Theory and practice of phase unwrapping

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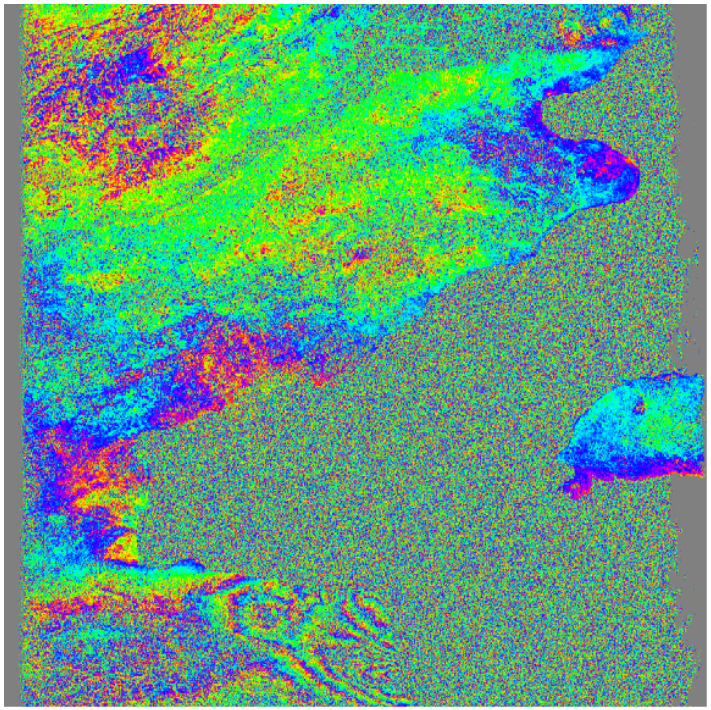


- What is phase unwrapping?
- Ways to unwrap.
- using SNAPHU in GMTSAR
- Reduce or avoid the problem

*or what to do when your interferogram looks like this.
(and what the code is really doing..)

Overview of unwrapping

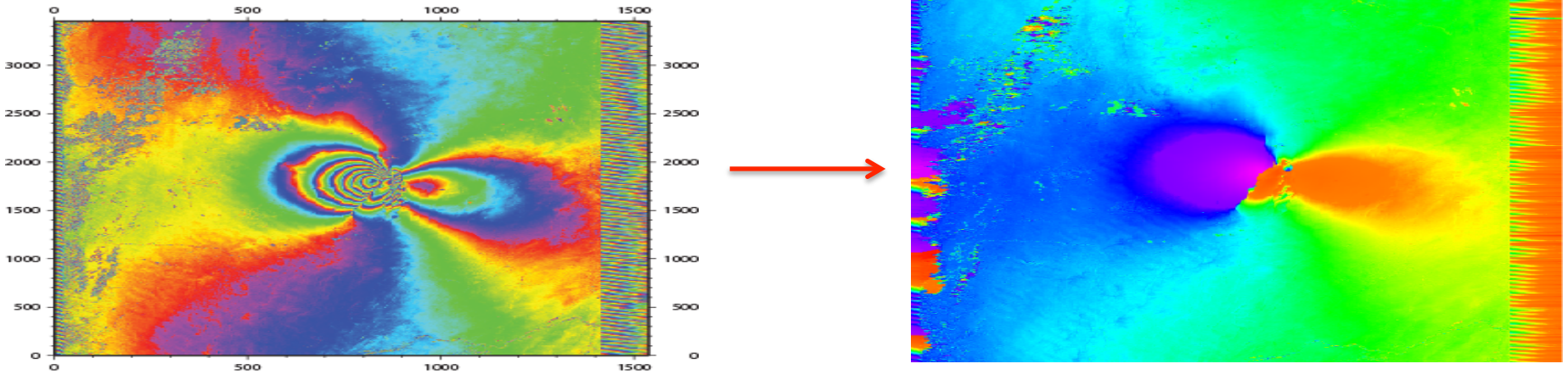
Given an interferogram(s)

- usually need to convert phase to useful units
- we know radar wavelength and geometry

Usually requires unwrapping

- unwrapping not always easy

Saudi 7/1/08-8/19/08 ALOS Bperp 20 m



Extracting the phase

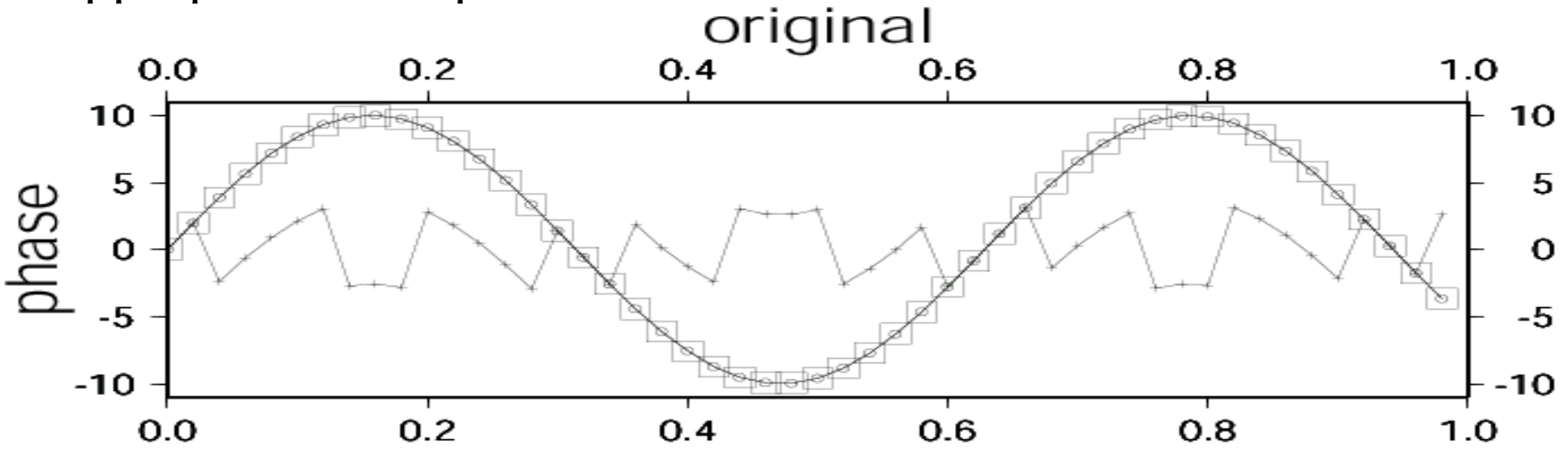
We can get only the wrapped phase*

$$\Phi(t) = \arctan(I(s(t)),R(s(t)))$$

where $-\pi < \Phi(t) \leq \pi$

We would like the continuous phase.

This appears simple. Look for 2π jumps and then add the appropriate multiple of 2π.



If the data are good, phase unwrapping is straightforward

- We could take derivative in complex version (e.g. Sandwell & Price)

Phase unwrapping

Assume that we have two signals taken at different times:

$g_1 = a_1 \exp(i4\pi R_1/\lambda)$

$g_2 = a_2 \exp(i4\pi R_2/\lambda)$

a1,a2 = amplitude

R_1, R_2 is range from antenna to surface

λ = wavelength

At a given point, assume $a_1 = a_2 = a$

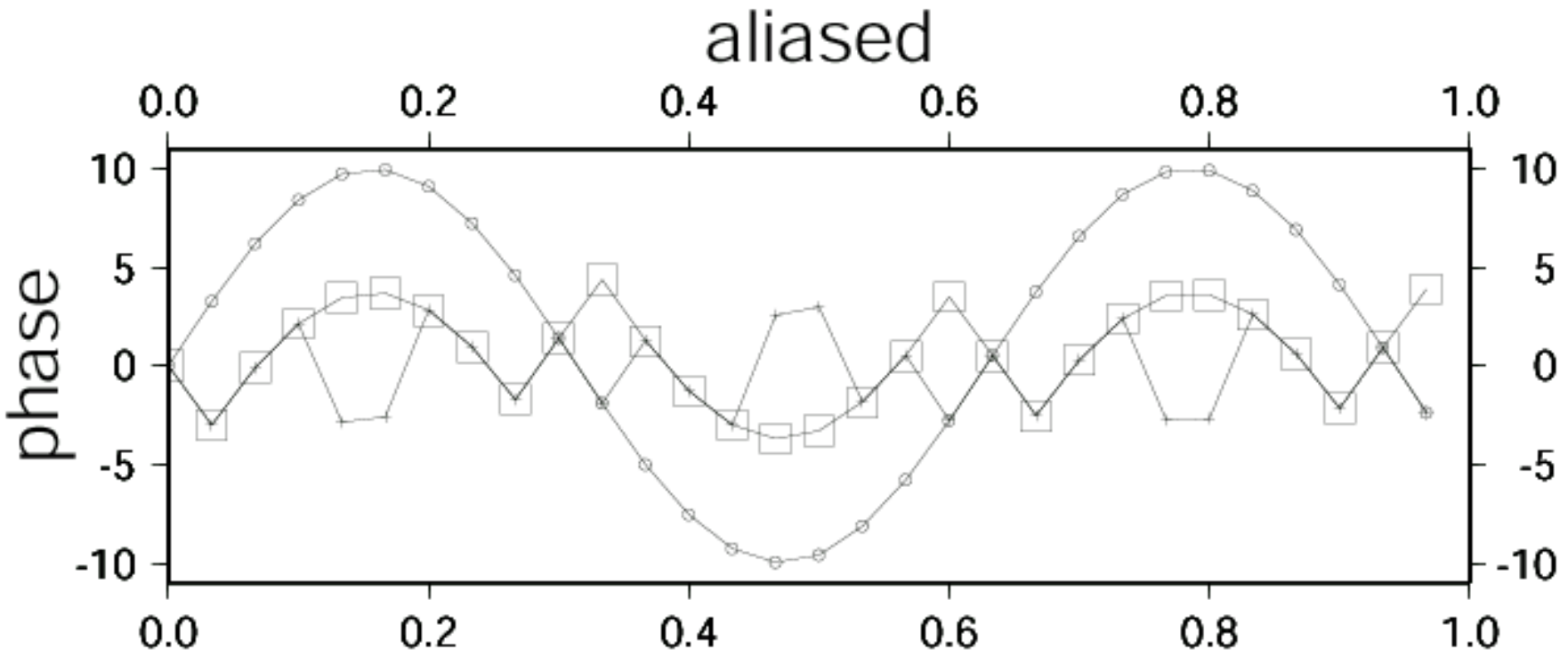
$(g1)(g2^*) = (a^2)\exp[i4\pi(R_1-R_2)] = s(t)$

The phase of this function is proportional to the effective difference in range, which in turn depends on satellite geometry, topography, atmosphere, soil moisture, or maybe even *deformation*.

Problem number 1: aliasing

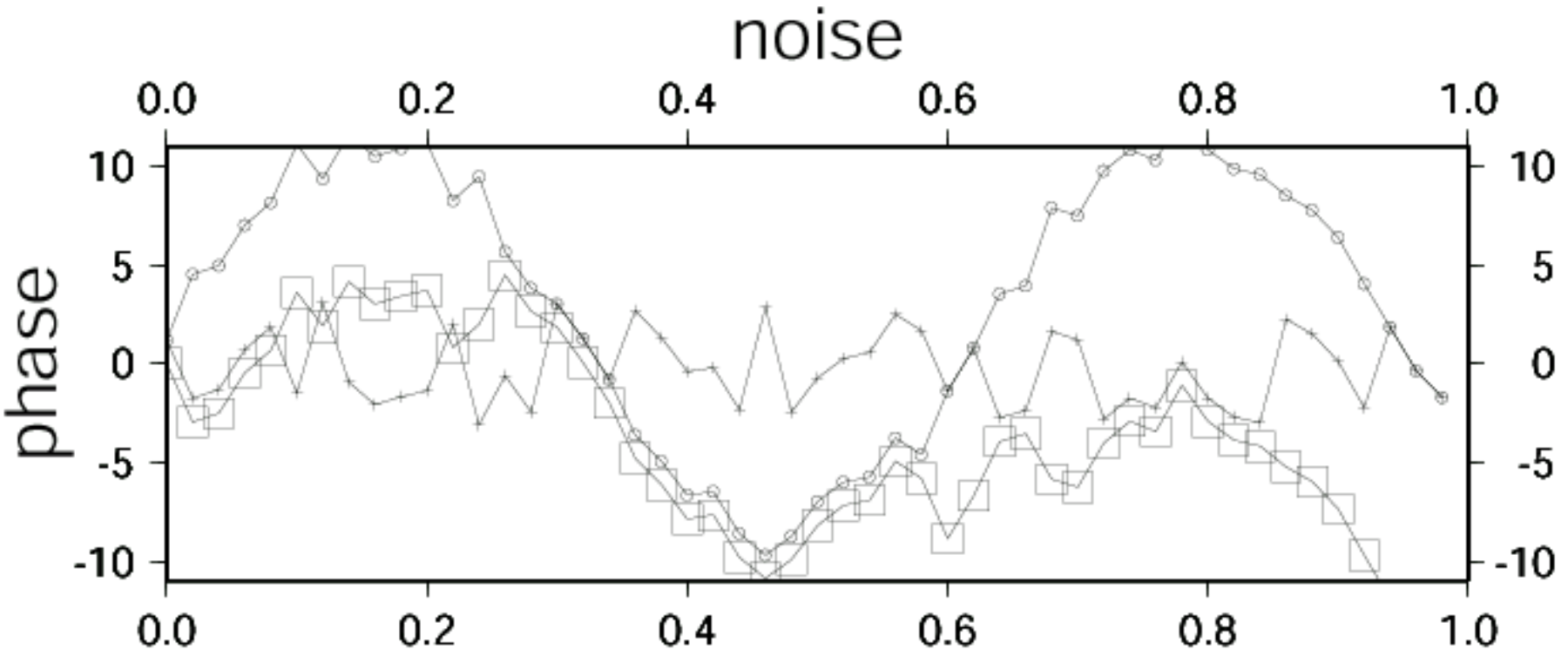
True phase changes by more than 1 cycle (2π radians) between samples.
Caused by:

- by large orbital separation
- steep topography
- large deformation (steep phase gradient)



Problem number 2:
Noise and/or gaps in the data

Changes on the surface (e.g., vegetation, snow, erosion) may cause the two images to de-correlate, introducing noise.



For images (and 3D) data the same problems exist but are more complicated

How to unwrap?
Want to find a function that when wrapped, is “close” to the observed data (whatever “close” is).

Two basic approaches:

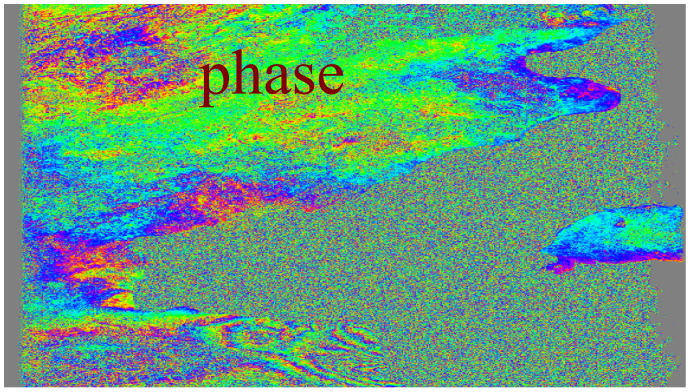
- Global methods that attempt to unwrap all pixels simultaneously.

- Local methods that solve along a path.

Note that the correlation data allows some estimation of how good the phase data is.

Identifying and resolving problems

1) Mask out areas of poor data.
    1) Use correlation as a guide (mask out all pixels with correlation less than a threshold value)
    2) Identify points of inconsistent data (residues)
2) Filter to reduce problems.
3) Use model to subtract expected phase.



