http://topex.ucsd.edu/gmtsar

Scripps Institution of Oceanography Technical Report

GMTSAR: An InSAR Processing System Based on Generic Mapping Tools (Second Edition)

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Table of Contents:

HW1

1) What is the illumination pattern for an aperture with a sign reversal at its center? What is $P(0)$? Is the function real or imaginary? Is the function symmetric or asymmetric?

The aperture is

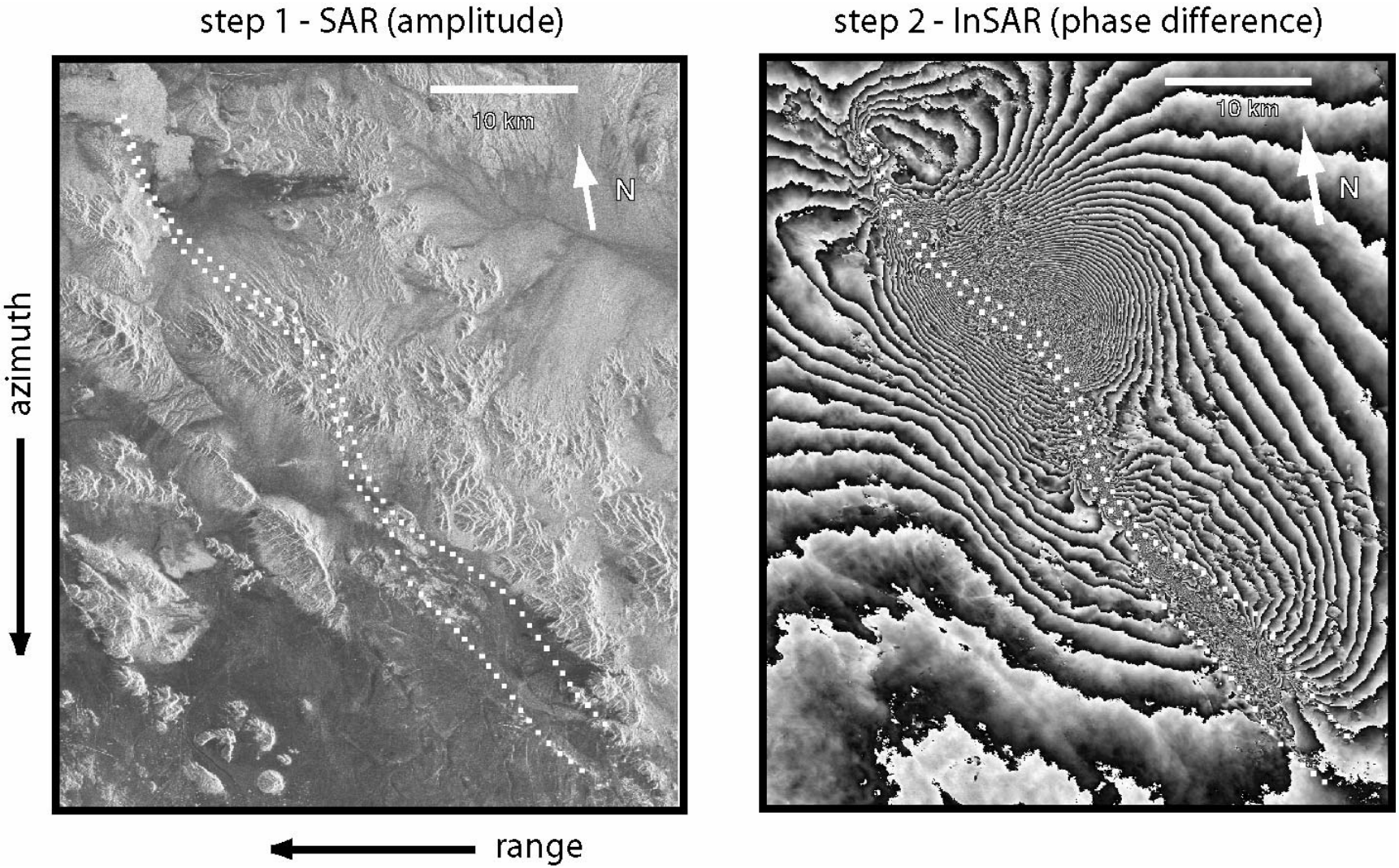
$$A(y) = \begin{cases} 0 & |y| > \frac{L}{2} \\ 1 & 0 < y \le \frac{L}{2} \\ -1 & -\frac{L}{2} \le y < 0 \end{cases}$$

2) What is the ground-range resolution of side-looking radar with a pulse length of $6\text{x}10^{-8}$ s and a look angle of 45°?

