http://topex.ucsd.edu/gmtsar

Scripps Institution of Oceanography Technical Report

GMTSAR: An InSAR Processing System Based on Generic Mapping Tools
(Second Edition)

David Sandwell\textsuperscript{(1)}, Rob Mellors\textsuperscript{(2)}, Xiaopeng Tong\textsuperscript{(1,3)}, Xiaohua Xu\textsuperscript{(1)}, Matt Wei\textsuperscript{(4)}, and Paul Wessel\textsuperscript{(5)}

May 1, 2011
Revised – June 1, 2016
Table of Contents:

Abstract

1. Introduction
   1.1 Objectives and limitations of GMTSAR
   1.2 Algorithms: SAR, InSAR and the need for precise orbits
      1.2.1 Proper focus
      1.2.2 Transformation from geographic to radar coordinates
      1.2.3 Image alignment
      1.2.4 Flattening interferogram - no trend removal

2. Software
   2.1 Standard products
   2.2 Software design

3. Processing Examples
   3.1 Two-pass processing
   3.2 Stacking and time series
   3.3 ScanSAR Interferometry

4. References

Appendix A. Principles of Synthetic Aperture Radar
Appendix B. SAR Image Formation and PRM-file
Appendix C. InSAR Summary
Appendix D. ScanSAR processor and interferometry
Appendix E. Sentinel TOPS-mode processing and interferometry
Appendix F. Geolocation accuracy for Pinon corner reflectors
HW1

1) What is the illumination pattern for an aperture with a sign reversal at its center? What is $P(0)$? Is the function real or imaginary? Is the function symmetric or asymmetric?

The aperture is

$$A(y) = \begin{cases} 
0 & |y| > \frac{L}{2} \\
1 & 0 < y \leq \frac{L}{2} \\
-1 & -\frac{L}{2} \leq y < 0
\end{cases}$$

2) What is the ground-range resolution of side-looking radar with a pulse length of $6 \times 10^{-8}$ s and a look angle of 45°?

3) (a) What is the period for a satellite in a circular orbit about the moon where the radius of the orbit is $1.9 \times 10^6$ m? The mass of the moon is $7.34 \times 10^{22}$ kg. (b) You are developing a SAR mission for the moon. The length of your SAR antenna is 10 m. What minimum pulse repetition frequency is needed to form a complete aperture? The circumference of the moon is $1.1 \times 10^7$ m. You will need the orbital period from problem (a).

4) Derive equation (A10)
amplitude and phase

step 1 - SAR (amplitude)

step 2 - InSAR (phase difference)
coherence and pixel matching
The illumination pattern on the screen is shown in the following diagram.

The first zero crossing, or angular resolution \( \theta_r \), of the sinc function occurs when the argument is \( \pi \) so \( \sin \theta_r = \frac{\lambda}{L} \) and for small angles \( \theta_r \approx \frac{\lambda}{L} \) and \( \tan \theta_r \approx \sin \theta_r \). Note that
Appendix A – Principles of Synthetic Aperture Radar

Fraunhofer diffraction

To understand why a synthetic aperture is needed for microwave remote sensing from orbital altitude one must understand the concepts of diffraction and resolution. Consider the projection pattern of coherent radiation after it passes through an aperture (Figure A1). First we'll consider a 1-D aperture and then go on to a 2-D rectangular aperture to simulate a rectangular SAR antenna. The 2-D case provides the shape and dimension of the footprint of the radar. Although we will develop the resolution characteristics of apertures as transmitters of radiation, the resolution characteristics are exactly the same when the aperture is used to receive radiation. These notes were developed from Rees [2001] and Bracewell [1978].

Figure A1 Diagram for the projection of coherent microwaves on a screen that is far from the aperture of length L.

We simulate coherent radiation by point sources of radiation distributed along the aperture between -L/2 and L/2. For simplicity we'll assume all the sources have the same amplitude, wavelength \( \lambda \), and phase. Given these sources of radiation, we solve for the illumination pattern on the screen as a function of \( \theta \). We'll assume that the screen is far enough from the aperture so that rays AP and OP are parallel. Later we'll determine how far away the screen needs to be in order for this approximation to hold. Under these conditions, the ray AP is slightly shorter than the ray OP by an amount \(-y \sin \theta\). This corresponds to a phase shift of \(-\frac{2\pi}{\lambda} y \sin \theta\). The amplitude of the illumination at point \( P \) is the integral over all of the sources along the aperture multiplied by their complex phase value

\[
P(\theta) = \int_{-L/2}^{L/2} A(y)e^{-i2\pi k y \sin \theta} \, dy
\]

where \( k = 1/\lambda \). This is called the Fraunhofer diffraction integral. The illumination across the aperture is uniform in both amplitude and phase so we set \( A(y) = 1 \). Now let \( s = 2\pi k \sin \theta \) so the fourier integral is easy to evaluate.

\[
P(s) = \int_{-L/2}^{L/2} e^{-is} \, dy = \frac{e^{-isL/2} - e^{isL/2}}{-is} = 2\sin(sL/2) = L \sin c(sL/2)
\]

(A2)

Replacing \( s \) with \( 2\pi \sin \theta / \lambda \) we arrive at the final result.

\[
P(\theta) = L \sin c\left(\frac{2\pi \sin \theta}{\lambda}\right)
\]

(A3)

The illumination pattern on the screen is shown in Figure A2.