Filtering

objective: improve signal-to-noise of fringes prior to unwrapping

static

- usually lowpass
- convolve with set of filter coefficients (boxcar, Gaussian, etc)

adaptive

-*Goldstein and Werner* [1998] spectral filter.
- effective but "can significantly change the structure of the interferogram"[*Baran et al*, 2003]

adaptive filter

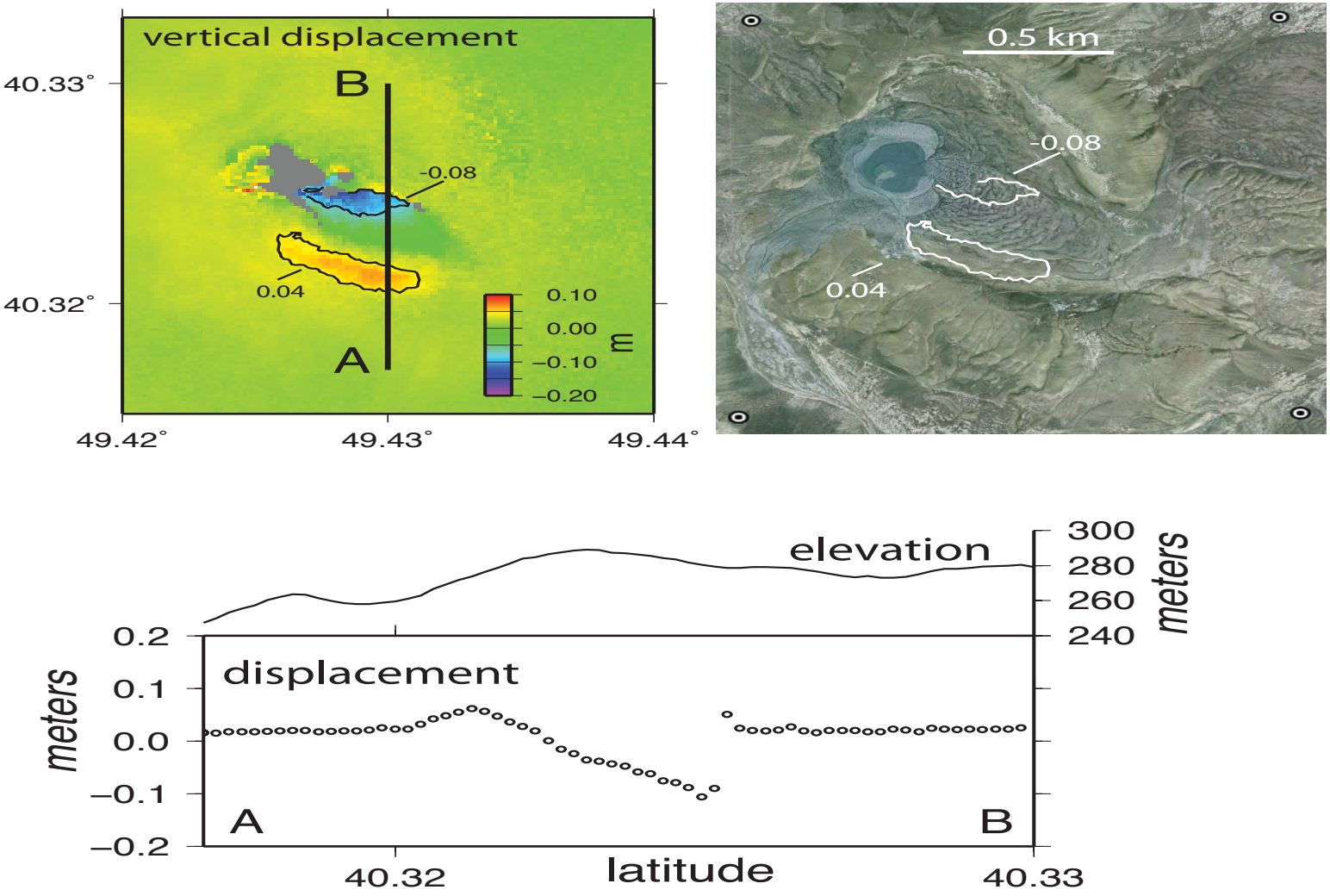
- filter parameter vary depending of properties of each patch

$$H(u,v) = (S\{|Z(u,v)|\}^{a})(Z(u,v))$$

Z(u,v) Fourier spectrum of small 2D patch of complex interferogram
(perhaps 32 by 32 pixels)

S smoothing (e.g. 3 by 3 pixels)
H output
α exponent (a = 0, no filter, a > 0 filters)

-take Fourier of small patch
-raise spectrum to power
- inverse FFT
-results depends on noise and phase!
- *Baran et al* [2003] use $\alpha = 1 - \gamma$ where γ is the coherence



An example of a signal that was dramatically changed by filtering. Spatially small signal with discontinuity.

Global

We want to find the function whose local derivatives "match" the observed derivatives given some measure :

$$e^p = [(\Phi_{i+1,k} - \Phi_{i,k}) - \Delta^x_{i,k}]^p + [(\Phi_{ik+1} - \Phi_{ik}) - \Delta^y_{ik})]^p$$

P = exponent

Φ = unknown function

Δ = derivatives of the observed phase (can calculate from the complex phase).

For p = 2, this is equivalent to the discrete version of Poisson's Equation

Two basic ways to solve:

Transform : FFT, DCT (discrete cosine transform-be careful with b.c.'s)

Matrix (will allow weighting but now nonlinear and requires iteration)

Can vary the exponent (i.e. don't have to use 2)

Transform-based methods

Fast, but do not allow weighting
FFT requires periodic conditions and extension of data.
Apply 2D Fourier transform:

$$\rho_{i,k} = (\Delta^x_{i,k} - \Delta^x_{i-1,k}) + (\Delta^y_{i,k} - \Delta^y_{i,k-1})$$

$$\Phi_{m,n} = \frac{P_{m,n}}{2\cos(\pi m / M) + 2\cos(\pi n / N) - 4}$$

Φ = Fourier transform of ϕ
P = Fourier transform of ρ

1. Calculate the $\rho_{i,k}$ from the data
2. Calculate the 2D FFT of $\rho_{i,k}$
3. Calculate $\Phi_{m,n}$ from the transformed $\rho_{i,k}$
4. Do inverse FFT

Local (path following)

Similar to the 1D approach

1.) Calculate the differences of the wrapped phase.

2.) Wrap the differences.

3.) Set the value of the first value.

3.) Integrate along all values.

Do this along a line throughout 2D area (in a zigzag back and forth along the rows, for example)
Works great if there is no noise.
With noise:

1) An error near the start of the path propagates along the whole path.

2) Answer may vary with path.

3) Need to identify bad pixels. How?

Identifying inconsistent data

Phase should reflect the topography.
We know that topographic surfaces are conservative

- Any points that violate this rule should be avoided.
- These points are known as residues.
- Any integration path that circles a residue will contain errors

    => need to make “branch cuts”

- A residue is a property of phase differences, not a single pixel.
- can be positive or negative
- Can be identified by examining groups of pixels

Wrapped differences should sum to zero around a loop.
If sum is not zero, then identify the loop as a _residue._

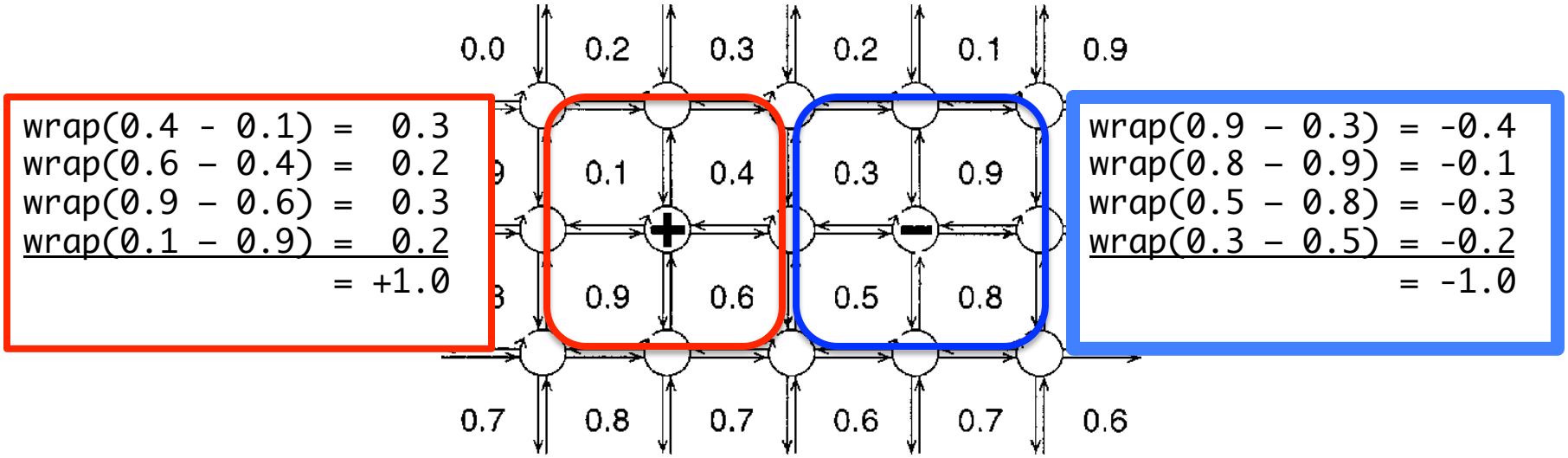


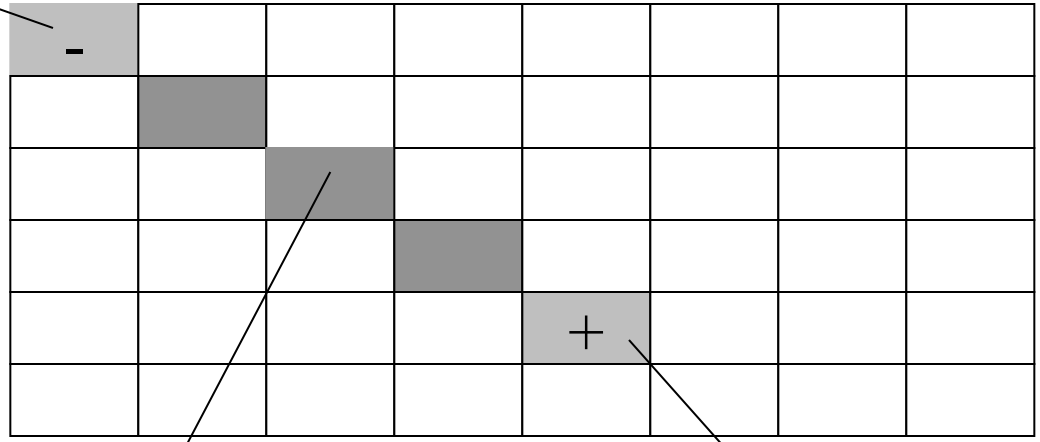
Fig. 1. Example network equivalent of the phase unwrapping problem. The numbers represent the 2-D array of phase samples (normalized to one cycle). Each 2 × 2 clockwise loop integral of wrapped phase gradients is a node in the network, and positive and negative residues result in supply and demand nodes. Neighboring nodes are connected by arcs, or possible flow paths. The amount of flow on an arc represents the difference (in cycles) between the unwrapped and the wrapped phase gradients associated with that arc. The net amount of flow out of a node must be equal to the node's surplus.

Chen, C. W., and H. A. Zebker (2000), Network approaches to two-dimensional phase unwrapping: intractability and two new algorithms, *J. Opt. Soc. Am. A, 17*, 401-414.

Path following with masking and residues

1.) Calculate correlation for phase data.
2.) Mask out all areas with correlation less than a certain threshold value.
3.) Go through all pixels and identify residue locations (upper left of 4 pixels).
4) Start with first residue, look for nearest residue. Draw a “line” of marked pixels between the two.
- if residues cancel, go to next residue and start new “tree”
- otherwise, look for next nearest and draw line
- can also “cancel” by connecting to edge.
- connected lines are called a tree.
5) path-integrate along remaining pixels.

Negative residue

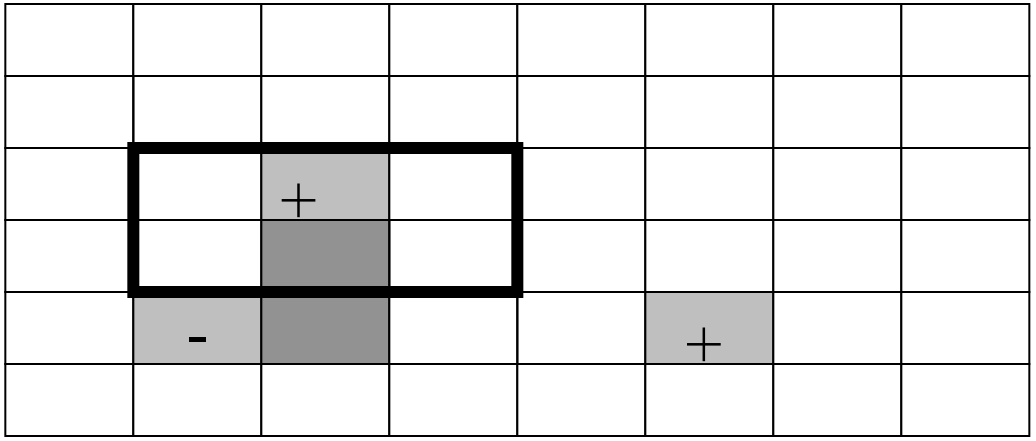


Branch cut

Positive residue

1) start

2) search in box



3) find second residue

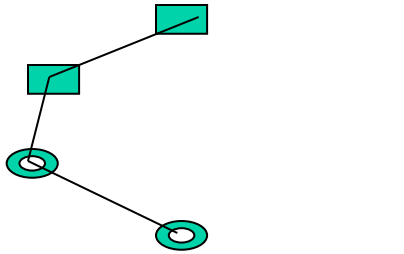
4) connect with branch cut

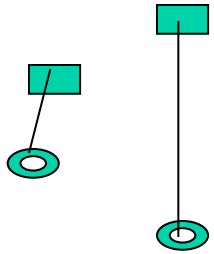
- Fast
- Need to start integration (seed point) in area of good data.
- Can connect close residues (‘dipoles’) first.
- Residues often lie in areas of layover.
- Regions can get isolated from each other with dramatically different phase (by multiples of 2p).
- Some implementations allows manual connecting of regions.

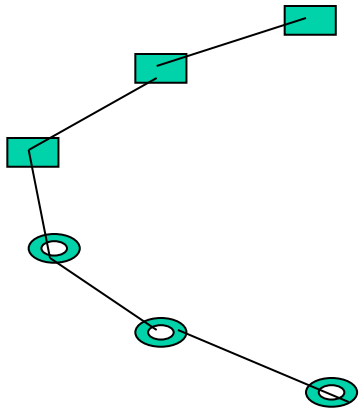
- Minimizes distance between residues; does not minimize number of cycles needed to “unwrap”.

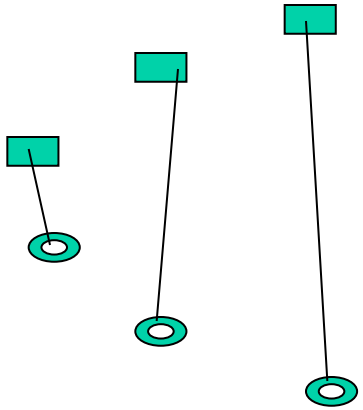
- Can also use quality rather than residues to define.

What is the best unwrapping….









Good – L0

Bad – L1

Negative residue

Positive residue

Residues caused by topographic layover with illumination from one side

Modifications

1) Minimum span (MST)
    1) Define cuts so that a tree cannot connect to itself.
    2) Connect all trees.
    3) Use knowledge of residues to guide integration.
    4) Complete unwrapping
2) Minimum cost (MCF)
    1) Uses flow to reduce cycles

Chen and Zebker's (2000)

1) A branch-cut algorithm minimizes the length of discontinuity by an (L^0) norm.
2) Flynn yields a L^1 solution.
3) Least-squares yield an L^2 solution.

C&Z claim low norms are best but this depends on the type of problem.
Unwrapping topography may differ from deformation phase.

For interferograms corrupted by noise, L^0 and L^1 algorithms yield similar solutions.
For layover, where discontinuities separate severe phase gradients, L^1 algorithms do not do well.

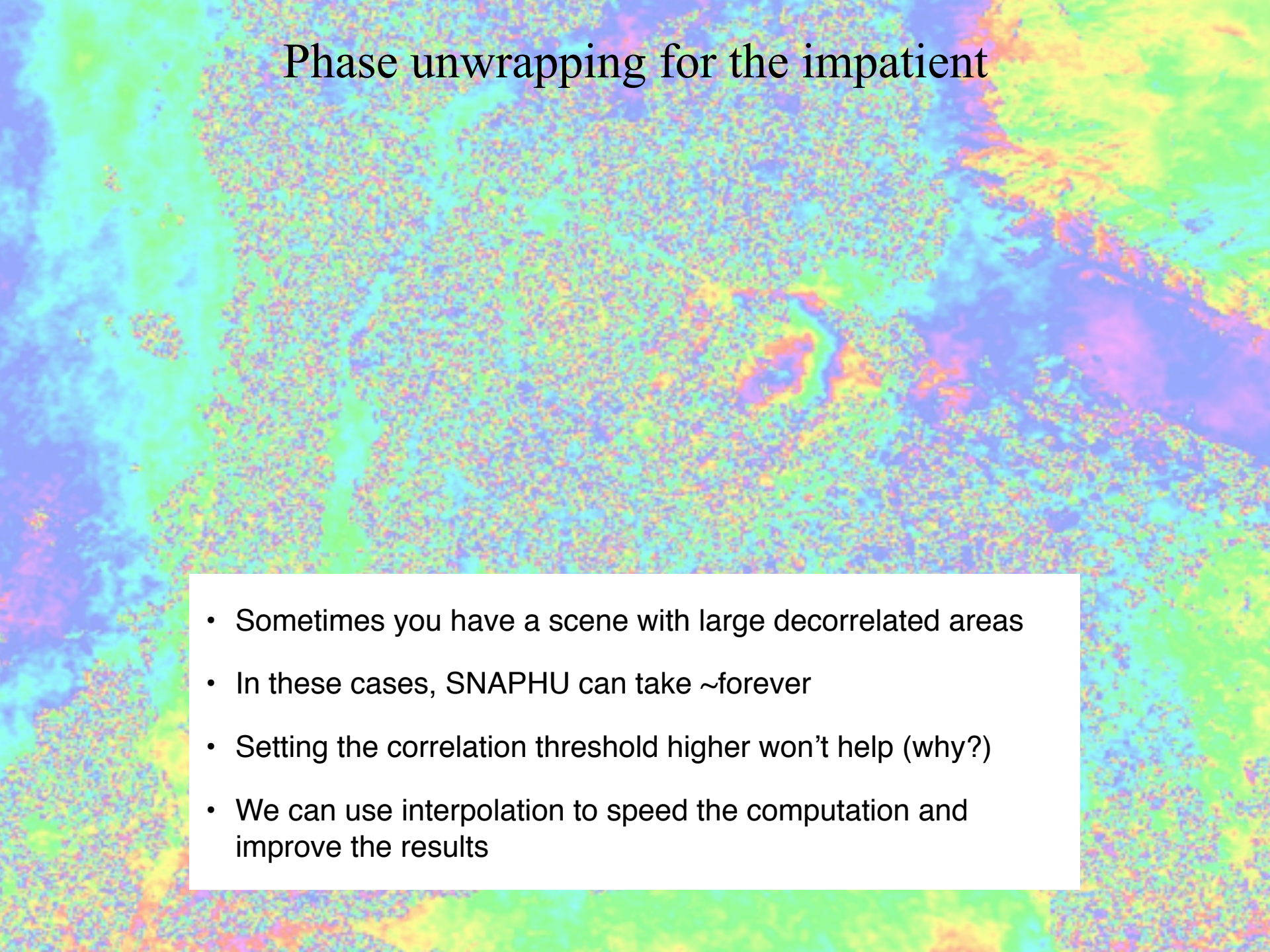
GMT5SAR uses SNAPHU, which is C&Z's algorithm

The ultimate L0 algorithm would minimize the total cut length.