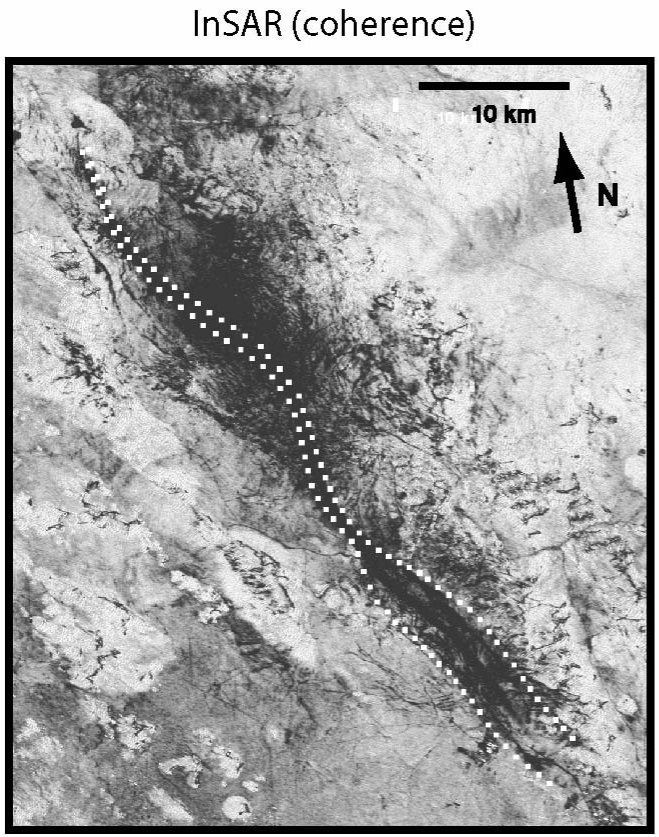
3) (a) What is the period for a satellite in a circular orbit about the moon where the radius of the orbit is $1.9\text{x}10^{6}$ m? The mass of the moon is $7.34\text{x}10^{22}$ kg. (b) You are developing a SAR mission for the moon. The length of your SAR antenna is 10 m. What minimum pulse repetition frequency is needed to form a complete aperture? The circumference of the moon is $1.1\text{x}10^{7}$m. You will need the orbital period from problem (a).

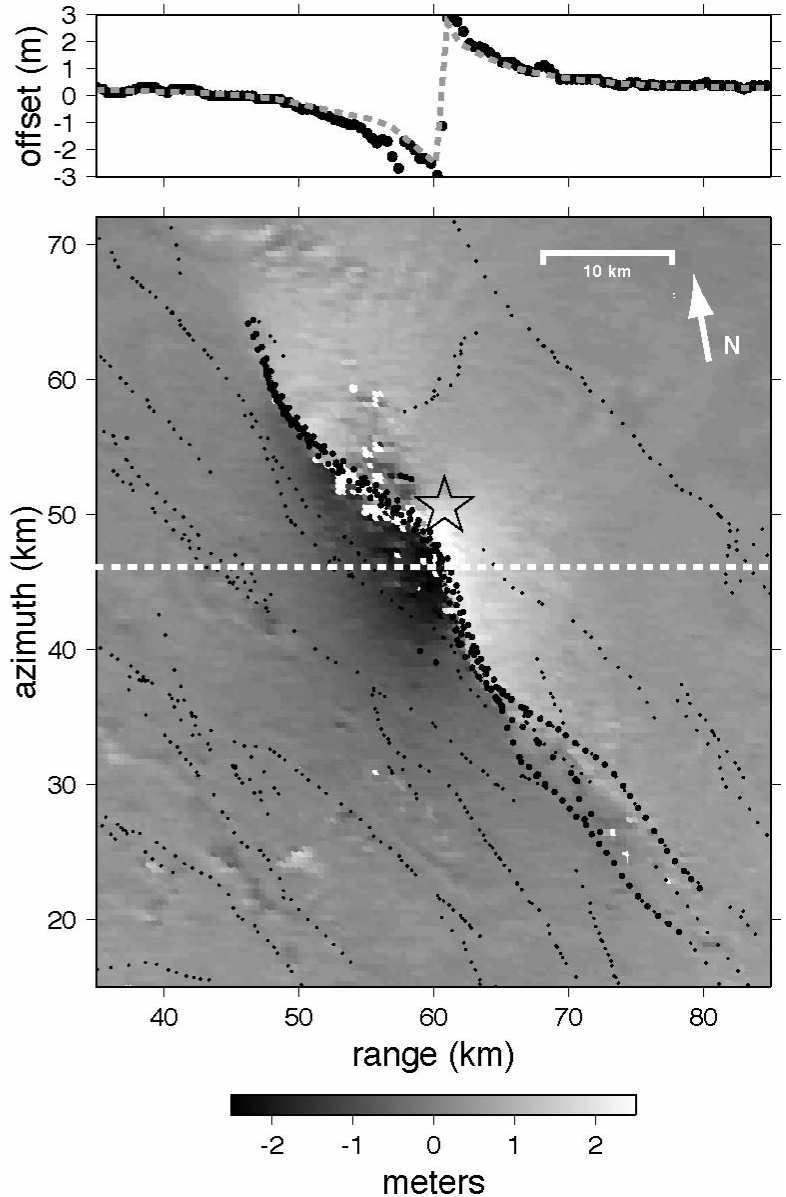
4) Derive equation (A10)