Figure A2. Sinc function illumination pattern for the aperture shown in Figure A1.

The first zero crossing, or angular resolution \( \theta_r \) of the sinc function occurs when the argument is \( \pi \) so \( \sin \theta_r = \frac{\lambda}{L} \) and for small angles \( \theta_r \approx \lambda/L \) and \( \tan \theta_r = \sin \theta_r \). Note that
resolution: optical vs. microwave

\[ D_s = 2H \sin \theta_r = 2H \frac{\lambda}{L} \]

\[ H = 800\text{km}. \]

*Optical:*
\[ L = 1m \]
\[ \lambda = 0.5\mu m \]
\[ D_s = 0.8m \]

*Microwave:*
\[ L = 10m \]
\[ \lambda = 0.23m \]
\[ D_s = 46,000m! \]
2-D Aperture

\[ P(\theta_x, \theta_y) = \int_{-L/2}^{L/2} \int_{-W/2}^{W/2} A(x, y) \exp \left[ i \frac{2\pi}{\lambda} (x \sin \theta_x + y \sin \theta_y) \right] dx dy \]

\[ P(\theta_x, \theta_y) = LW \ \text{sinc} \left( \frac{\pi W \sin \theta_x}{\lambda} \right) \ \text{sinc} \left( \frac{\pi L \sin \theta_y}{\lambda} \right) \]
range resolution

\[ R_r = \frac{C \tau}{2 \sin \theta} \]

- \( \theta \) - look angle
- \( H \) - spacecraft height
- \( \tau \) - pulse length
- \( C \) - speed of light (sound)
azimuth resolution

\[ L \quad - \quad \text{length of radar antenna} \]
\[ \rho \quad - \quad \text{nominal slant range} \quad \frac{H}{\cos \theta} \]
\[ \lambda \quad - \quad \text{wavelength of radar} \]

unfocussed

\[ R_a = \rho \sin \theta_r = \frac{\rho \lambda}{L} \]

focussed

\[ R_a' = \frac{\lambda H}{2R_a \cos \theta} = \frac{L}{2} \]
Pulse Repetition Frequency

Minimum PRF (Lower Bound)
- PRF needs to be high enough to sample the entire Doppler spectrum to avoid aliasing
- PRF defines the Nyquist frequency
- Maximum Doppler shift must be less than the Nyquist

Maximum PRF (Upper Bound)
- Echo from far range of first pulse must return before the echo from near range of second pulse

\[
\Delta f = \frac{V \sin \theta_a}{c} \Rightarrow \Delta f = \frac{V}{L}
\]

\[
PRF > 2\Delta f = \frac{2V}{L}
\]

\[
t_2 < t_1 + \frac{1}{PRF} \Rightarrow PRF < \frac{c}{2H} \left( \sec \theta_2 - \sec \theta_1 \right)^{-1}
\]
HW1

1) What is the illumination pattern for an aperture with a sign reversal at its center? What is $P(0)$? Is the function real or imaginary? Is the function symmetric or asymmetric?

The aperture is

$$A(y) = \begin{cases} 
0 & \text{if } |y| > \frac{L}{2} \\
1 & 0 < y \leq \frac{L}{2} \\
-1 & -\frac{L}{2} \leq y < 0
\end{cases}$$

2) What is the ground-range resolution of side-looking radar with a pulse length of $6 \times 10^{-8}$ s and a look angle of 45°?

3) (a) What is the period for a satellite in a circular orbit about the moon where the radius of the orbit is $1.9 \times 10^6$ m? The mass of the moon is $7.34 \times 10^{22}$ kg. (b) You are developing a SAR mission for the moon. The length of your SAR antenna is 10 m. What minimum pulse repetition frequency is needed to form a complete aperture? The circumference of the moon is $1.1 \times 10^7$ m. You will need the orbital period from problem (a).

4) Derive equation (A10)
Table of Contents:

Abstract

1. Introduction
   1.1 Objectives and limitations of GMTSAR
   1.2 Algorithms: SAR, InSAR and the need for precise orbits
      1.2.1 Proper focus
      1.2.2 Transformation from geographic to radar coordinates
      1.2.3 Image alignment
      1.2.4 Flattening interferogram - no trend removal

2. Software
   2.1 Standard products
   2.2 Software design

3. Processing Examples
   3.1 Two-pass processing
   3.2 Stacking and time series
   3.3 ScanSAR Interferometry

4. References

5. Problems

Appendix A. Principles of Synthetic Aperture Radar
Appendix B. SAR Image Formation
Appendix C. InSAR
Appendix D. ScanSAR processor and interferometry
Appendix E. Geolocation accuracy for Pinon corner reflectors
Appendix F. Installation of GMTSAR