Chen and Zebker (C&Z) show that the L0 problem is equivalent to an NP-complete problem and therefore hard for efficient algorithms to solve completely.

- Therefore, the L0 branch-cut algorithm is a good place to start.
- Major problem is that cuts close on themselves.
- Total tree length is upper bound on total discontinuity length.

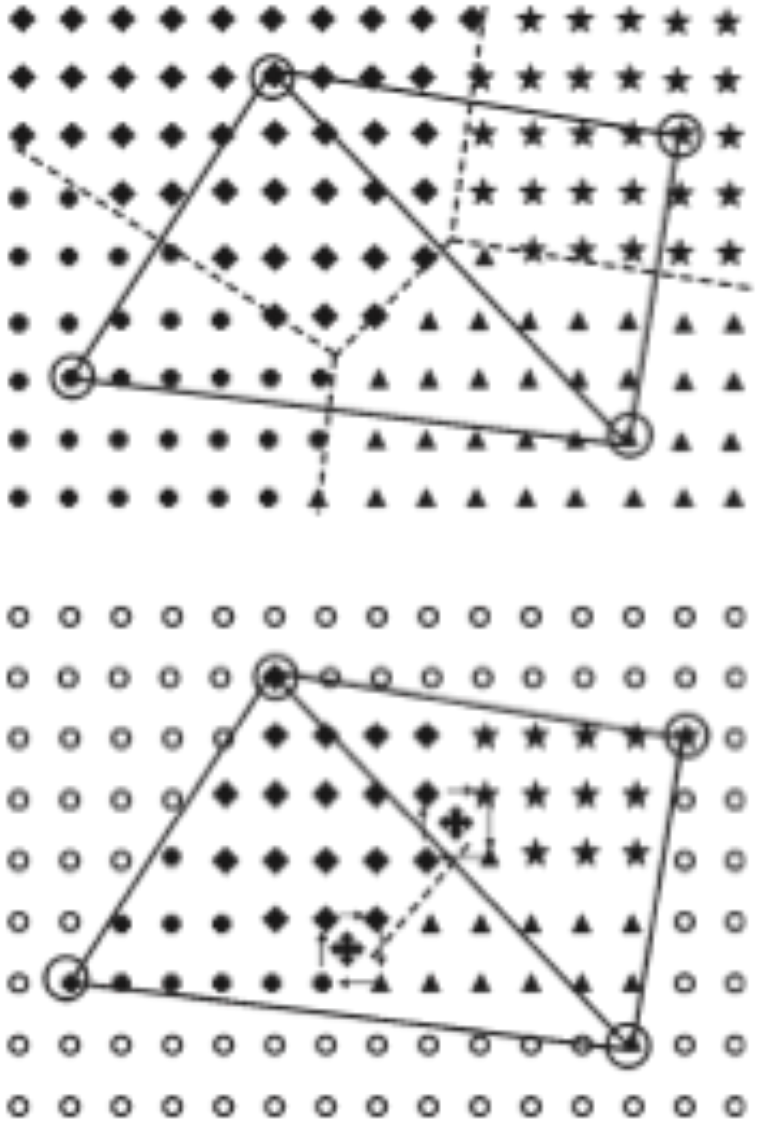
Phase unwrapping for the impatient



- Sometimes you have a scene with large decorrelated areas
- In these cases, SNAPHU can take ~forever
- Setting the correlation threshold higher won't help (why?)
- We can use interpolation to speed the computation and improve the results

Nearest Neighbor Interpolation

- We can think of the correlated pixels as a sparse dataset
- Nearest neighbor interpolation preserves the topology of any loops containing residues
- This means the unwrapped, masked result should be the same, whether or not we interpolate first
- Reference:
  P.S. Agram and H.A. Zebker, "Sparse two-dimensional phase unwrapping using regular grid methods," *IEEE Geosci. Rem. Sens.*, 2009.

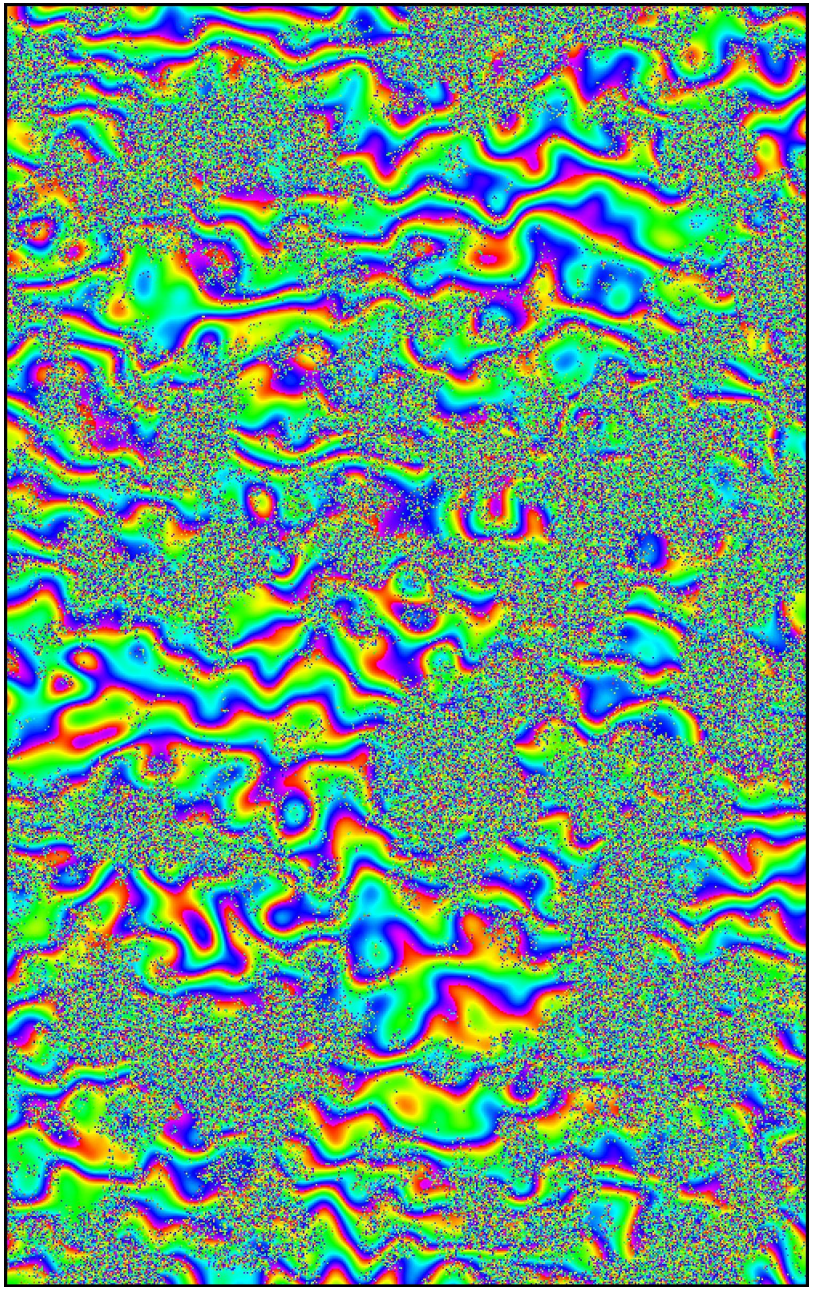


Implementation

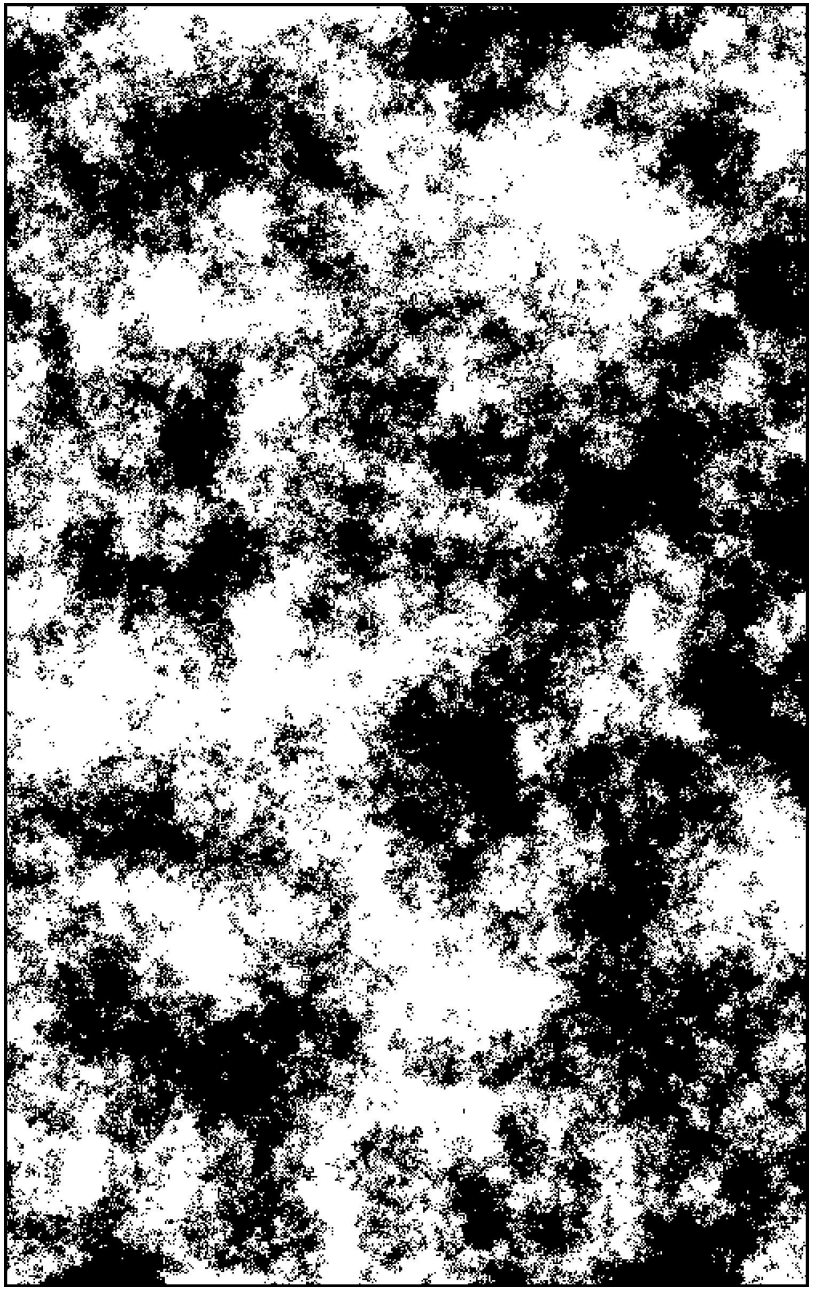
```
# get x,y bounds
set minx = `grdinfo -C $in.grd |cut -f 2`
set maxx = `grdinfo -C $in.grd |cut -f 3`
set nx = `grdinfo -C $in.grd |cut -f 10`
set boundsx = "$minx $maxx"
set miny = `grdinfo -C $in.grd |cut -f 4`
set maxy = `grdinfo -C $in.grd |cut -f 5`
set ny = `grdinfo -C $in.grd |cut -f 11`
# for some reason we have to reverse these two
set boundsy = "$maxy $miny"

# first convert to ascii
grd2xyz $in.grd -S -V > $in.gmt

# run gdal, then convert back to grd
gdal_grid -of GTiff -txe $boundsx -tye $boundsy -outsize $nx $ny -
l $in -a nearest $in.gmt $out.tiff
gdal_translate -of GMT -ot Float32 $out.tiff $out.grd

# fix the grd header metadata
grdedit $out.grd -T #(note: must be pixel node registration for
snaphu)
grdedit $out.grd -R$minx/$maxx/$miny/$maxy
```