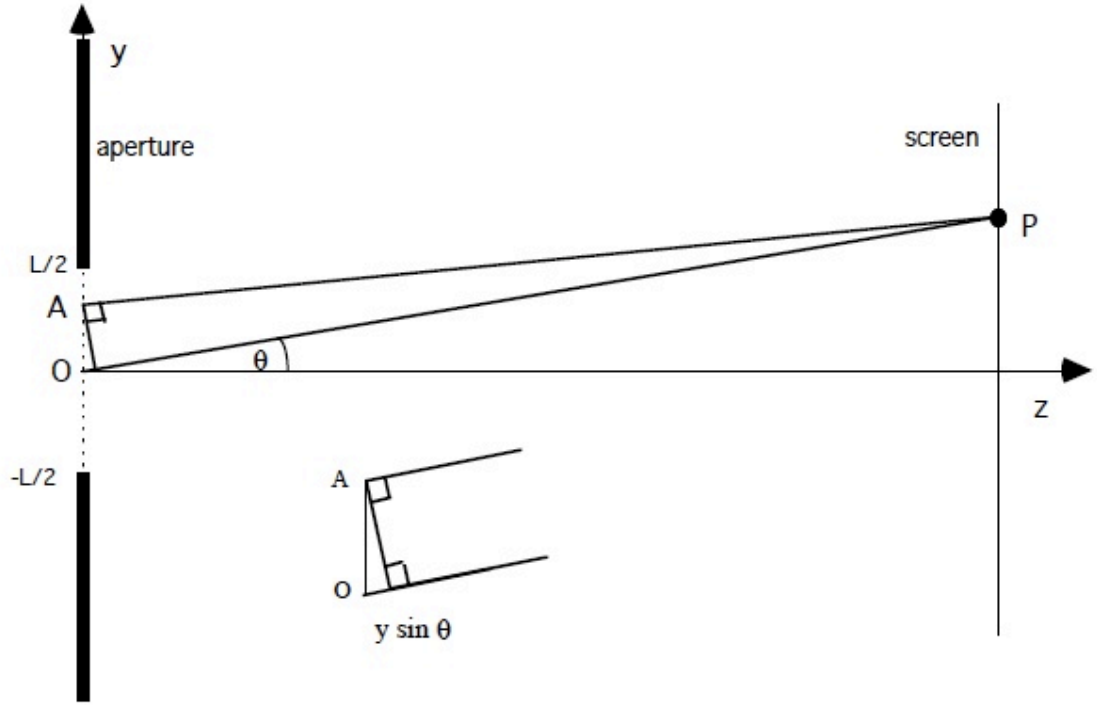
amplitude and phase



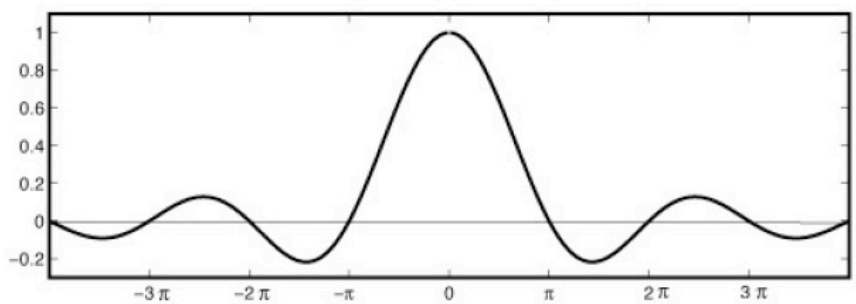
coherence and pixel matching







The illumination pattern on the screen is shown in the following diagram.



The first zero crossing, or angular resolution θ_r of the sinc function occurs when the argument is π so $\boxed{\sin\theta_r = \frac{\lambda}{L}}$ and for small angles $\theta_r \cong \lambda / L$ and $\tan\theta_r \cong \sin\theta_r$. Note that

Appendix A – Principles of Synthetic Aperture Radar

Fraunhoffer diffraction

To understand why a synthetic aperture is needed for microwave remote sensing from orbital altitude one must understand the concepts of diffraction and resolution. Consider the projection pattern of coherent radiation after it passes through an aperture (Figure A1). First we'll consider a 1-D aperture and then go on to a 2-D rectangular aperture to simulate a rectangular SAR antenna. The 2-D case provides the shape and dimension of the footprint of the radar. Although we will develop the resolution characteristics of apertures as transmitters of radiation, the resolution characteristics are exactly the same when the aperture is used to receive radiation. These notes were developed from *Rees* [2001] and *Bracewell* [1978].

