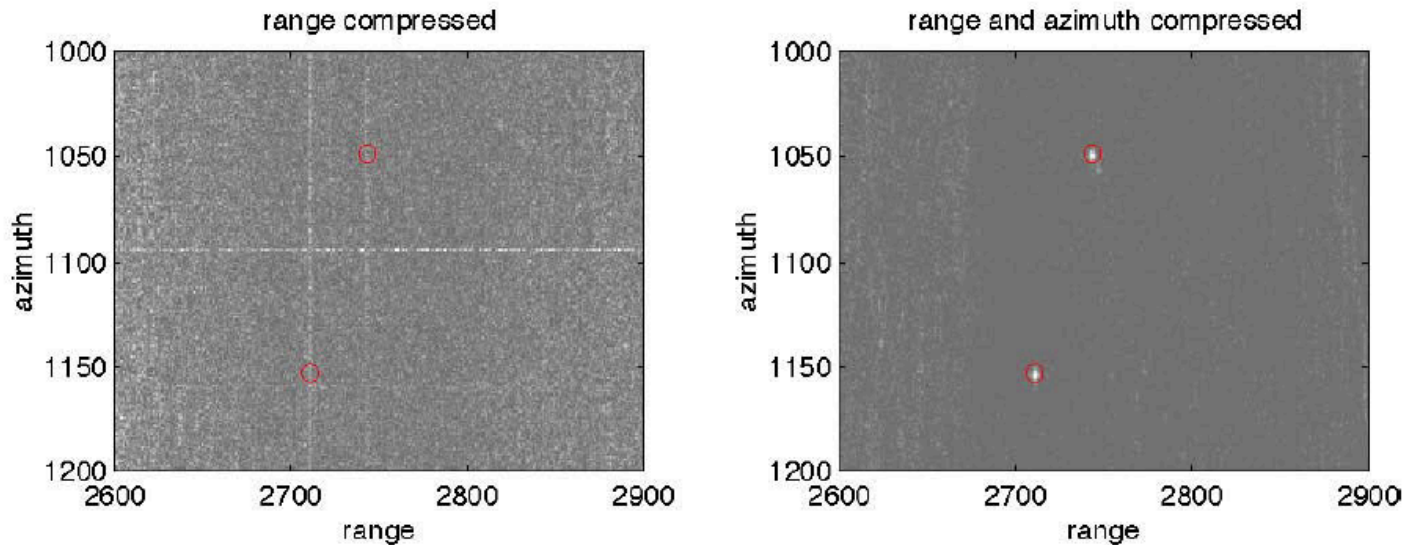
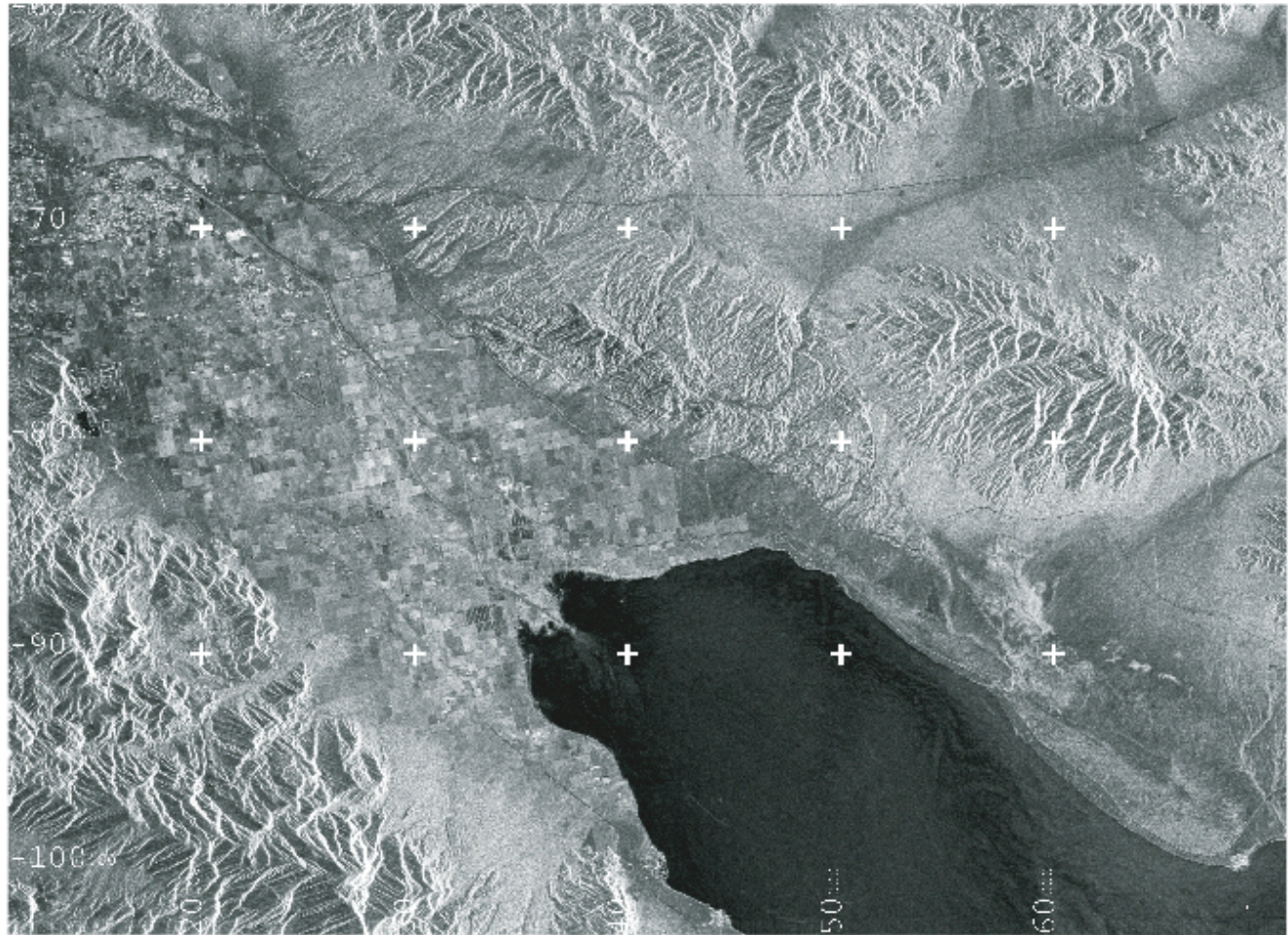


