Principles and Theory of Radar Interferometry

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UNAVCO Short Course

Outline of Tutorial

MORNING SESSION

I. Fundamentals of Interferometry
- A. Young's double slit experiment
- B. Variations to Young's experiment
- C. Extension to radar interferometry

II. Radar Imaging Fundamentals
- A. Basic Principles of Remote Sensing, Radar, and SAR
- B. Range and Azimuth Compression
- C. Doppler and its Implications
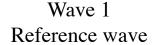
Outline of Tutorial

AFTERNOON SESSION

I. Properties of radar images

II. Geometric Aspects of Interferometry and Interferometric Phase
    A. Interferometry for Topographic Mapping
    B. Interferometry for Deformation Mapping

III. Interferometric Correlation
    A. SNR and Interferometric Correlation
    B. Geometric, Temporal and Volumetric Decorrelation
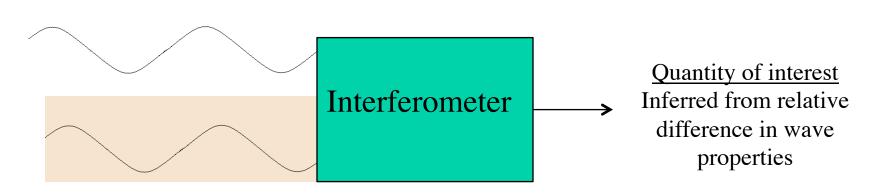   C. Other Error Sources

What is interferometry?

Interferometry is a family of techniques in which waves, usually electromagnetic, are superimposed in order to extract information about the relative change in wave properties.

Wave 1
Reference wave

Interferometer

Quantity of interest
Inferred from relative difference in wave properties

Wave 2
Modified wave

Waves typically modified by interaction with media, multiple reflections etc.

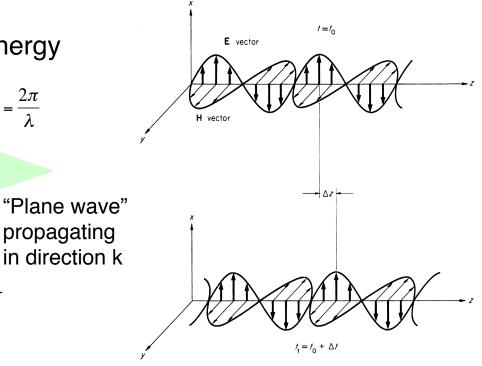# Wave Properties of Light

- Light waves are electromagnetic energy that propagates

$$k = \frac{2\pi}{\lambda}$$

"Plane wave" propagating in direction k
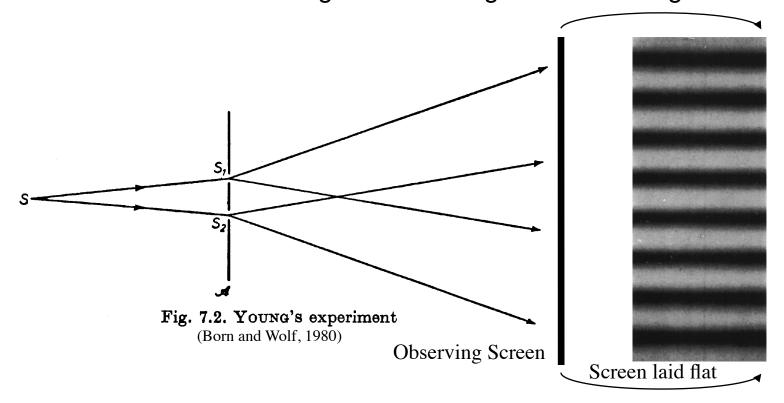
Energy propagates $\longrightarrow$

Amplitude   "Phase"   frequency

$$E(x,y,z,t) = E_0 e^{j\phi(x,y,z,t)} = E_0 e^{j(\omega t - \vec{k}\cdot\vec{r})}$$

5

Young's double slit experiment

- In Young's experiment, a point source illuminates two separated vertical slits in an opaque screen. The slits are very narrow and act as line sources. For this case, the pattern of intensity variations on the observing screen is bright/dark banding.