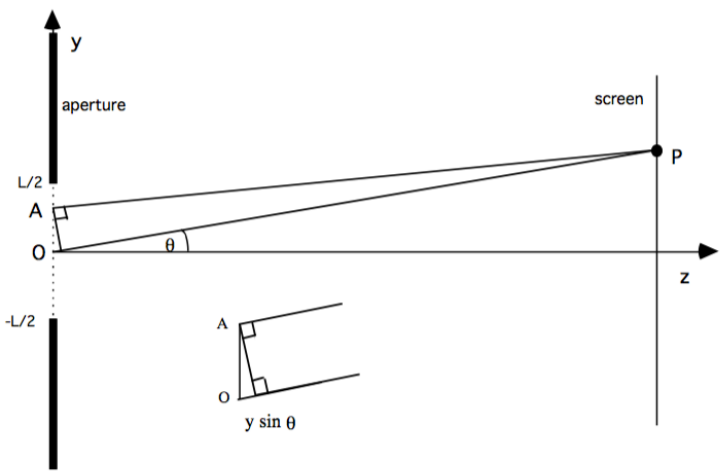


Figure A1 Diagram for the projection of coherent microwaves on a screen that is far from the aperture of length L.

We simulate coherent radiation by point sources of radiation distributed along the aperture between -*L*/2 and *L*/2. For simplicity we'll assume all the sources have the same amplitude, wavelength λ, and phase. Given these sources of radiation, we solve for the illumination pattern on the screen as a function of θ. We'll assume that the screen is far enough from the aperture so that rays AP and OP are parallel. Later we'll determine how far away the screen needs to be in order for this approximation to hold. Under these conditions, the ray AP is slightly shorter than the ray OP by an amount $-y\sin\theta$. This corresponds to a phase shift of $\frac{-2\pi}{\lambda}y\sin\theta$. The amplitude of the illumination at point *P* is the integral over all of the sources along the aperture multiplied by their complex phase value

$$P(\theta) = \int_{-L/2}^{L/2} A(y)e^{-i2\pi yk\sin\theta}dy \qquad \text{(A1)}$$

where $k = 1/\lambda$. This is called the Fraunhoffer diffraction integral. The illumination across the aperture is uniform in both amplitude and phase so we set $A(y) = 1$. Now let $s = 2\pi k\sin\theta$ so the fourier integral is easy to evaluate.

$$P(s) = \int_{-L/2}^{L/2} e^{-isy}dy = \frac{e^{-isL/2} - e^{isL/2}}{-is} = \frac{2}{s}\sin(sL/2) = L\,\mathrm{sinc}(sL/2) \qquad \text{(A2)}$$

Replacing s with $2\pi\sin\theta/\lambda$ we arrive at the final result.

$$P(\theta) = L\,\mathrm{sinc}\left(\frac{L\pi\sin\theta}{\lambda}\right) \qquad \text{(A3)}$$

The illumination pattern on the screen is shown in Figure A2.

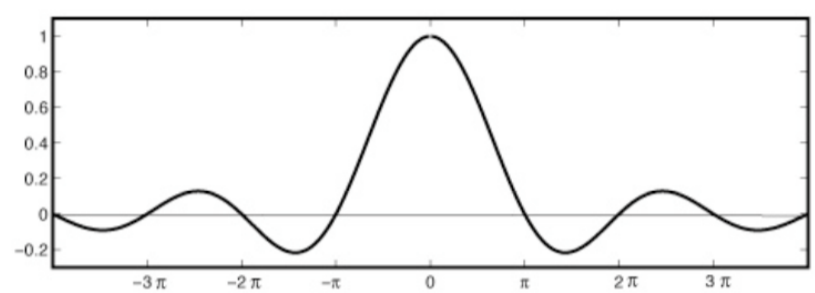
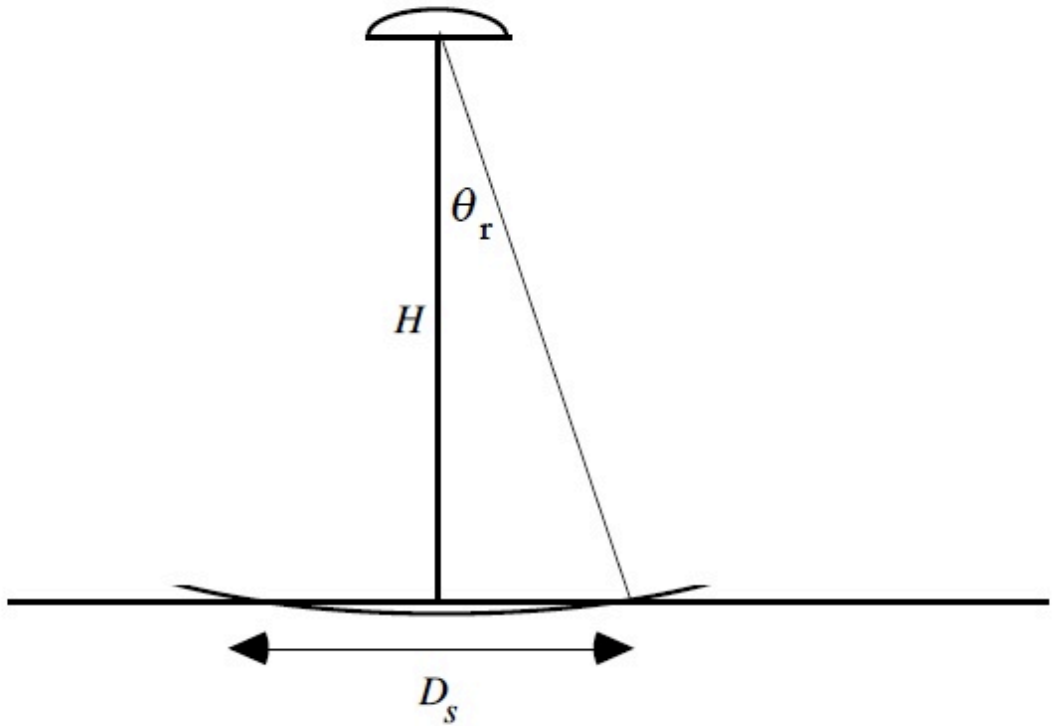