Figure B8. Range compressed (left) and fully focussed image (right). The two bright reflectors are 3-m corner cubes at Pinon Flat observatory. These were deployed in 1996 and therefore provide stable calibration points for ERS-1, ERS-2, Envisat, ALOS and all future satellites.

# More Examples



1) This is an image of radar backscatter from a stack of ERS SAR data. The flight path is top to bottom and the radar looks from the right. The area is the Salton Sea and Cochella Valley, and the tic marks are spaced at 10 km. The satellite is 7159717m from the center of the Earth, the local Earth radius is 6371593 m, and the range to the center of the image is 850148 m. Calculate the look angle to the center of the image. Identify areas of layover. What is the minimum mountain slope in the areas of layover? Why is the Salton Sea dark?

# Precise Orbit

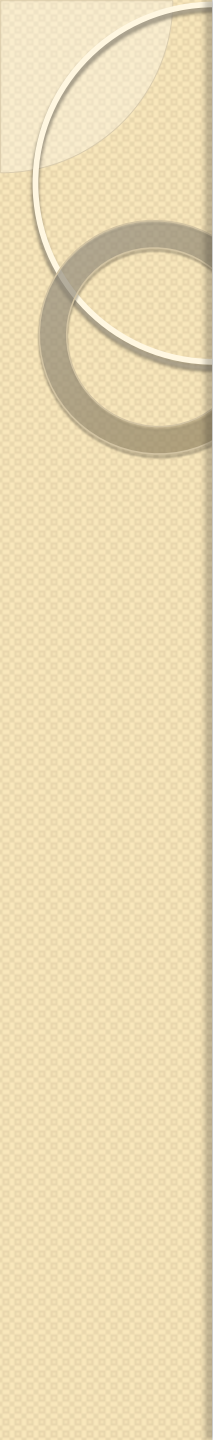
- Proper focus requires accurate estimates of Doppler centroid and Doppler rate.
- Transformation from geographic to radar coordinates without ground control.
- Accurate initial estimates for image alignment. TOPS requires geometric alignment to 1/200 pixel = 7 cm.
- InSAR baseline estimation – Removal of topographic curvature phase.

$$R(s) = R_o + \dot{R}_o(s - s_o) + \frac{\ddot{R}_o}{2}(s - s_o)^2 + \dots$$
$$f_{DC} = \frac{-2\dot{R}}{\lambda} \qquad f_R = \frac{2\ddot{R}}{\lambda}$$



Questions?