Fig. 7.2. Young's experiment
(Born and Wolf, 1980)

Observing Screen

Screen laid flat

Interference Concept

- Interference occurs when the phase of two different waves is not aligned. The observed intensity, I, is the time average of the sum of the wave fields
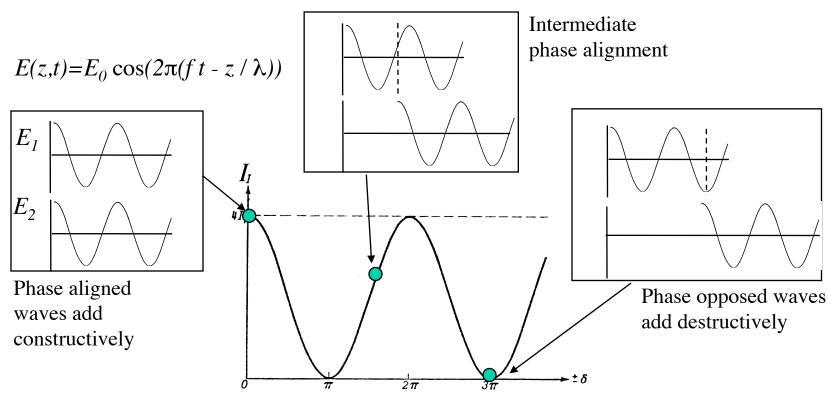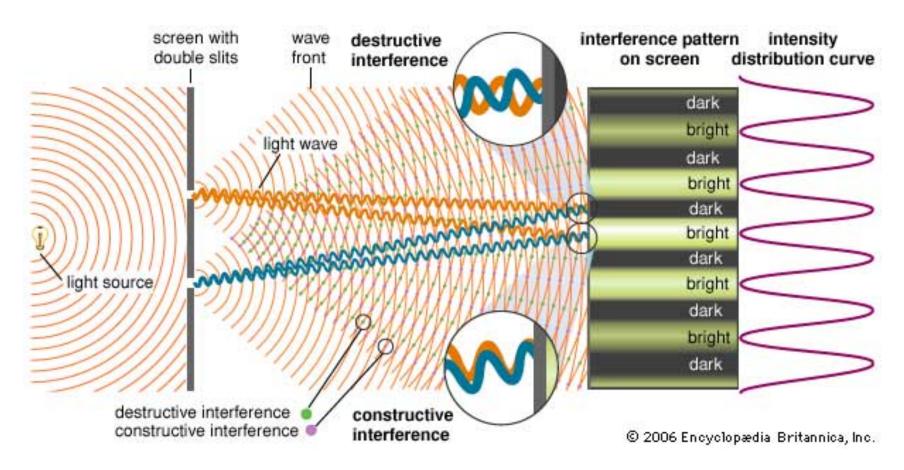
$$E(z,t) = E_0 \cos(2\pi(f t - z / \lambda))$$

Intermediate phase alignment

E_1

E_2

I_I

Phase aligned waves add constructively

Phase opposed waves add destructively

Fig. 7.1. Interference of two beams of equal intensity; variation of intensity with phase difference.
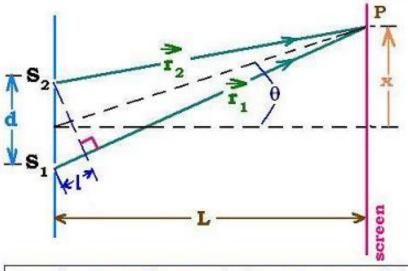
Wave interference

© 2006 Encyclopædia Britannica, Inc.