Figure A2. Sinc function illumination pattern for the aperture shown in Figure A1.

The first zero crossing, or angular resolution θ_r of the sinc function occurs when the argument is π so $\sin\theta_r = \frac{\lambda}{L}$ and for small angles $\theta_r \cong \lambda/L$ and $\tan\theta_r \cong \sin\theta_r$. Note that

resolution: optical vs. microwave



$$D_s = 2H\sin\theta_r = 2H\frac{\lambda}{L}$$

$$H = 800km.$$

$$Optical:$$

$$L = 1m$$

$$\lambda = 0.5\mu m$$

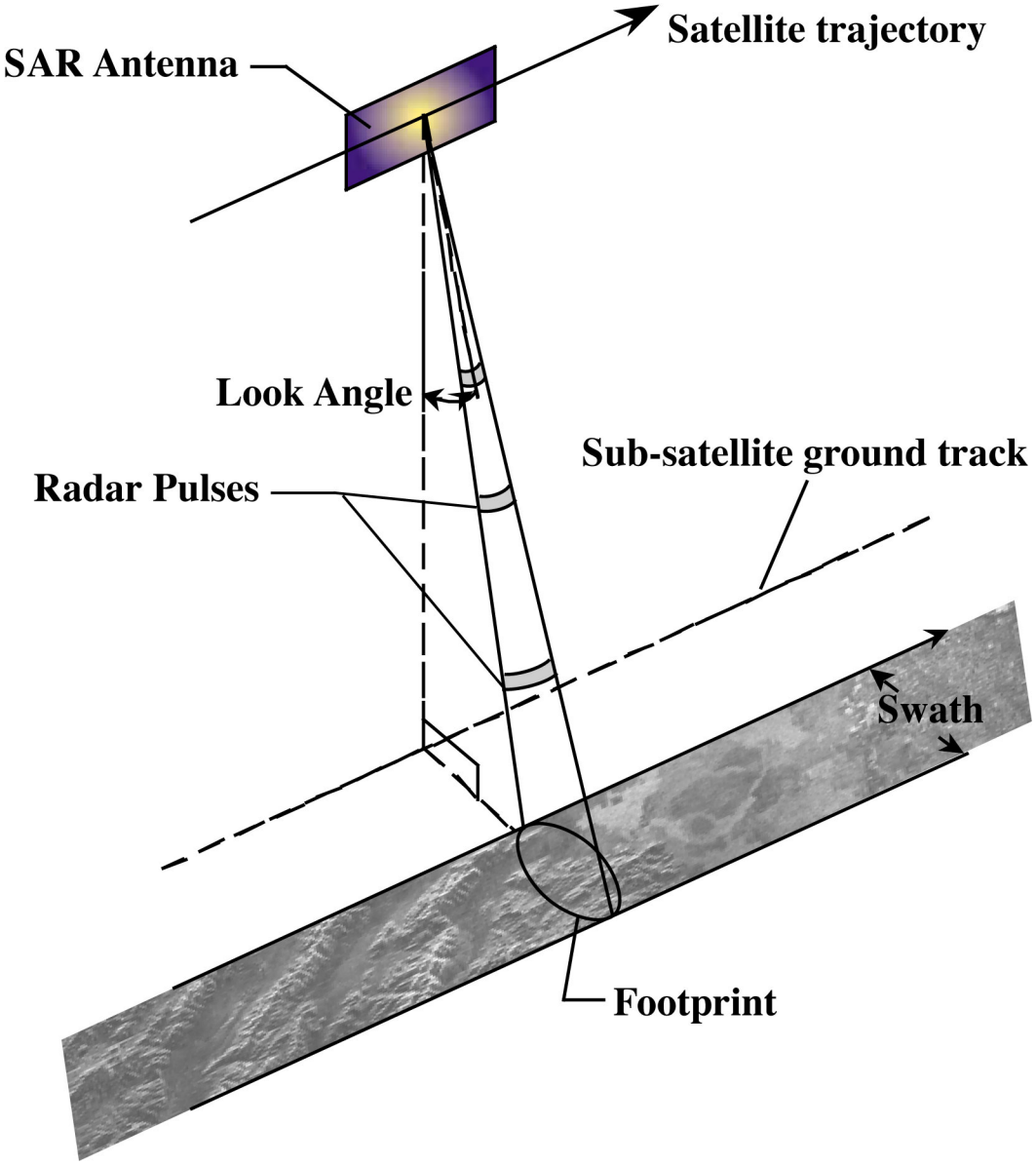
$$D_s = 0.8m$$

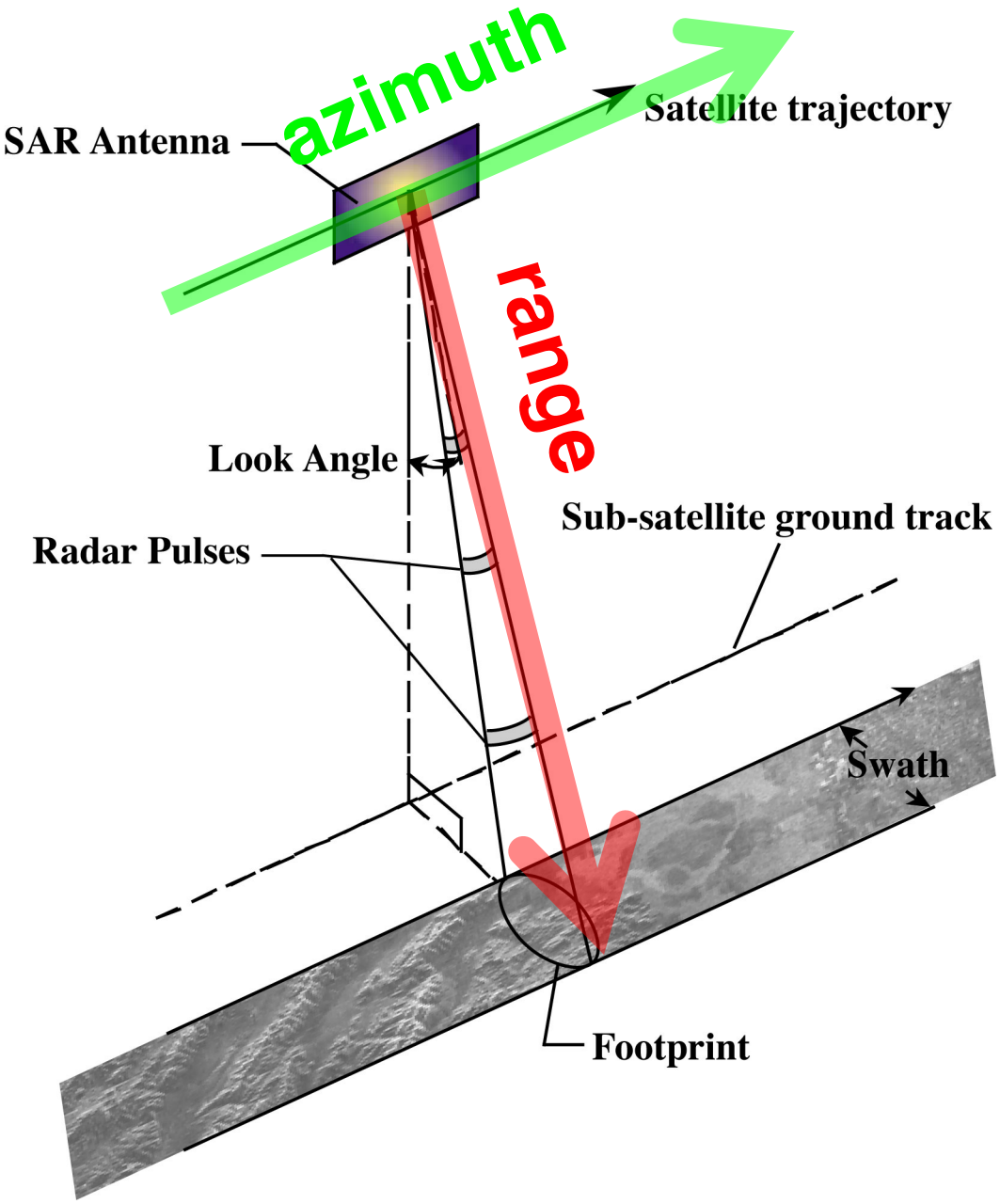
$$Microwave:$$

$$L = 10m$$

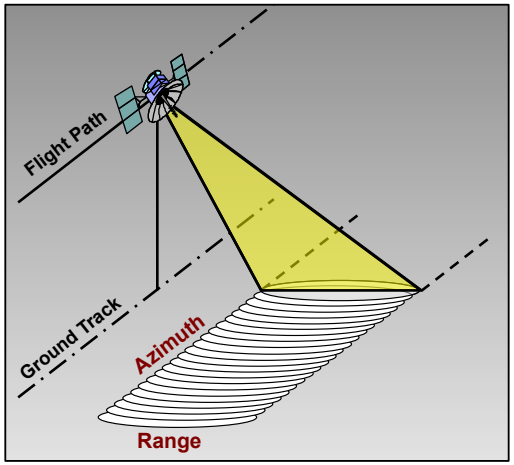
$$\lambda = 0.23m$$