Synthetic example

Phase

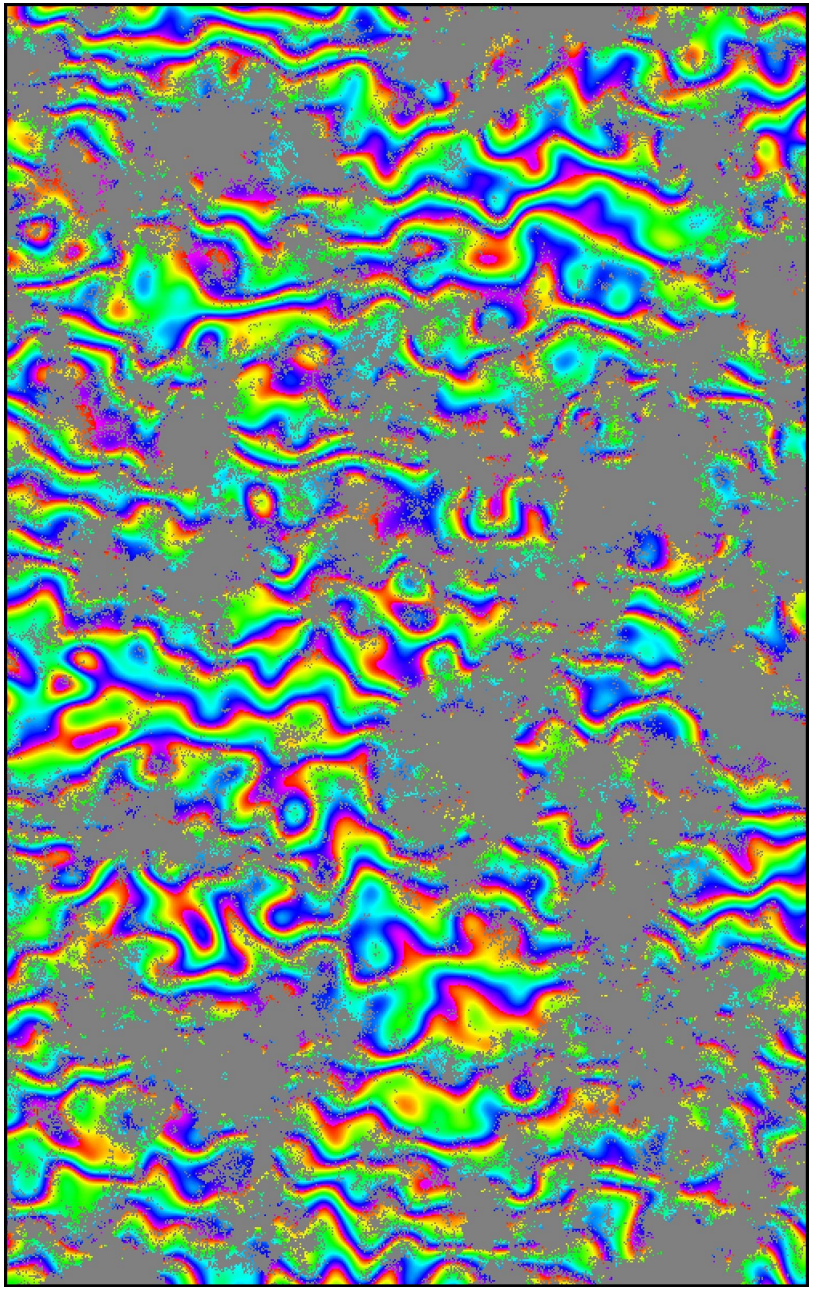


Correlation

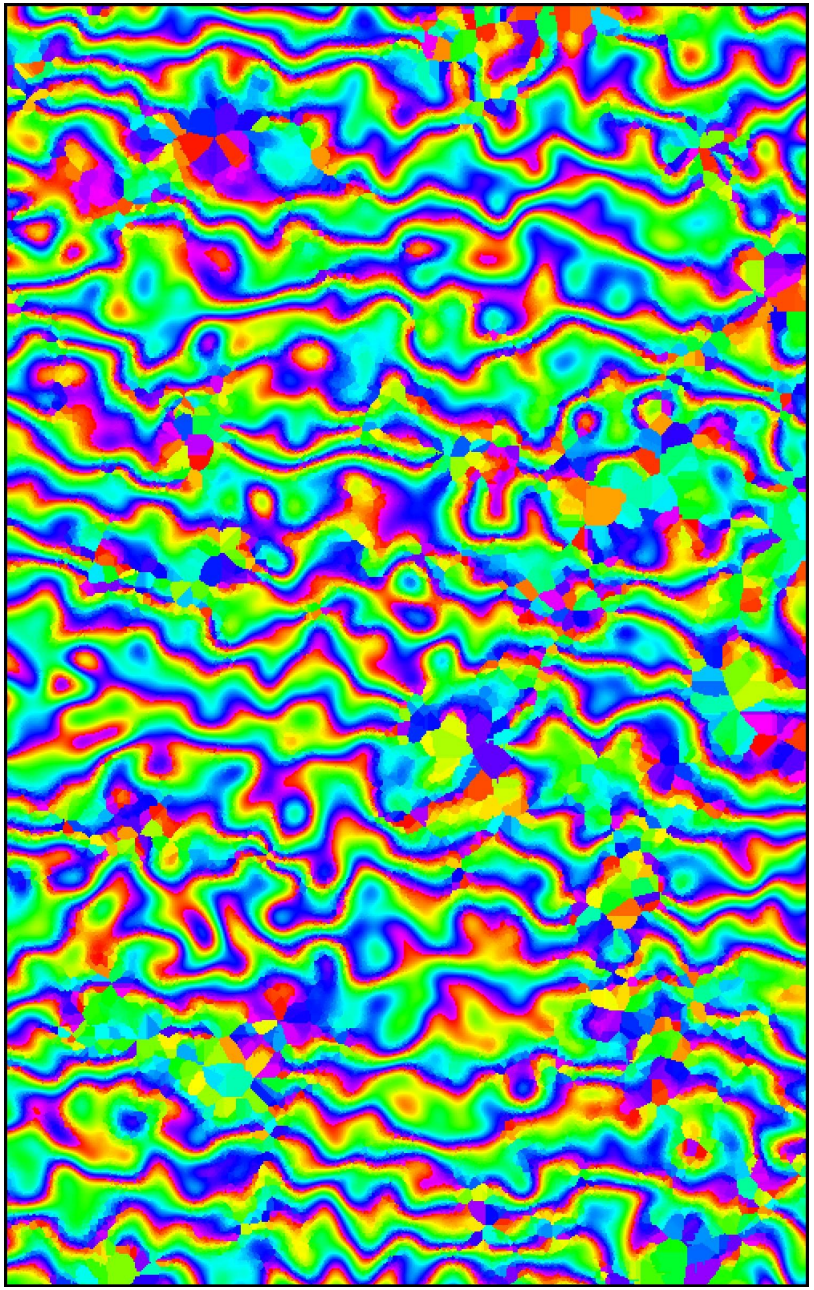


Mask, then interpolate

Masked

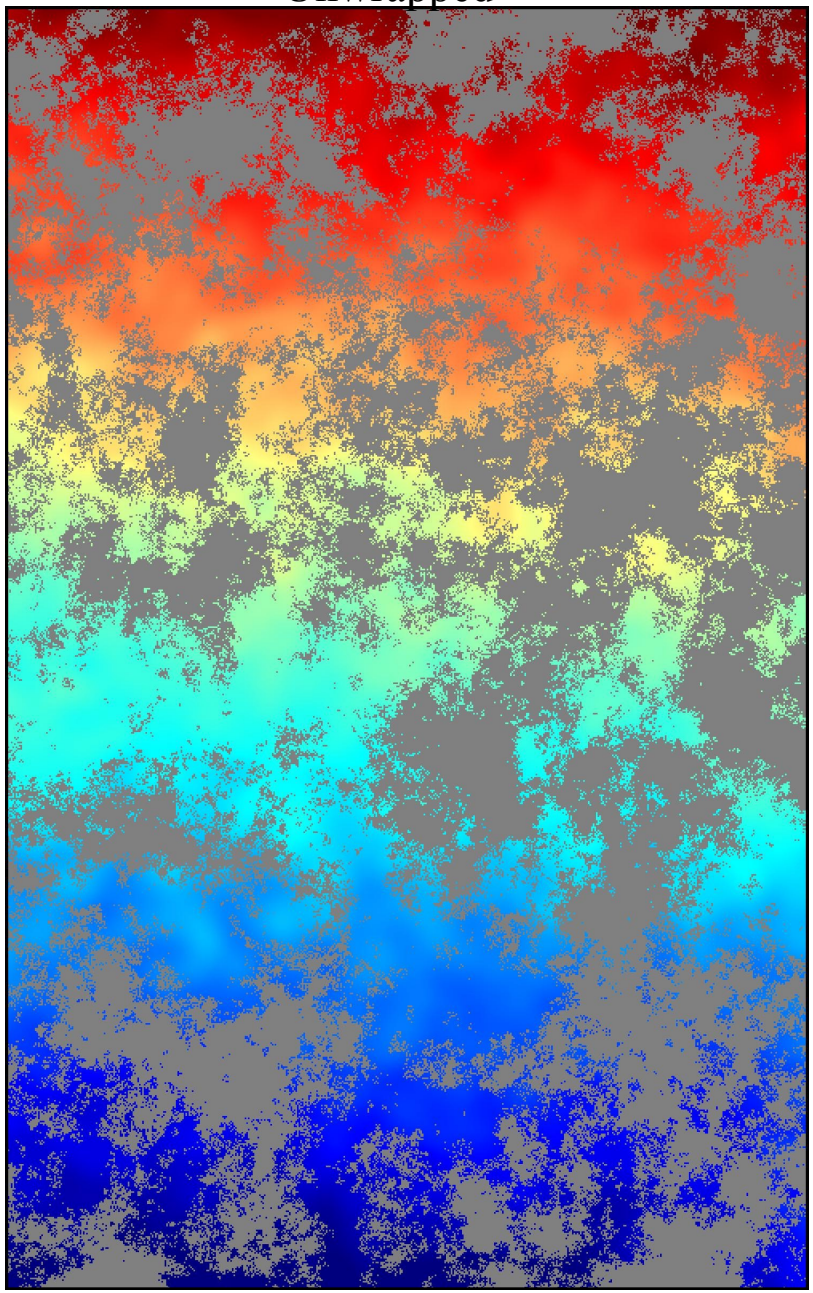


Filled

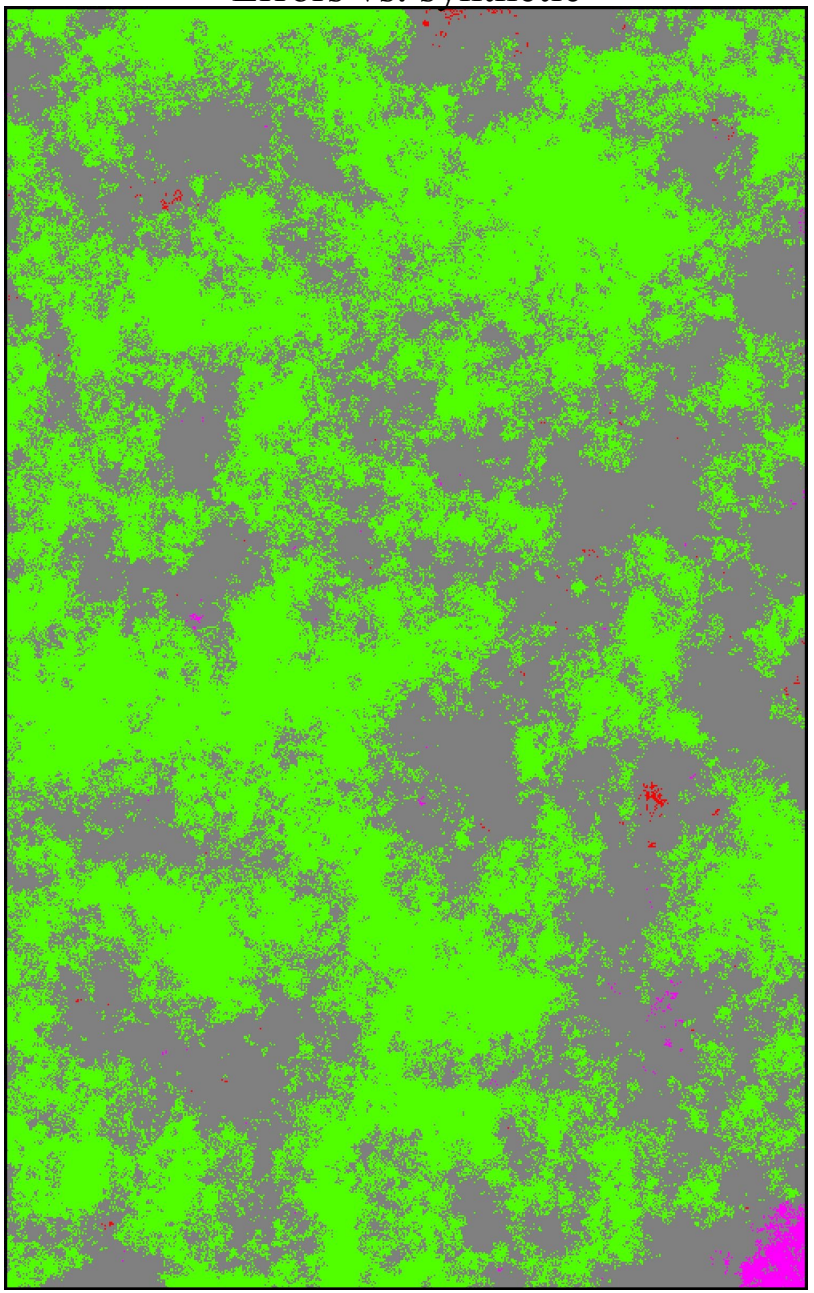


Basic unwrapping: 5.5 sec

Unwrapped

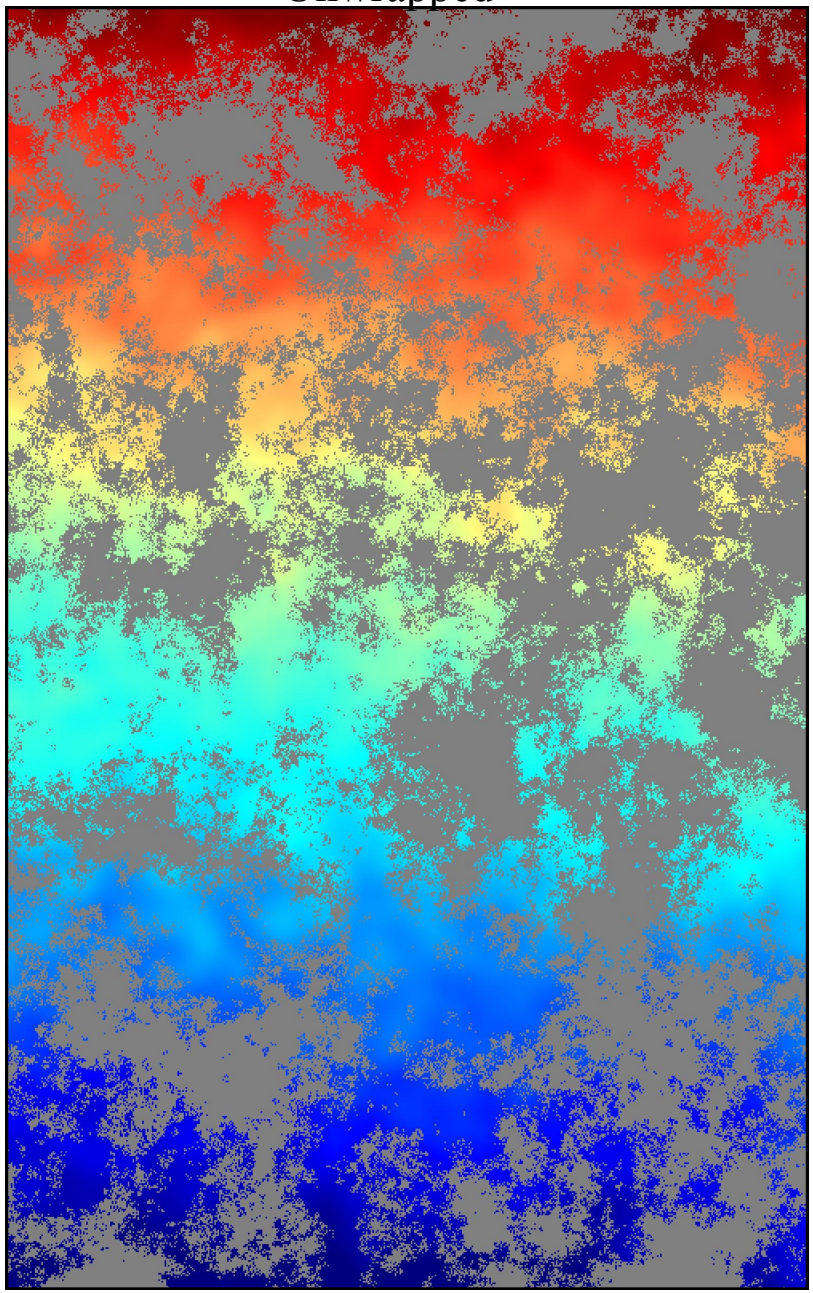


Errors vs. synthetic

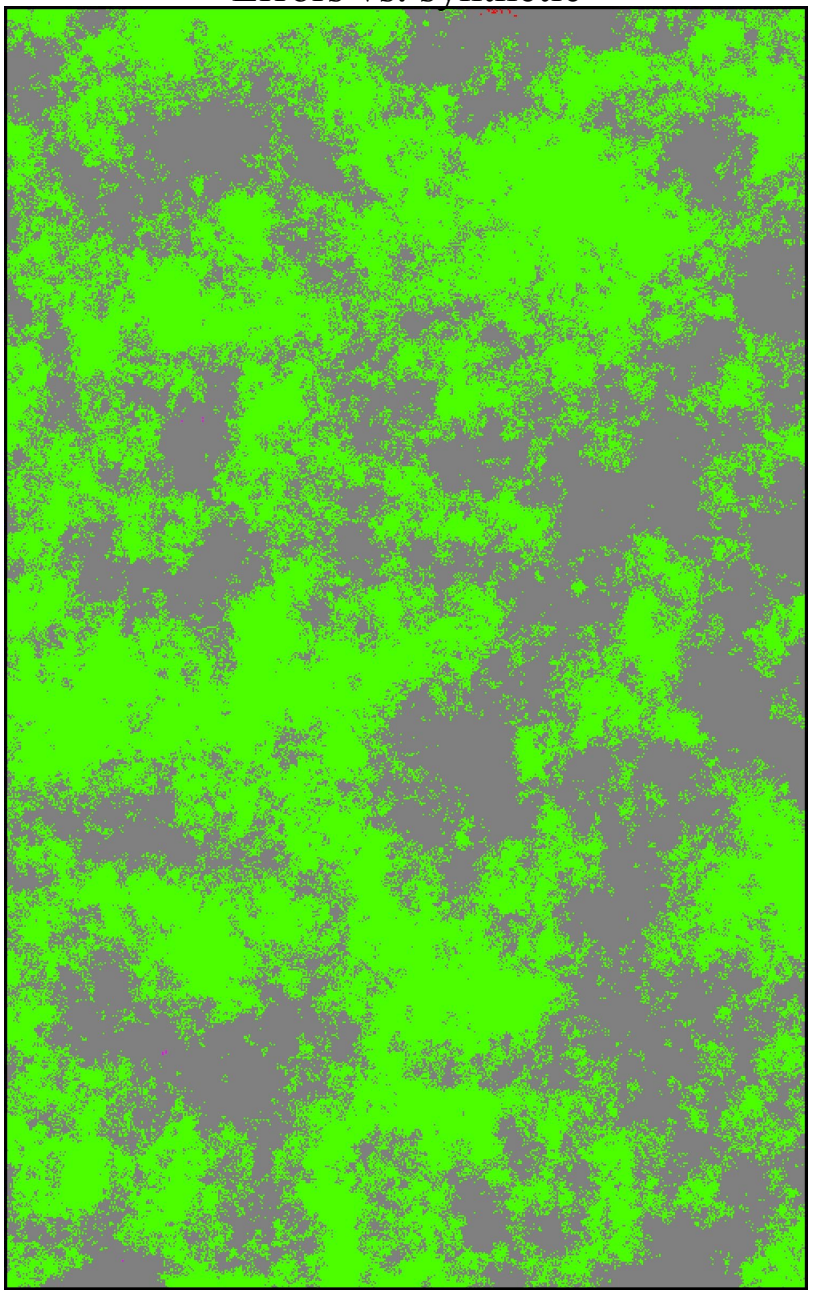


Interpolation-assisted: 0.9 sec (6x speedup!)