Band spacing depends on
1. wavelength
2. Spacing between the two slits

The math

L >> d => Lines from each slit to P are parallel

$$\Rightarrow \quad \sin\theta = \frac{x}{L} = \frac{l}{d}$$
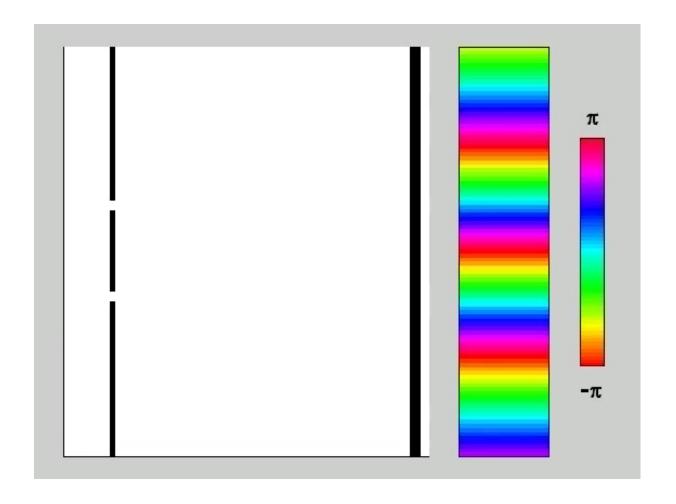
$$l = d \cdot \frac{x}{L}$$ Path difference

$$\Delta\phi = 2\pi \frac{l}{\lambda}$$ Phase difference

Baseline / slit separation

Animation demonstrating the relationship between
baseline and phase difference.

Generalizing Young's interferometer

- Two slits ensure that the interfering waves have same wavelength (derived from one source).

- On the screen, we observe (amplitude) the sum of the two waves.

- Individual wave information is lost.

- Two waves should be of similar amplitude.

What if we can vary the experimental setup?

Variant 1: Hypothetical Phase detector

- What if phase could be directly estimated?
- Can we use only one source at a time?

$$A_1 \cdot e^{-j\vec{k}\cdot\vec{r}_1} + A_2 \cdot e^{-j\vec{k}\cdot\vec{r}_2}$$ Original interferometer

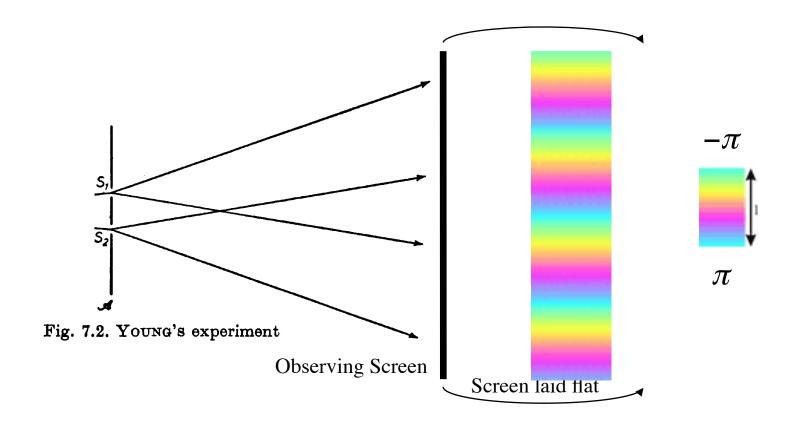$$A_1 \cdot e^{-j\vec{k}\cdot\vec{r}_1} \times \mathrm{conj}\left(A_2 \cdot e^{-j\vec{k}\cdot\vec{r}_2} \right)$$ If direct phase measurements were possible.

- No more dependence on <u>amplitude</u>
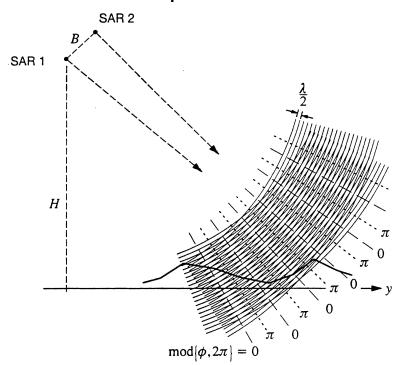
Phase difference image

Fig. 7.2. YOUNG's experiment

Observing Screen

Screen laid flat

$-\pi$

π

- Both sources operated at different times.
- Fringes generated by combining phase later.

- Radar Interferometry is a simple extension of the Young's interferometry concept

- Radar has a coherent source much like a laser

- The two radar (SAR) antennas act as coherent point sources

- SAR image is equivalent to a phase and amplitude detector.