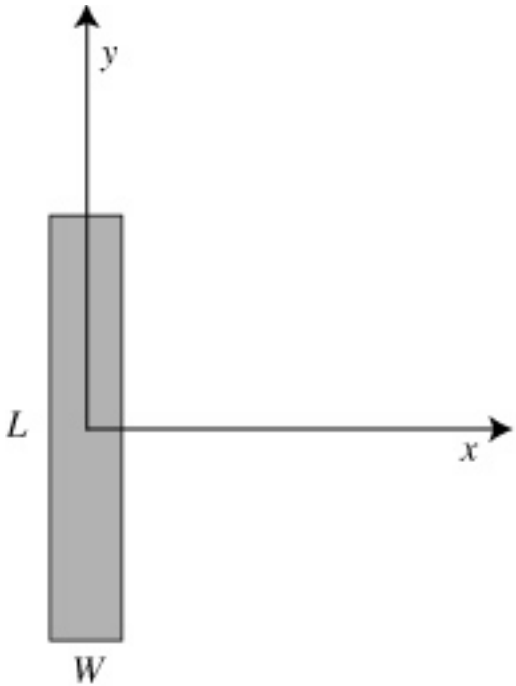
$$D_s = 46{,}000m!!!!!!$$

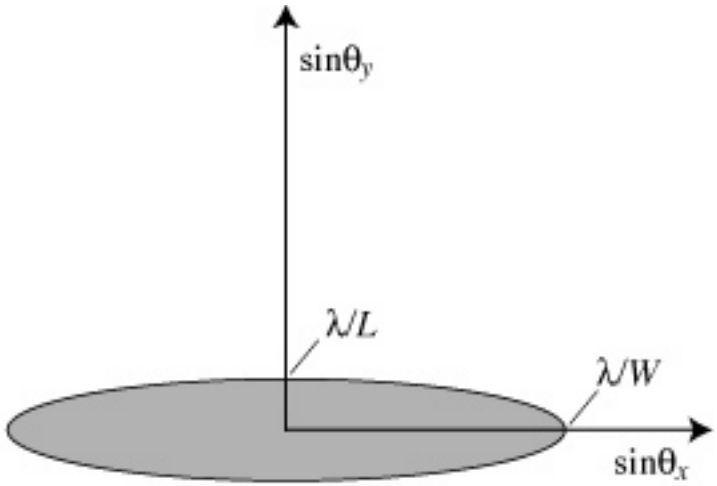

Satellite trajectory
SAR Antenna
Look Angle
Sub-satellite ground track
Radar Pulses
Swath
Footprint


azimuth
Satellite trajectory
SAR Antenna
range
Look Angle
Sub-satellite ground track
Radar Pulses
Swath
Footprint