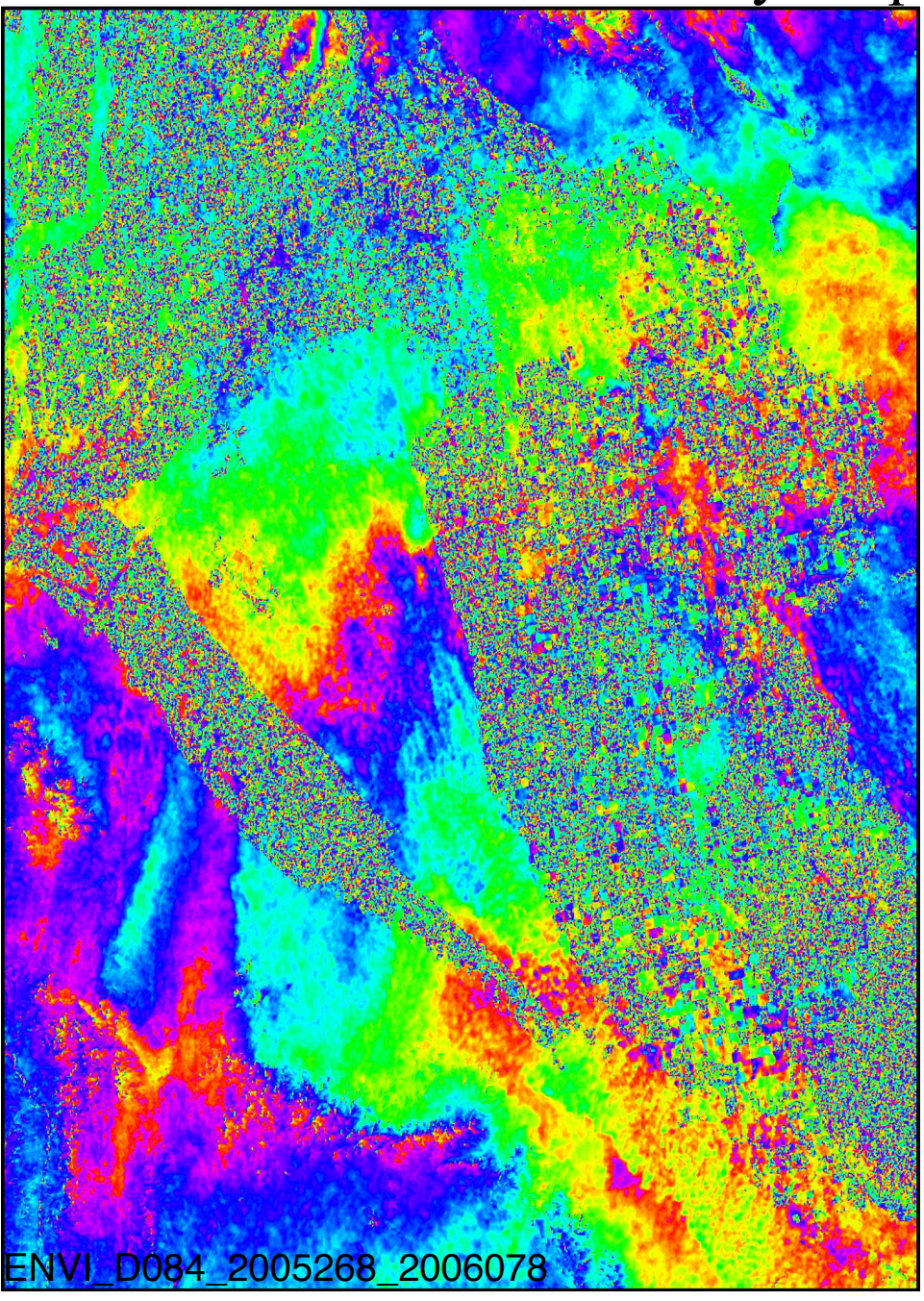
Unwrapped

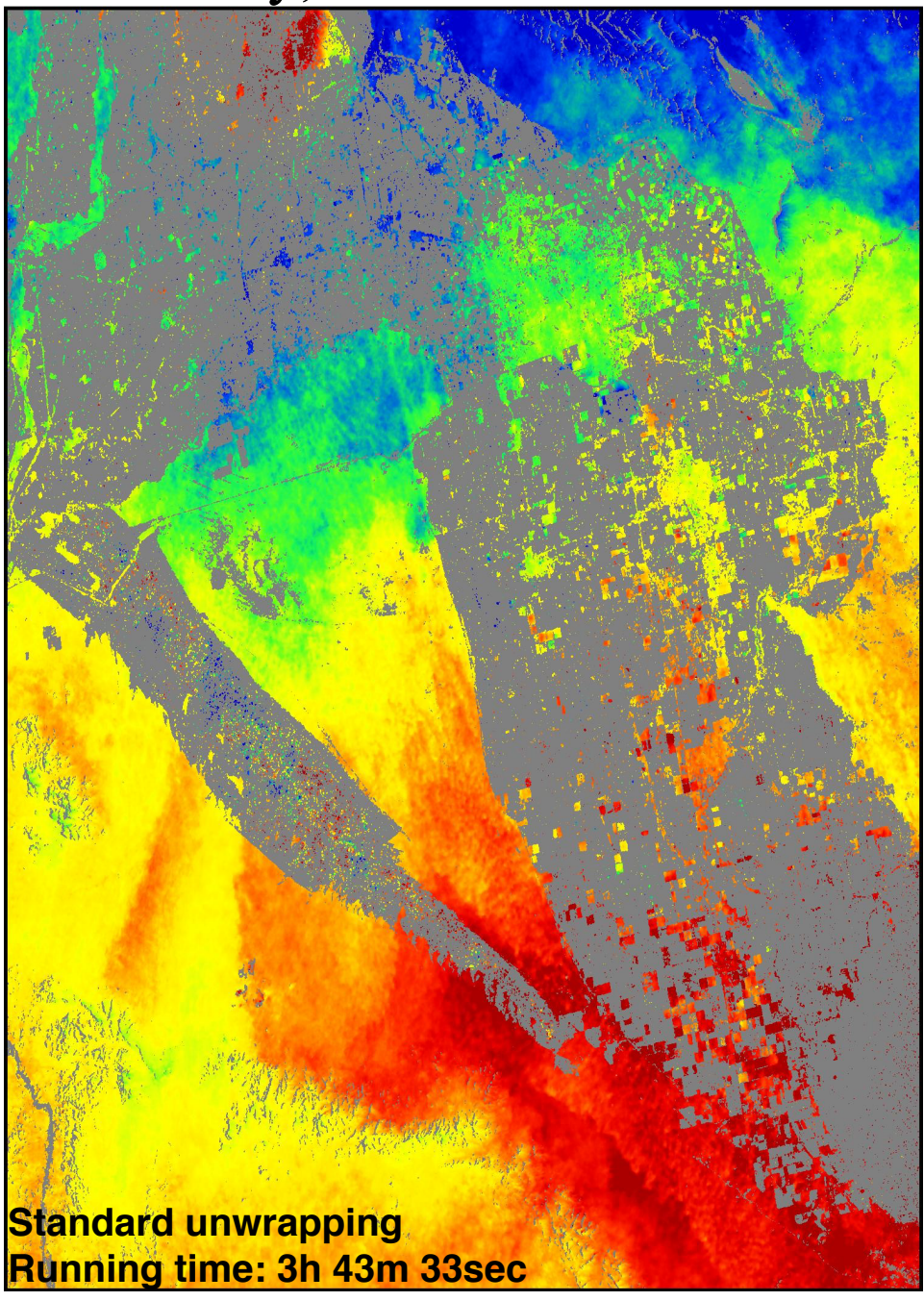


Errors vs. synthetic



Case study: Imperial Valley, CA





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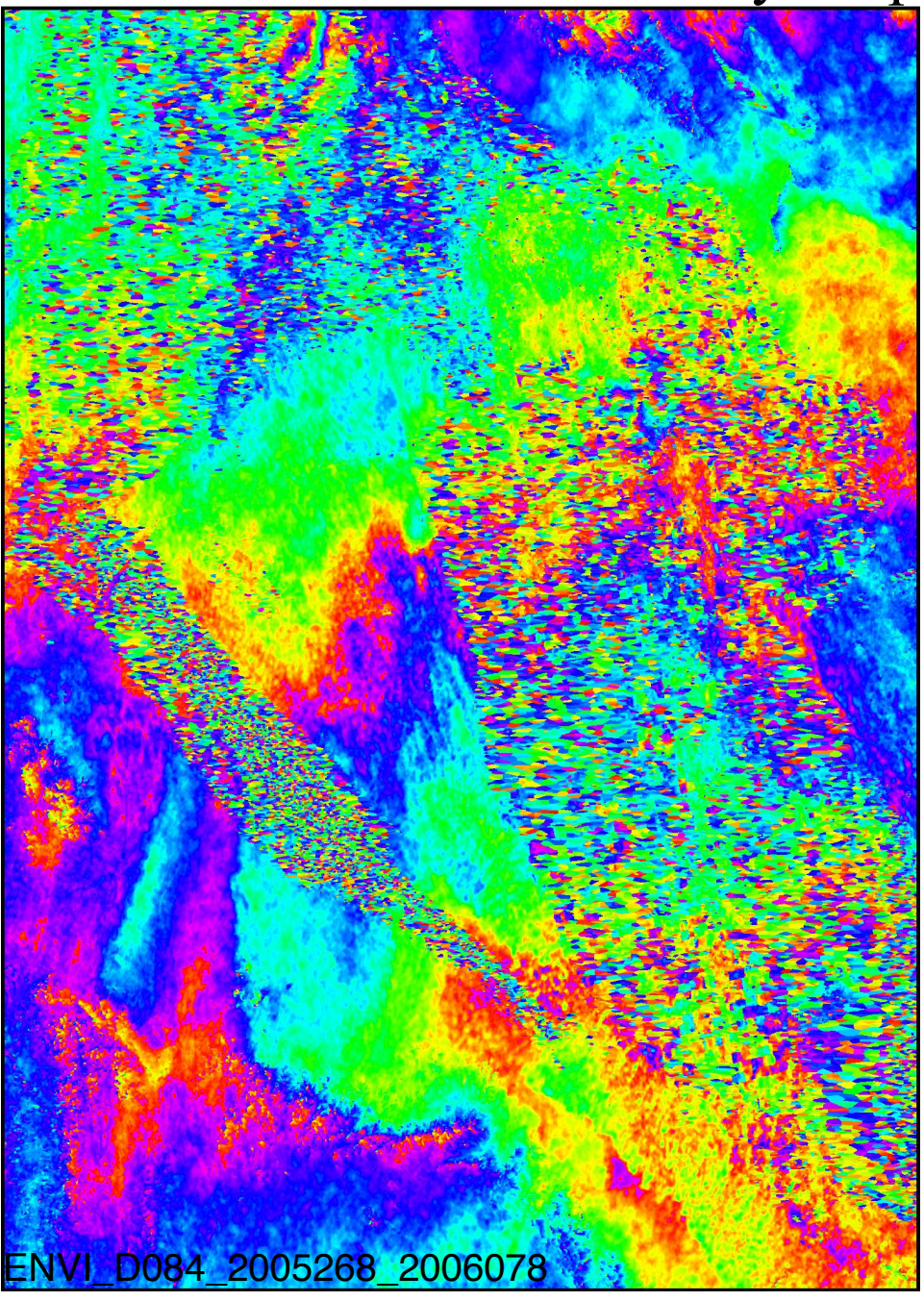

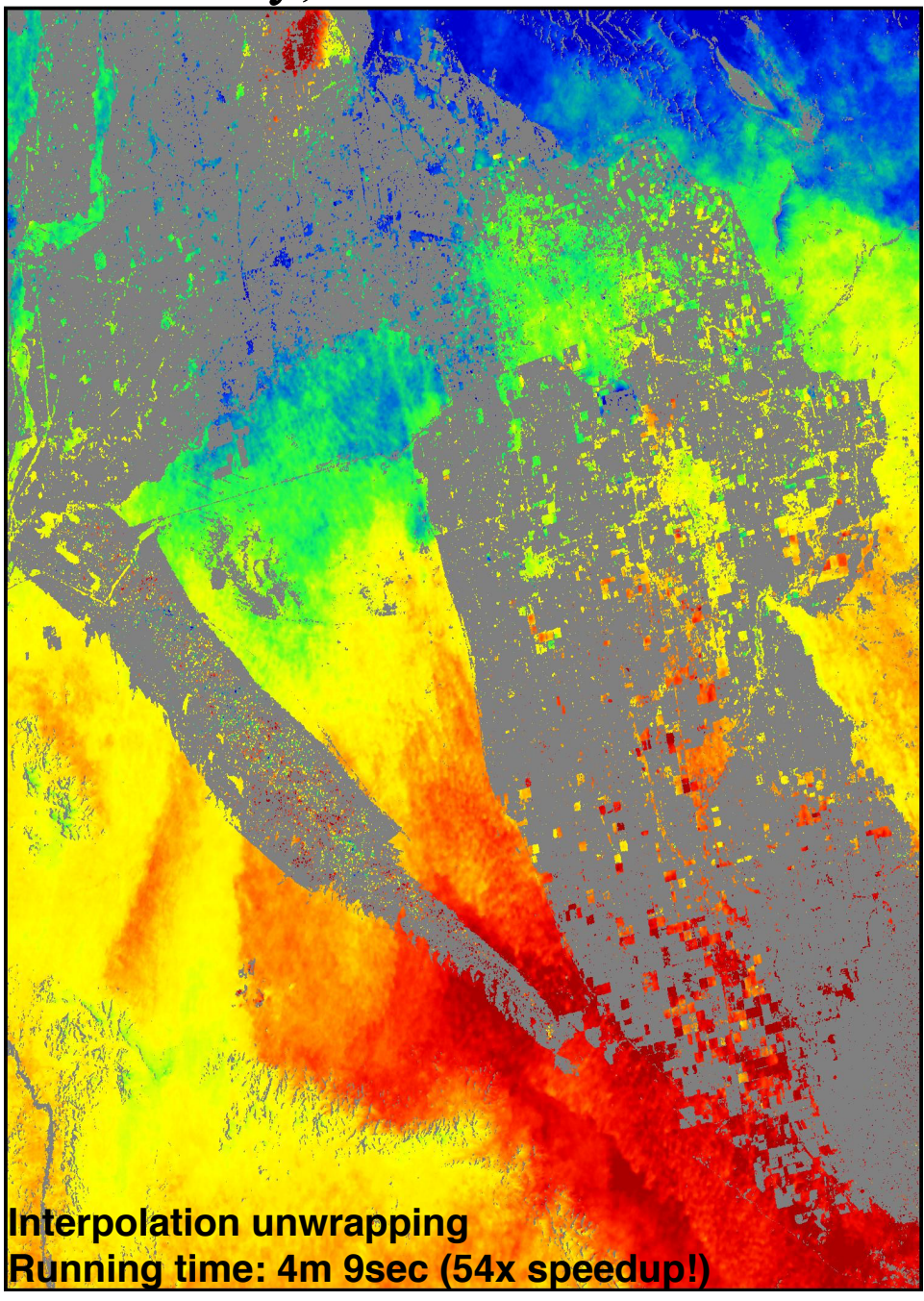

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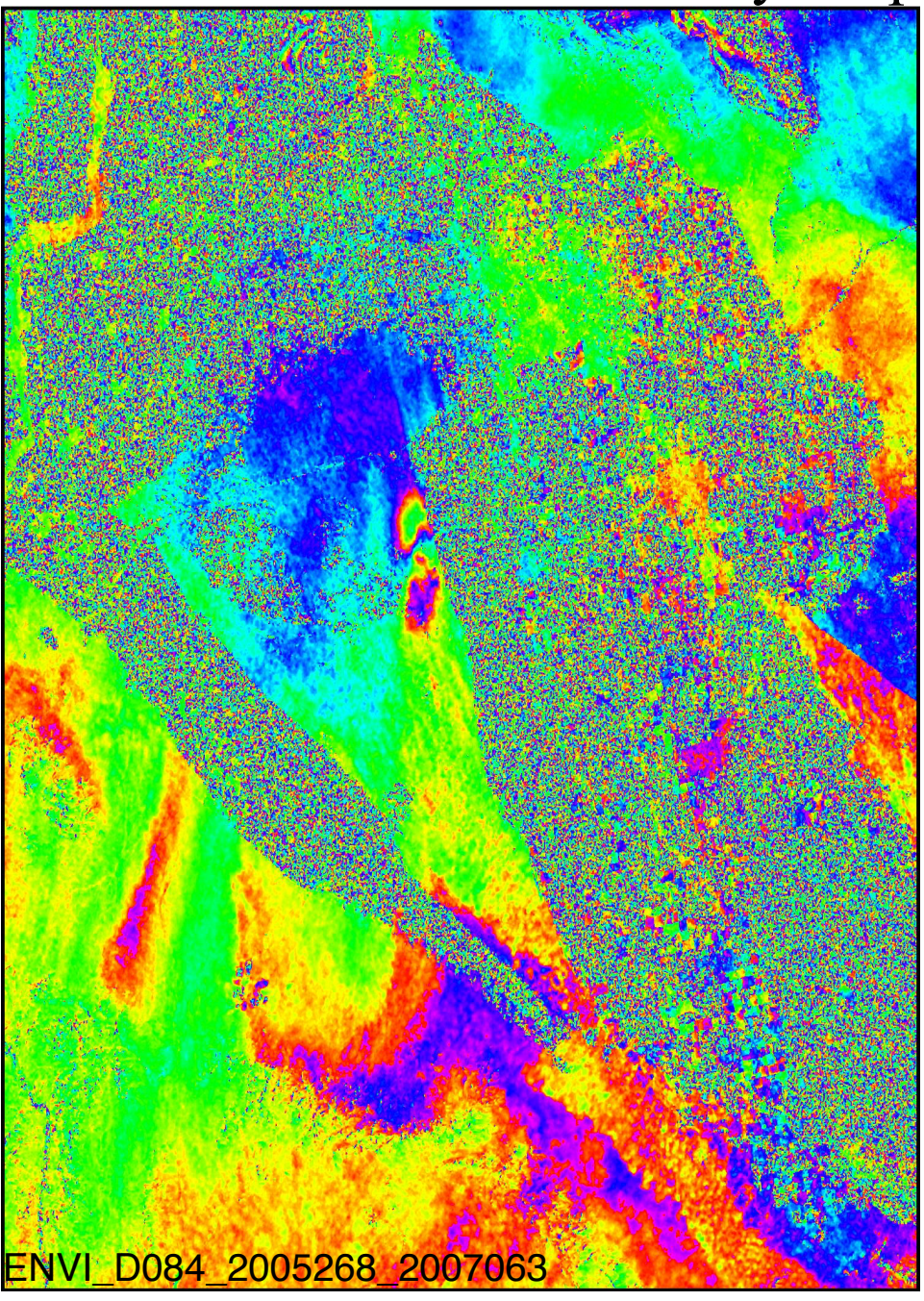