# HW2

## *Problems*

- 1) Explain why the raw signal data are provided as complex numbers. Why are the numbers restricted to the range 0-31?
- 2) Why is the SAR processing done in patches rather than all at once? What is the minimum possible patch size?
- 3) Most SAR instruments emit a frequency modulated chirp rather than a short pulse. Why? Write a Matlab program to deconvolve the FM chirp for ERS and reproduce the impulse response function shown in Figure B4.
- 4) Make a plot of the real and imaginary parts of the function given in (B5) for a time interval of -2 to 2 seconds. Use  $R_o = 850km$ ,  $\dot{R}_o = 0$ ,  $\ddot{R}_o = \frac{V^2}{R_o}$  where  $V = 7125ms^{-1}$ .
- 5) Derive equation (B12)
- 6) Derive equation (B13). (You may need to look back at Appendix A.)

# Demodulation and Digitization

- Carrier Frequency (GHz) & Band width (MHz)
- Hermitian Symmetry of Real Fourier Transformed signal
- Shift Theorem of Fourier Transform
- Real  $\rightarrow$  Complex

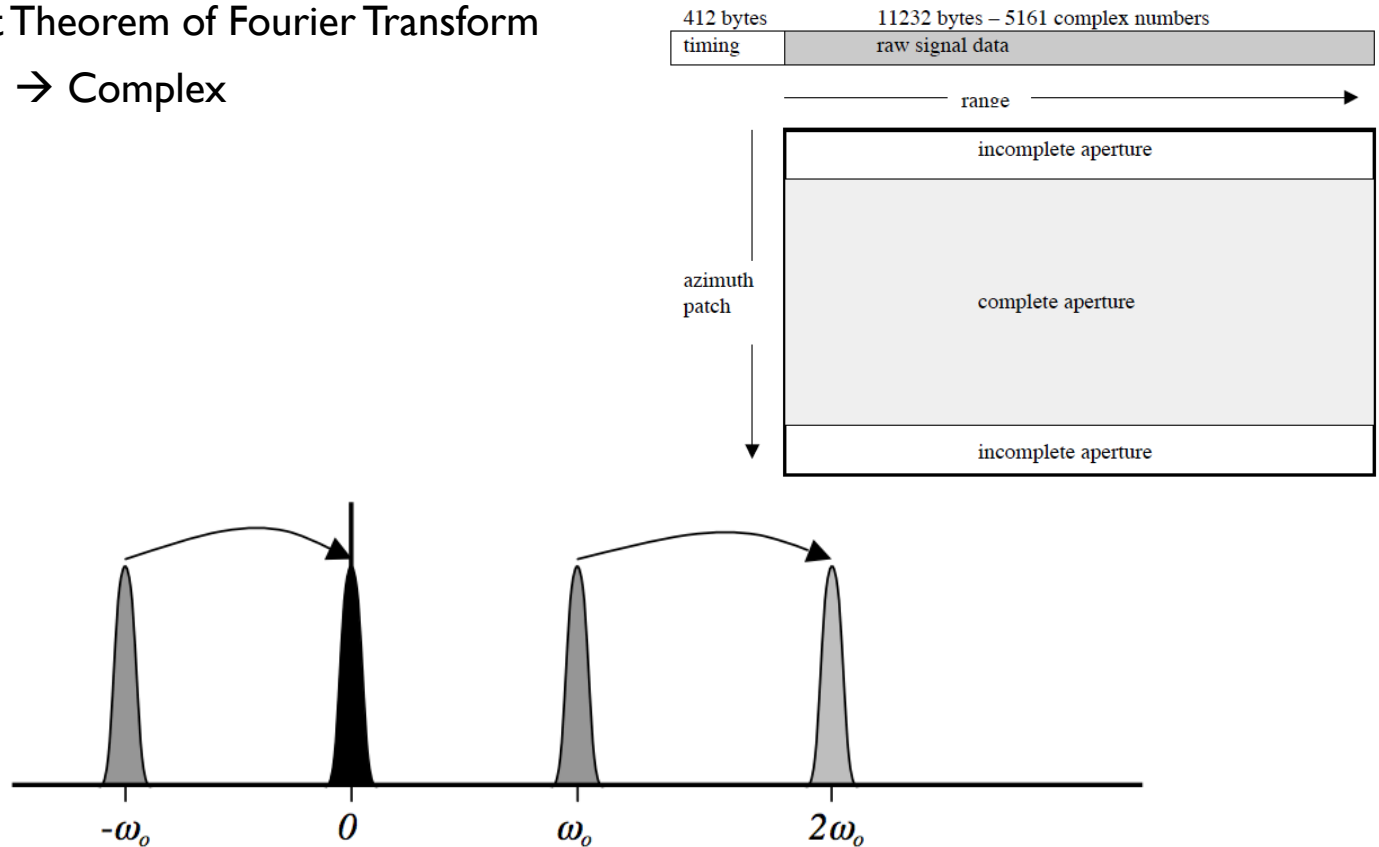


Figure B1. Diagram showing the power spectrum of the radar signal before and after shifting by the carrier frequency.