2-D Aperture

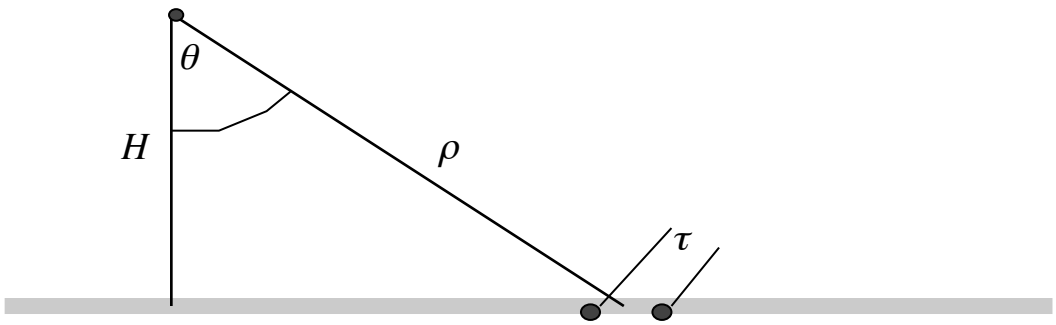






$$P(\theta_x,\theta_y) = \int_{-L/2}^{L/2}\int_{-W/2}^{W/2} A(x,y)\exp\left[i\frac{2\pi}{\lambda}\left(x\sin\theta_x + y\sin\theta_y\right)\right]dxdy \quad \longrightarrow \quad P(\theta_x,\theta_y) = LW\ \mathrm{sinc}\left(\frac{\pi W\sin\theta_x}{\lambda}\right)\ \mathrm{sinc}\left(\frac{\pi L\sin\theta_y}{\lambda}\right)$$

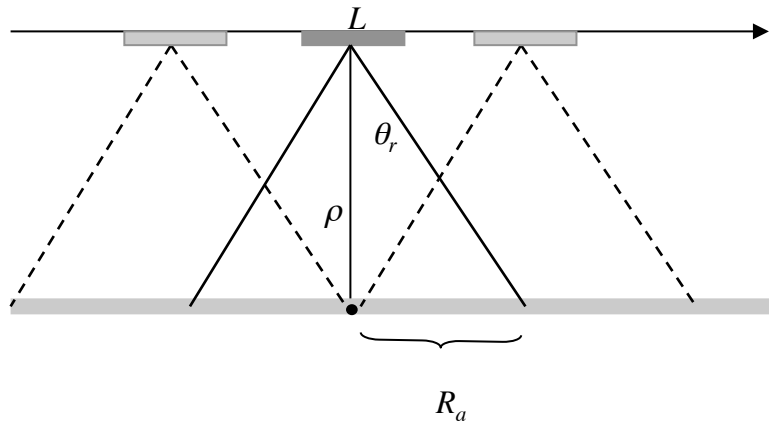
range resolution



θ - look angle
H - spacecraft height
τ - pulse length
C - speed of light (sound)

$$R_r = \frac{C\tau}{2\sin\theta}$$

azimuth resolution



L - length of radar antenna
ρ - nominal slant range $H/\cos\theta$
λ − wavelength of radar

unfocussed

$$R_a = \rho \sin\theta_r = \rho\lambda / L$$

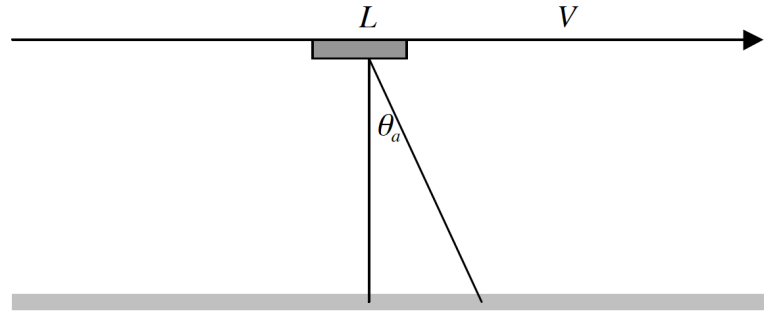
focussed

$$R_a^{'} = \frac{\lambda H}{2R_a \cos\theta} = \frac{L}{2}$$

Pulse Repetition Frequency

Minimum PRF (Lower Bound)

- PRF needs to be high enough to sample the entire Doppler spectrum to avoid aliasing
- PRF defines the Nyquist frequency
- Maximum Doppler shift must be less than the Nyquist

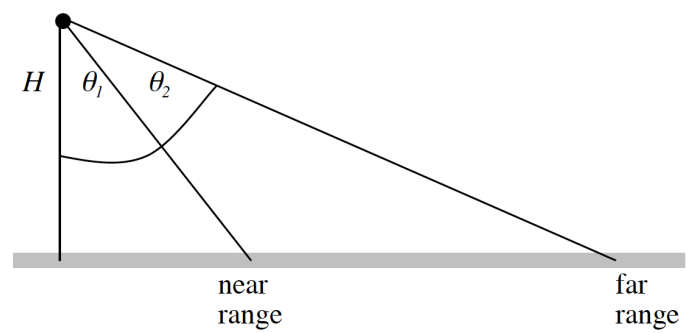


$$\frac{\Delta f}{f_0} = \frac{V \sin\theta_a}{c} \Rightarrow \Delta f = \frac{V}{L}$$

$$PRF > 2\Delta f = \frac{2V}{L}$$

Maximum PRF (Upper Bound)

- Echo from far range of first pulse must return before the echo from near range of second pulse



$$t_2 < t_1 + 1/PRF \Rightarrow$$

$$PRF < \frac{c}{2H}\left(\sec\theta_2 - \sec\theta_1\right)^{-1}$$

HW1

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