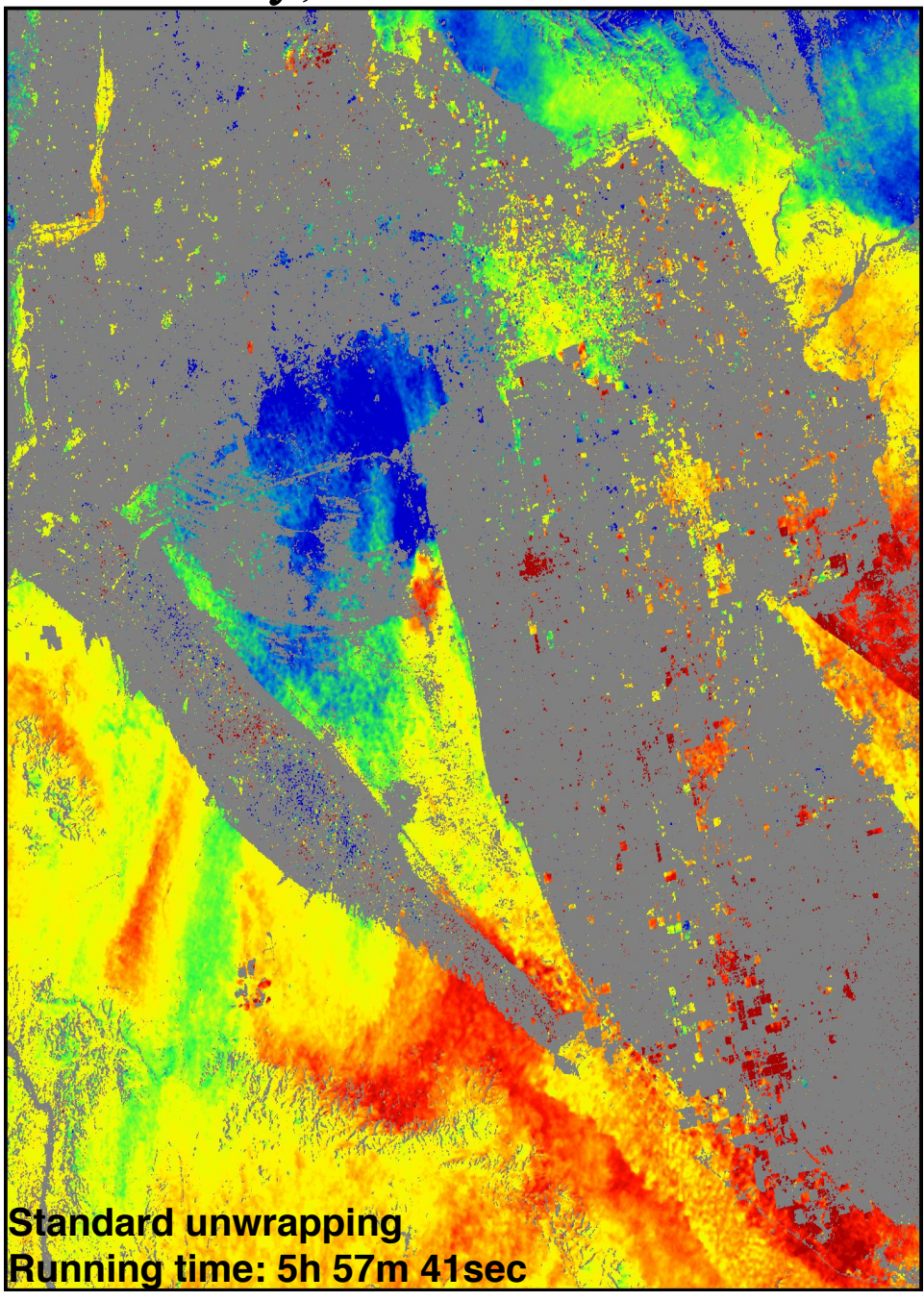

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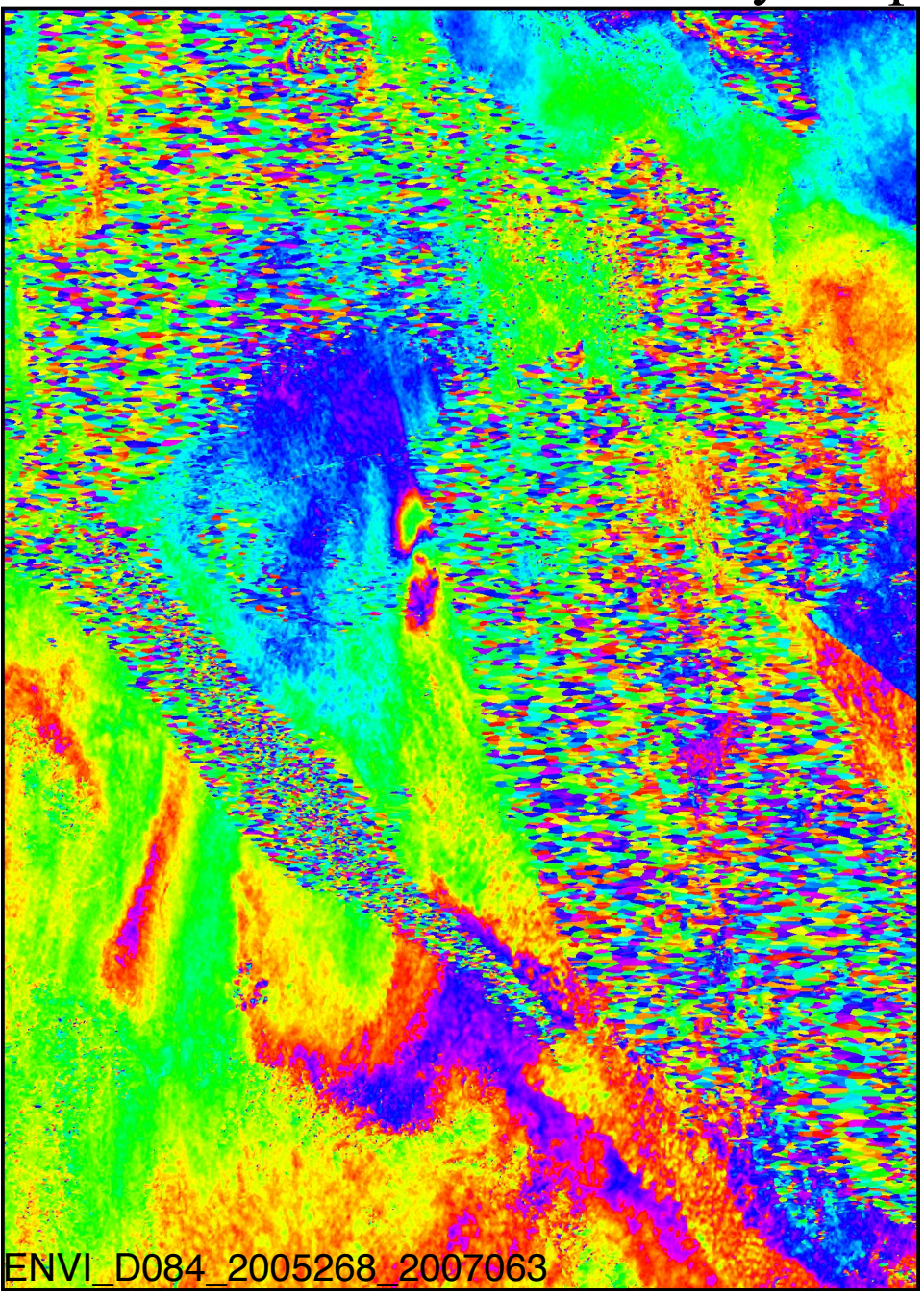
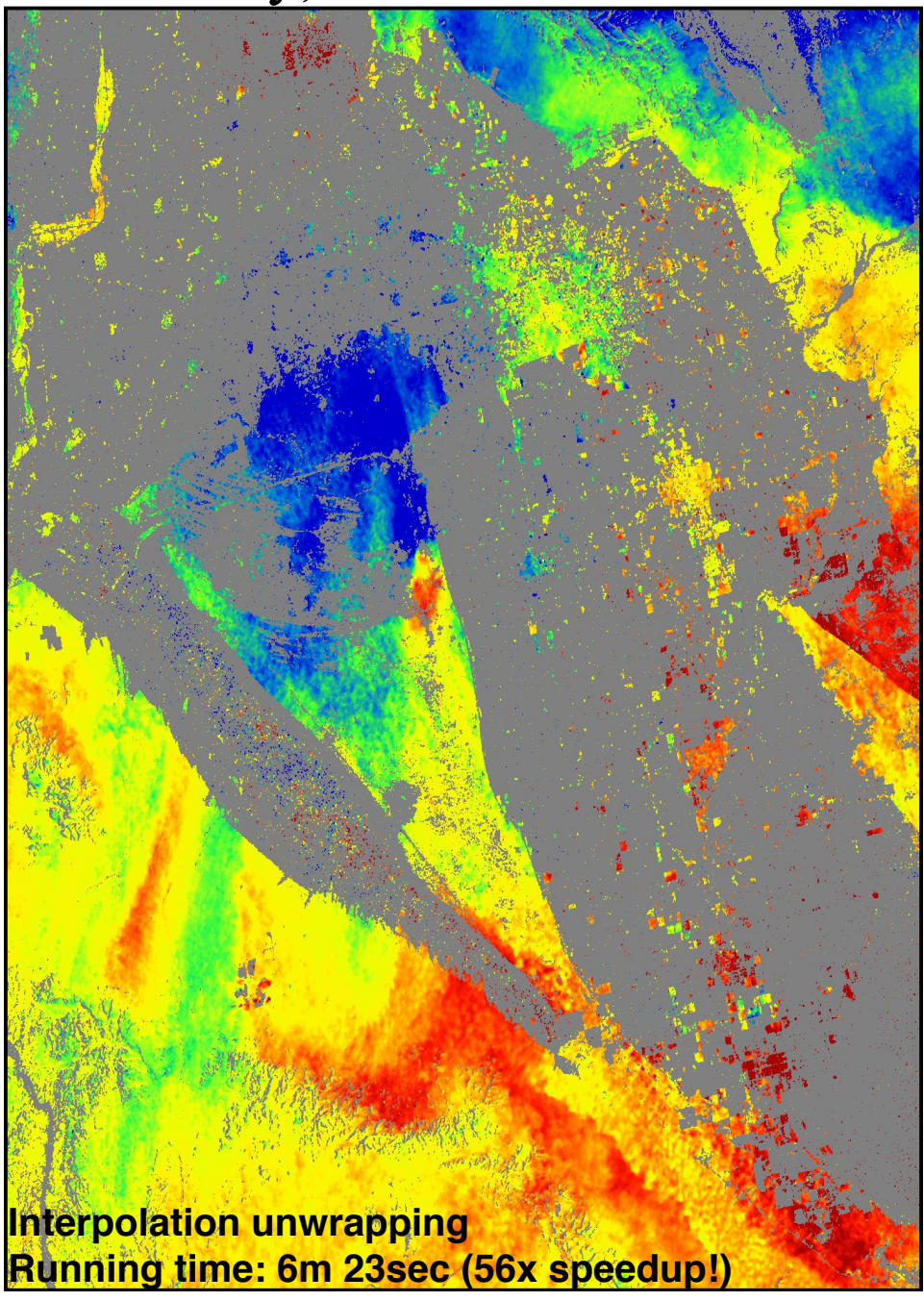




Evaluating model and phase unwrapping

- How do we evaluate?
  - Look at it – does it look okay?
  - May be impractical for large datasets and automatic processing.
  - Subtract out estimated model of deformation before unwrapping.
  - Can be done iteratively
  - After unwrapping, add in to regain original signal.
  - Can bias results with assumptions in model

Example

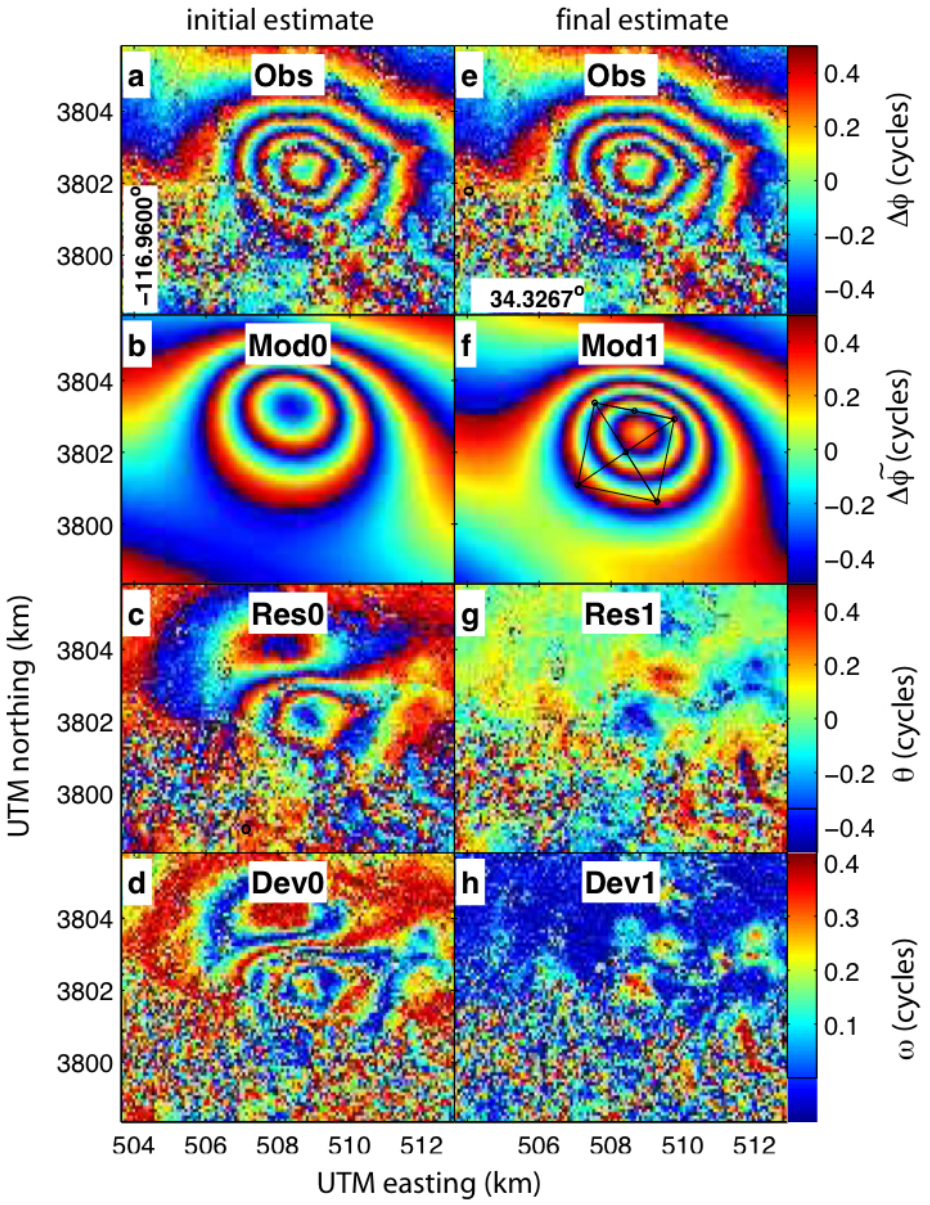
Initial **Final**

Observed

Modeled

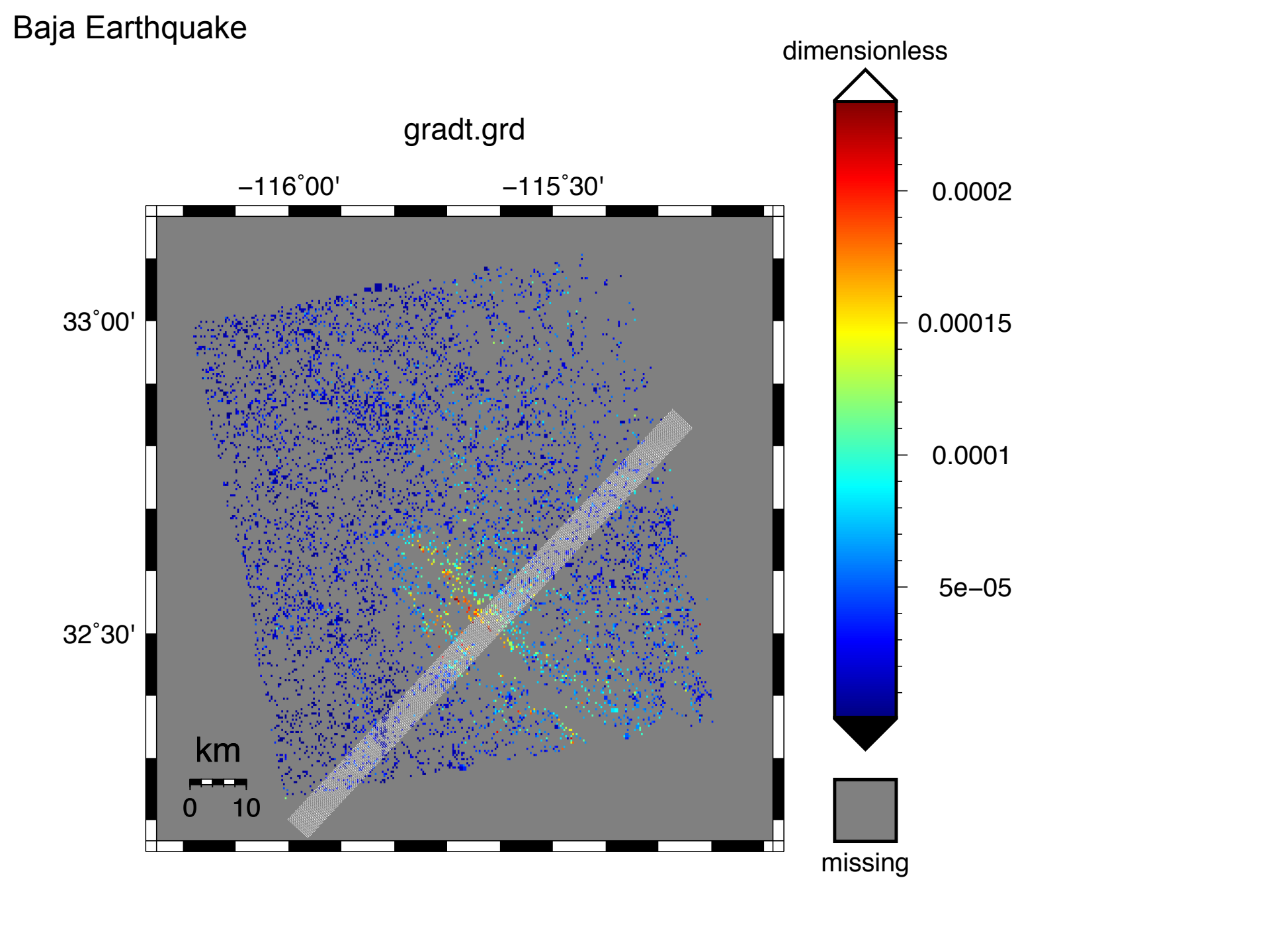
Residual

Deviations

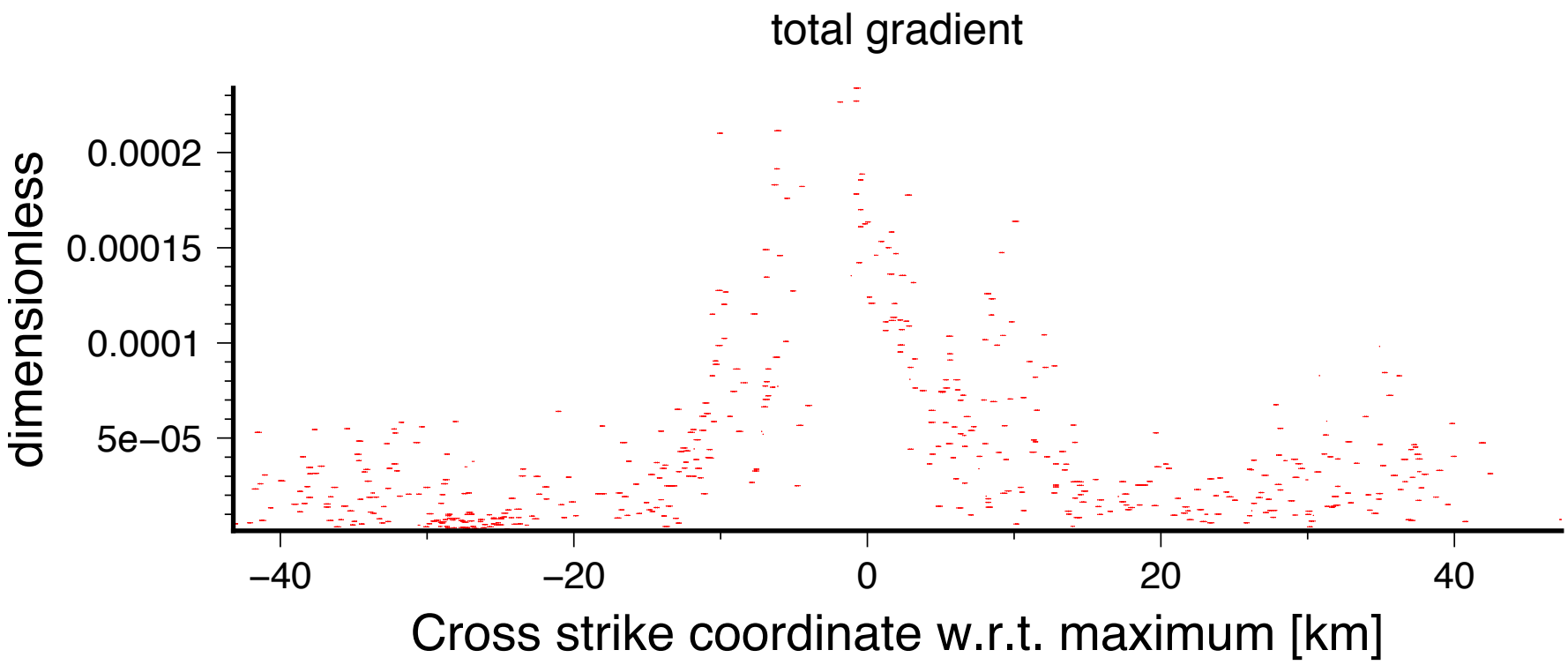


cycles

0.2796 cy **0.1483 cy**

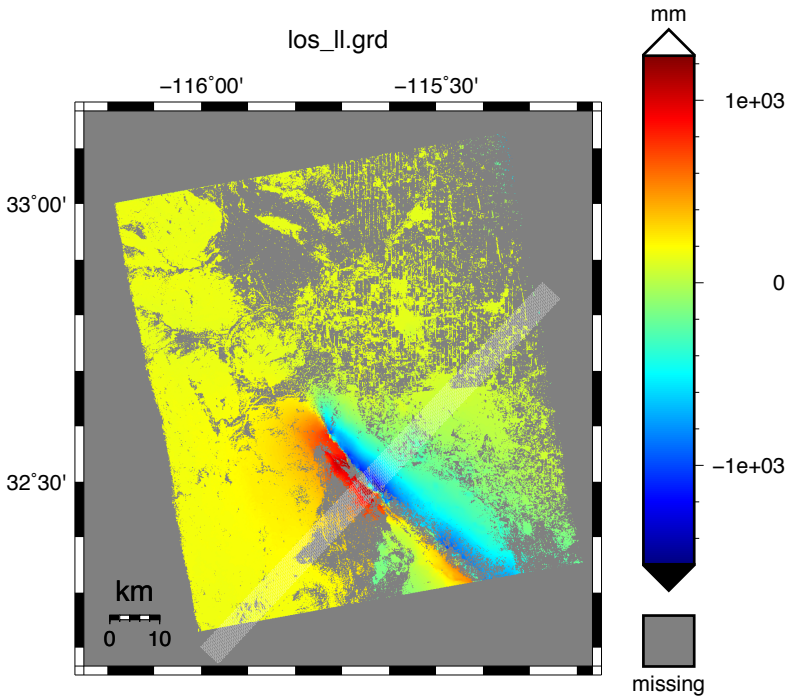

Baja Earthquake
gradt.grd
−116˚00'
−115˚30'
33˚00'
32˚30'
km
0
10
dimensionless
0.0002
0.00015
0.0001
5e−05
missing

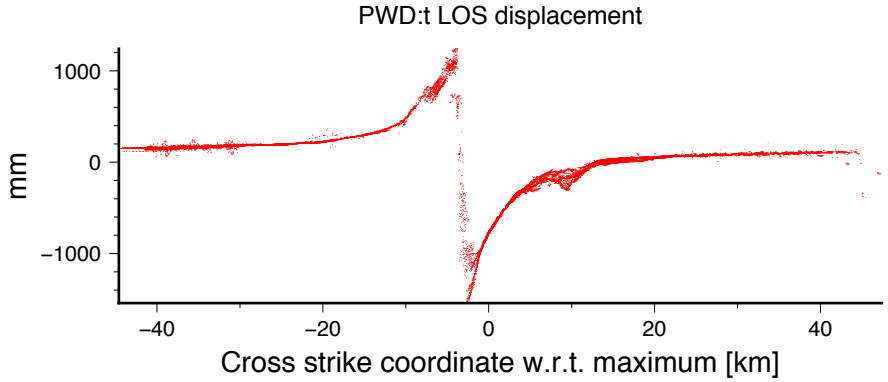
Baja Earthquake



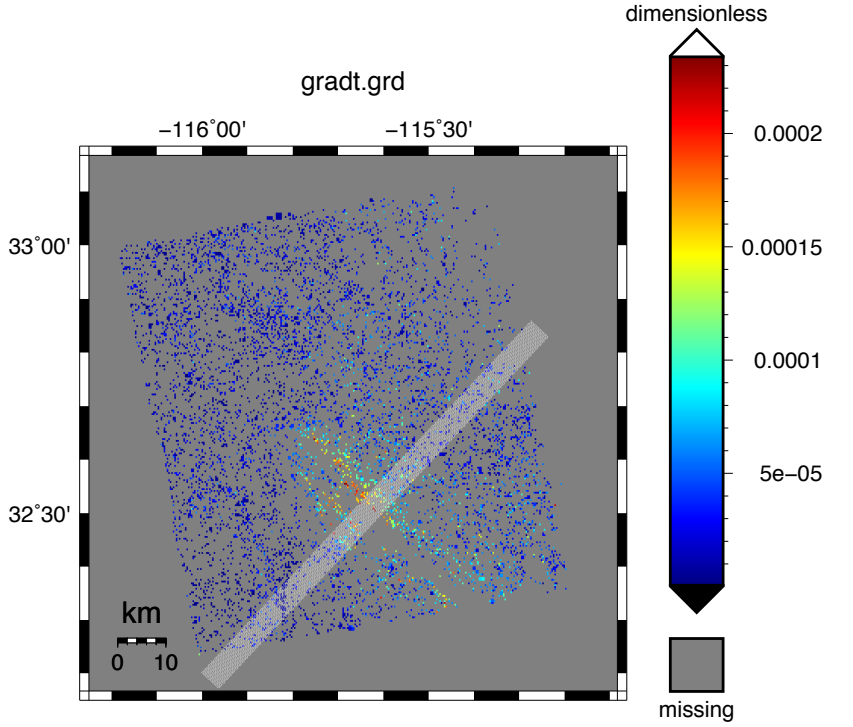
Baja Earthquake

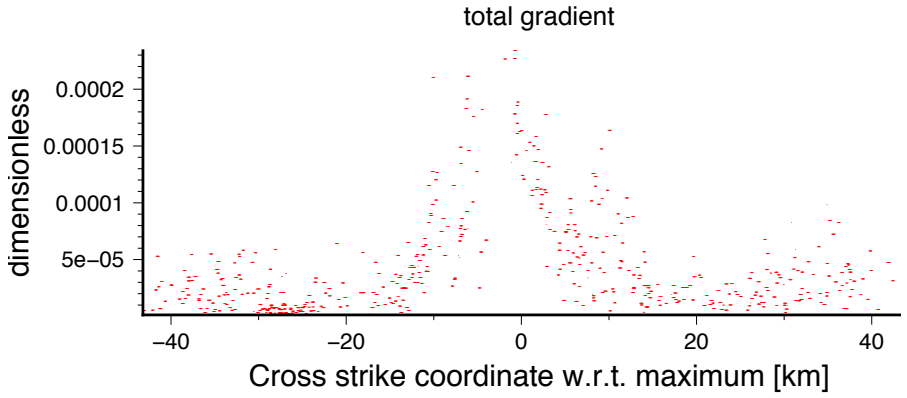
LOS displacement (mm)



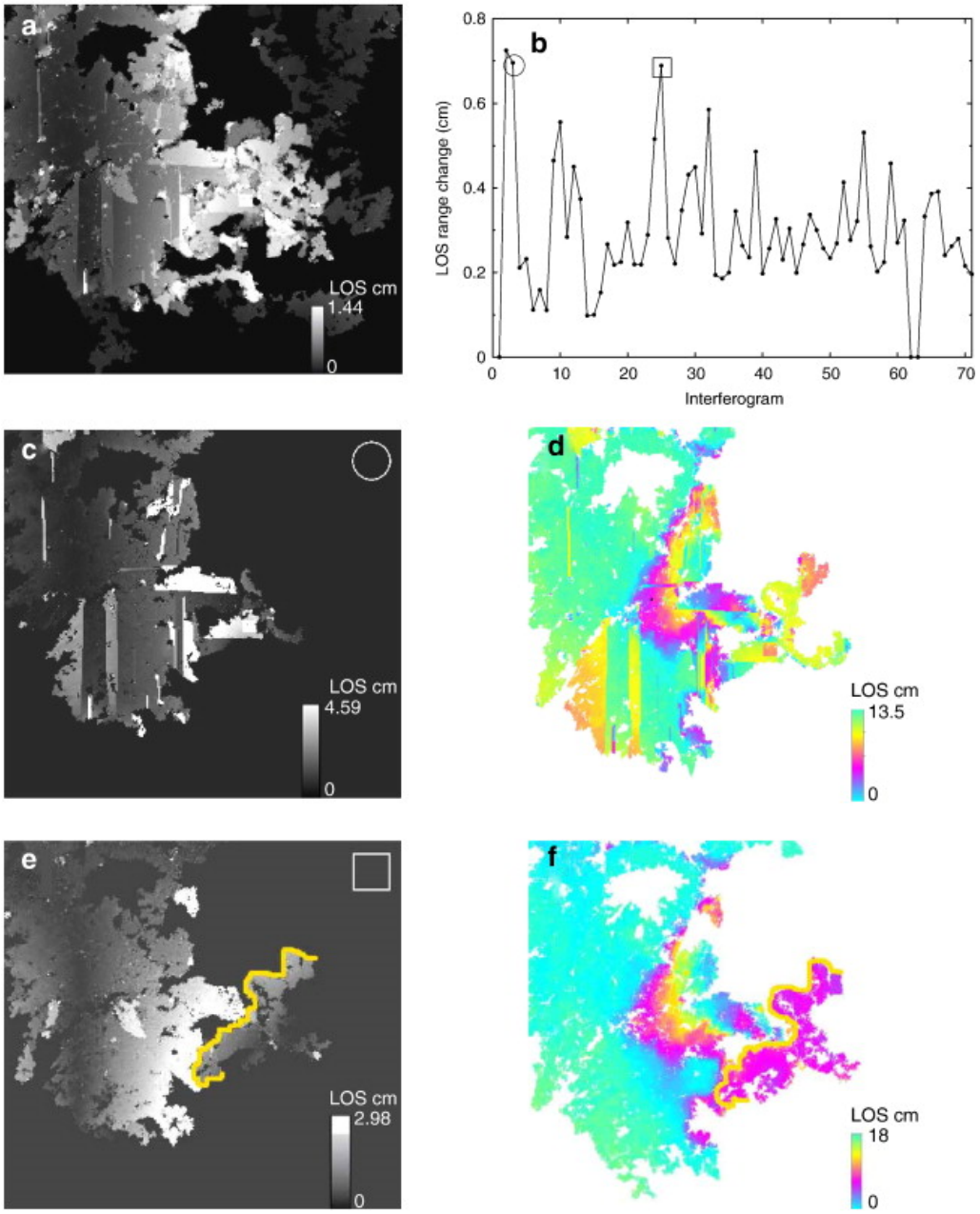


range gradient (dimensionless)





How can we tell if unwrapping is working? Look at it



Lopez-Quiroz et al. [2009]

Removing topography (and deformation)

If an accurate topographic model is available, then many of these problems can be alleviated during calculation of the interferogram.

- Reduce need for unwrapping.
- Deformation model can also be included.
- Can be done iteratively.
- Also true for large deformations (maybe done in an iterative fashion)

The future:
persistent scatterers
3D unwrapping

•The PS technique leads to widely spaced pixels. Phase relationships between these pixels may be challenging to define.

•If we have a time series of interferograms, phase unwrapping becomes a 3d problem.

Geocoding: One method

- "Fly" satellite along path with known orbit
- Find point (azimuth) of nearest approach to each DEM pixel with known latitude and longitude.
- Distance to point yields range
- This provides a mapping of range, azimith to latitude, longitude.

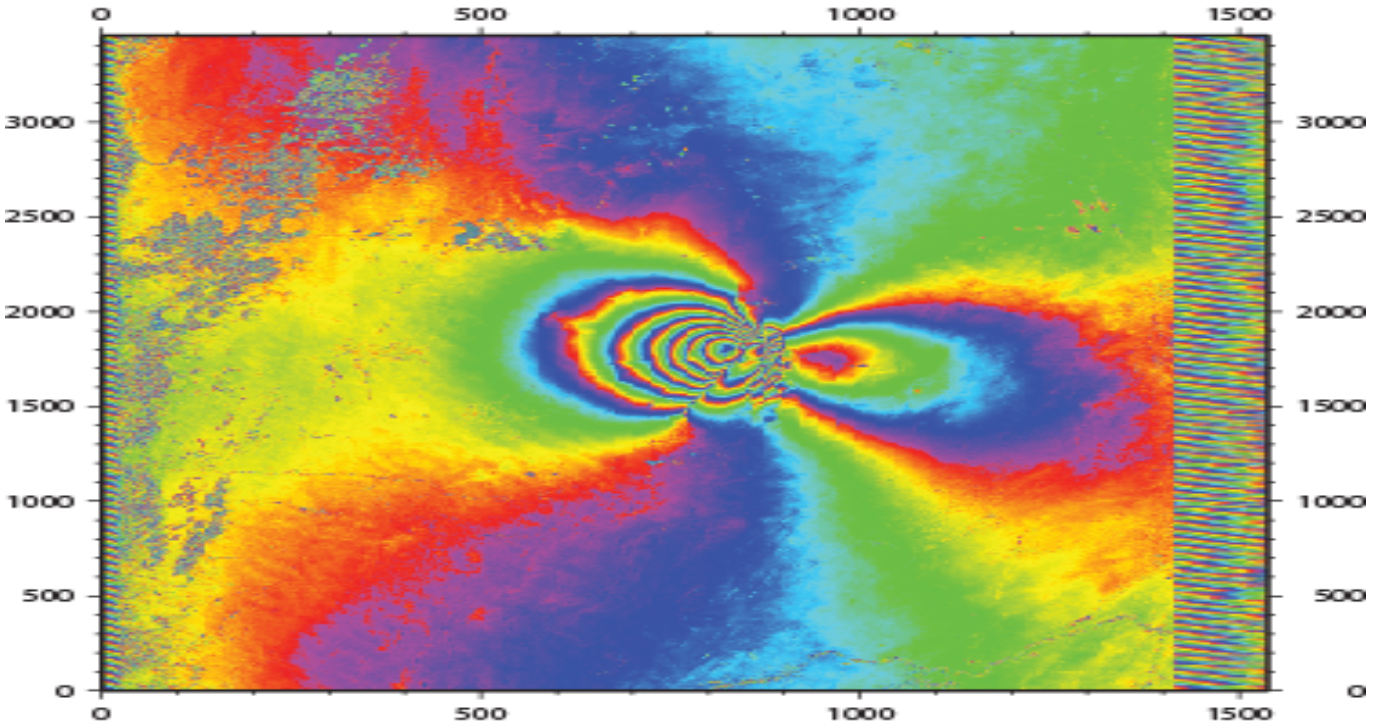
Flynn's minimum discontinuity

1) Identify lines of discontinuity (fringe lines)
    1) Difference between adjacent pixels > π
    2) Magnitude of discontinuity defined by number of multiples of 2π needed to fix.
2) Add multiples of 2π to eliminate lines of discontinuities that form loops.
3) Checks to see if operation creates more discontinuities than removes.
4) Continues in an iterative fashion.
5) At end, no more discontinuities can be removed without adding more.
6) Complicated algorithm (i.e. I looked at it and got a headache)

Comments:

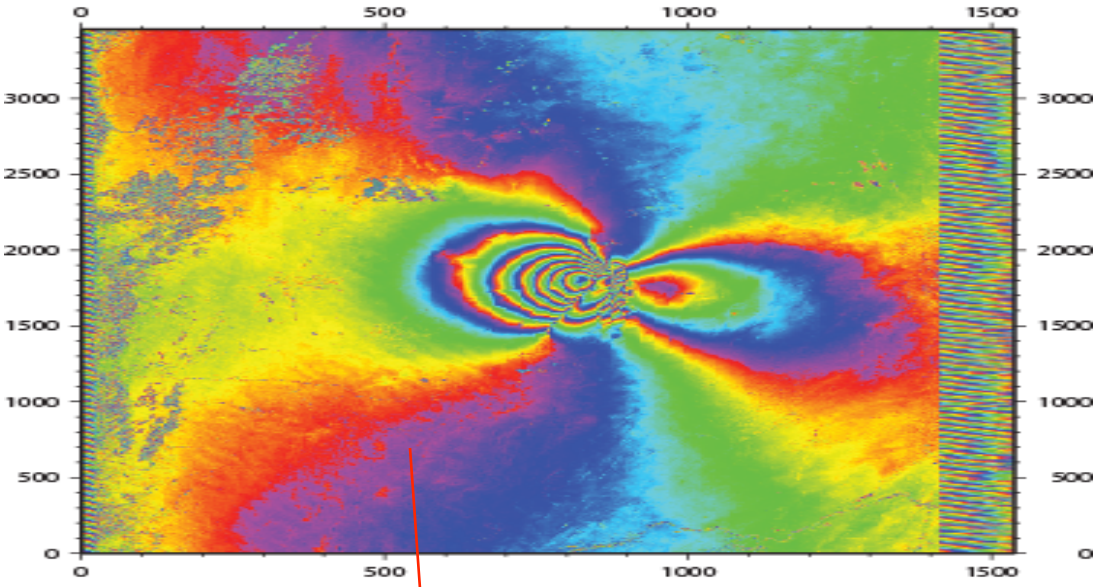
- Slow. Masking helps.

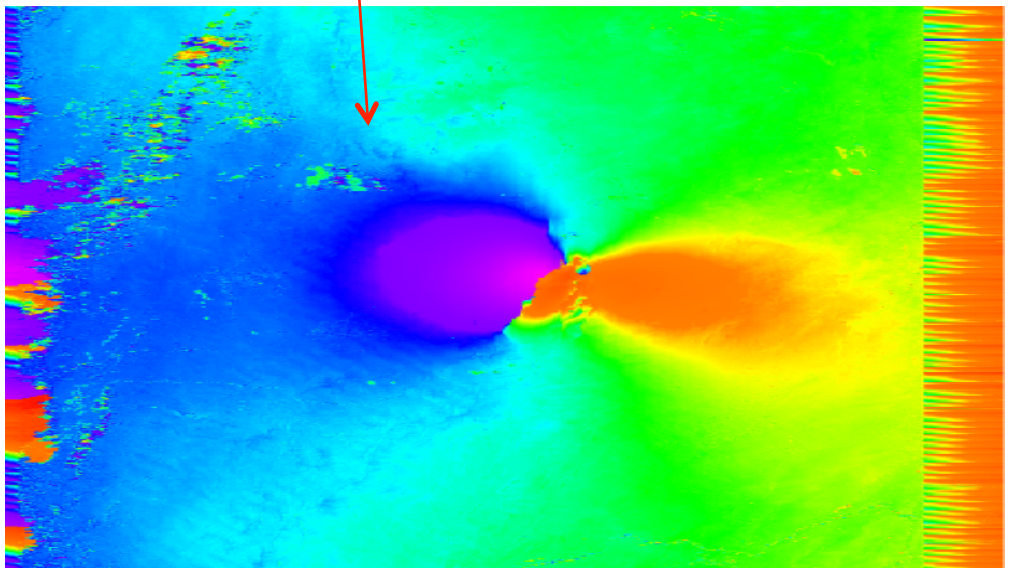
sometimes filtering is not necessary



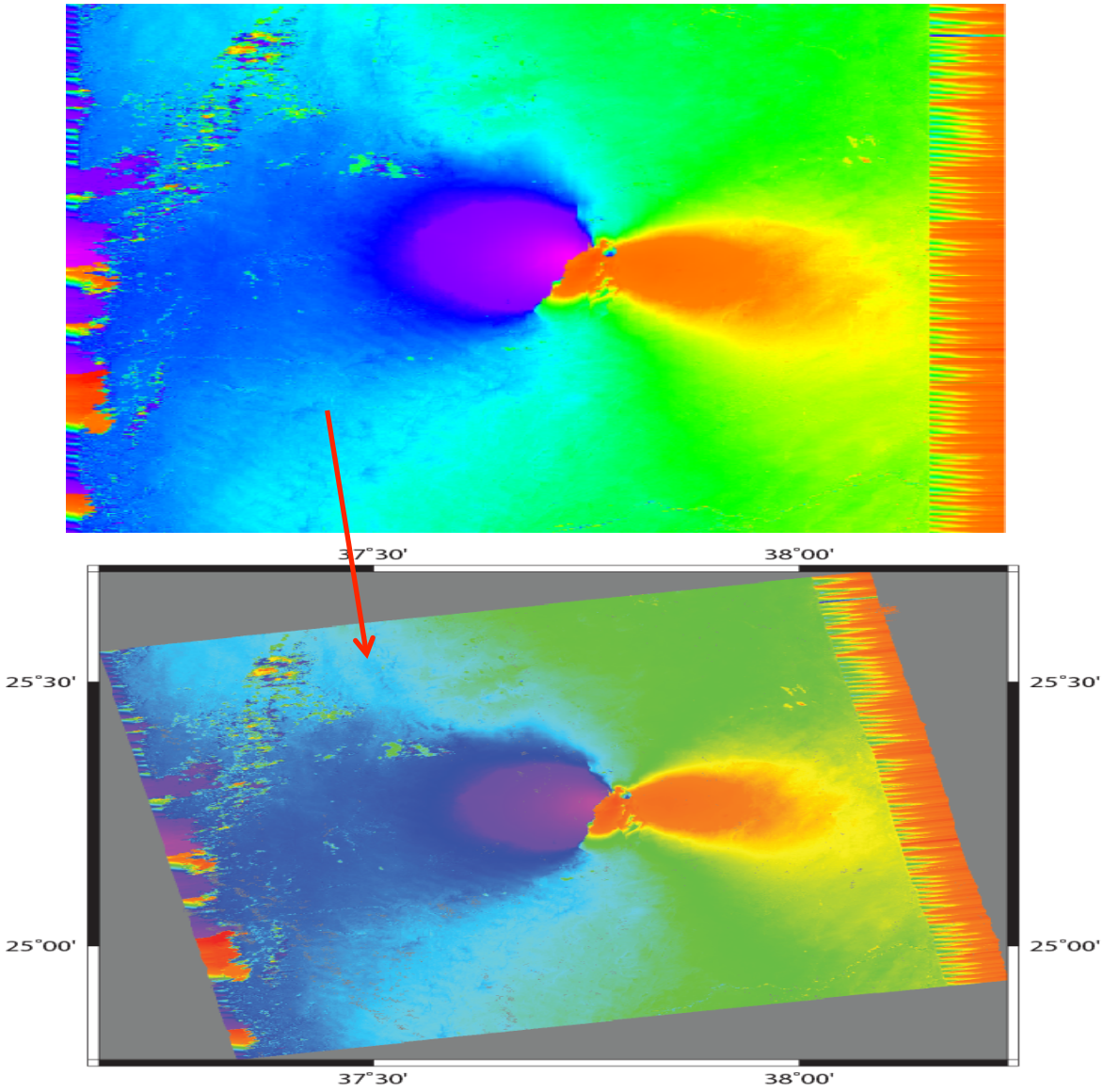
Saudi
7/1/08-8/19/08
ALOS
Bperp 20 m
(low coherence masked out)

want to eliminate “fringes”





Geocoding



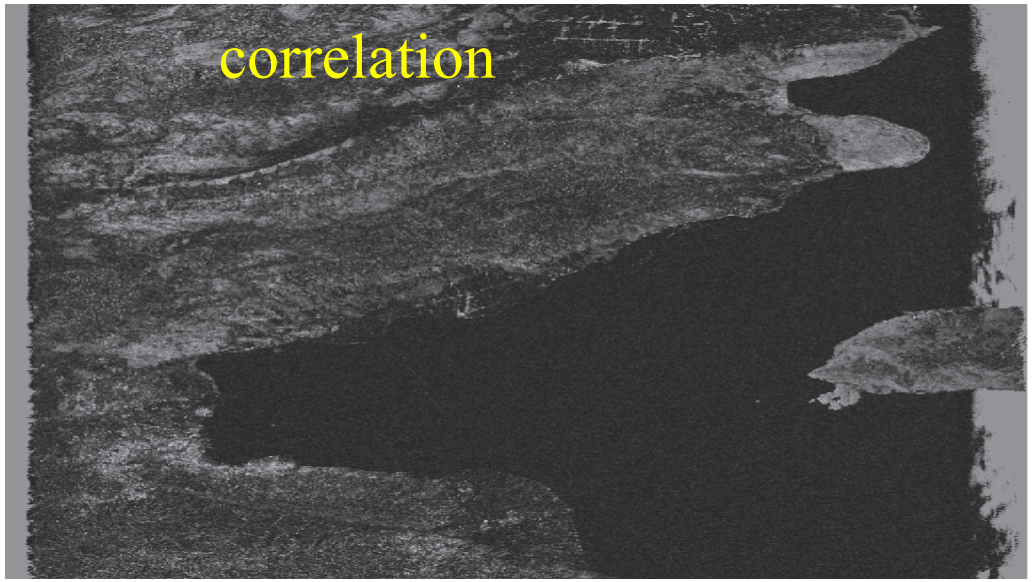
•An elegant and easy solution, but….
doesn't work very well with noise.
•Tends to underestimate true phase when noise exists
(it's a least-square fit).
•No easy way to add weighting short of iterating.
Matrix methods solve:

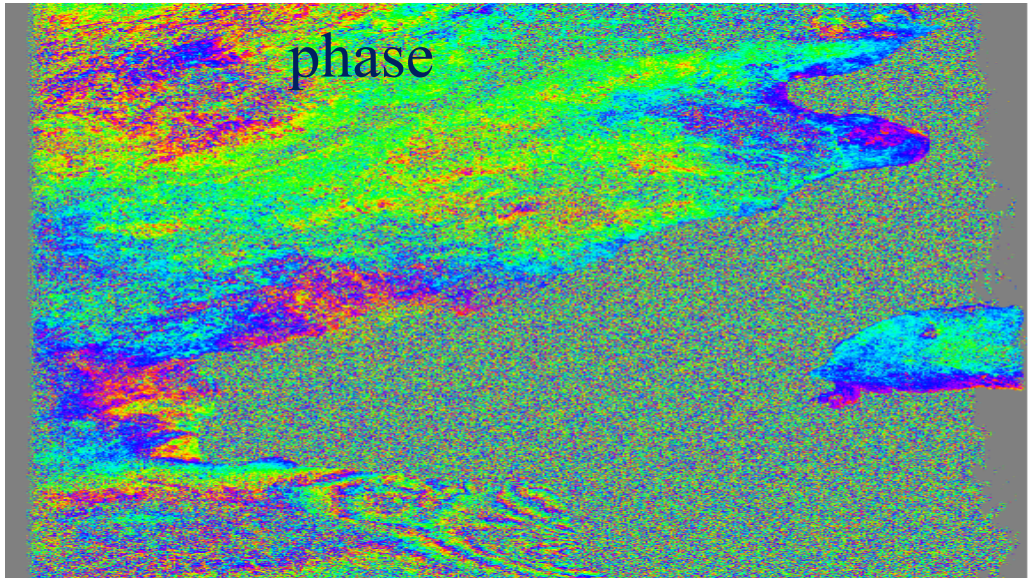
$$\mathbf{Ax} = \mathbf{b}$$

With weighting: $\mathbf{WAx} = \mathbf{Wb}$

W = matrix of weights
A = operator
B = set of observed phase values
x = unknown function
These work better than the transform methods but like all global solutions, do not provide a good fit anywhere.

Using correlation





azimuth

range

Haiti
ALOS L-band
(23 cm)
ascend
T447, F249

3/9/09-1/25/10

Bperp = 780
(gmtsar)