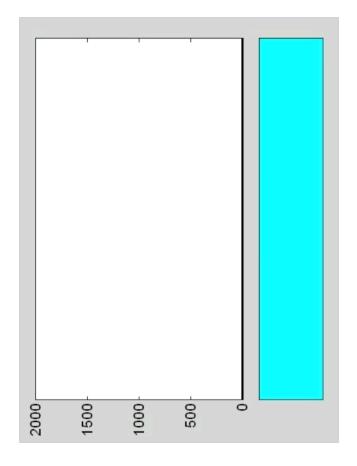- Two SAR images can then be used to generate interferograms.

Variant 2: Topography/Deformation

- What if the screen were not flat?
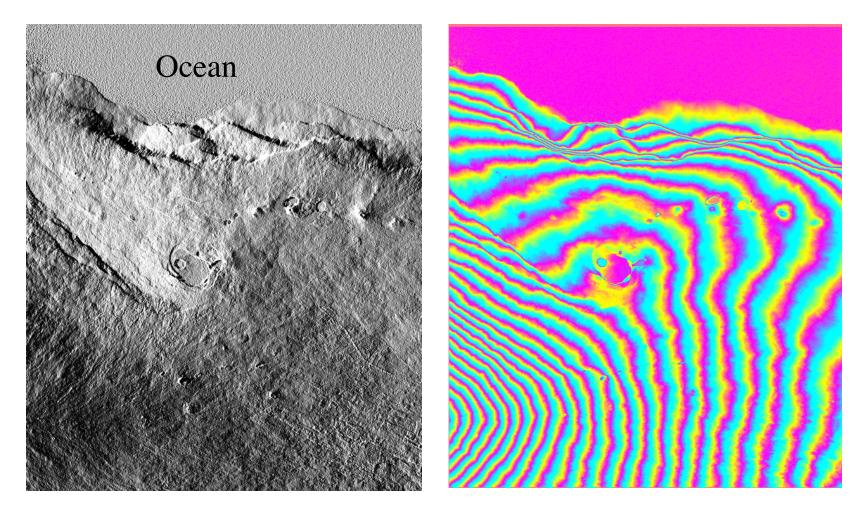- What if the screen deformed between two images?

Animation
Observed phase

-

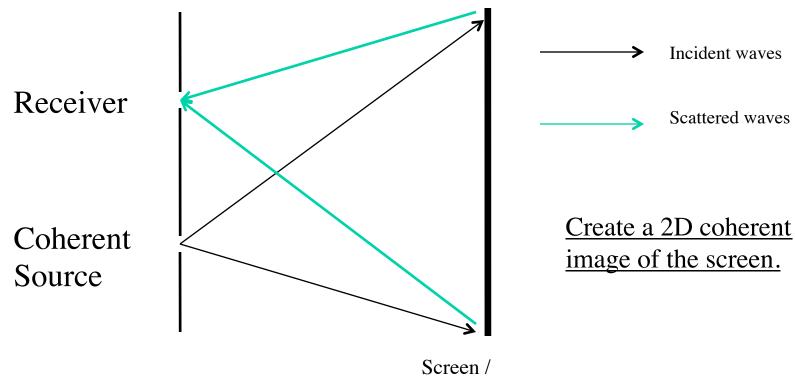Flat screen phase

as a function of
screen topography

Ocean

- The surface topography modulates the interference pattern.

Variant 3: Coherent Imaging system

- Practical problem
  - we can't set up phase detectors everywhere.
- Possible solution

Receiver

Coherent
Source

Incident waves

Scattered waves

Create a 2D coherent
image of the screen.

Screen /
scattering surface

2D imaging system

- Wavelength/ frequency needs to be constant for preserving phase information

- Source and receiver combination used to create a 2D image of the screen / scattering surface

- Often source and receiver are co-located

$$\Delta\phi(x,y) = \phi_1(x,y) - \phi_2(x,y)$$

$$\phi_i = \text{source} \rightarrow \text{pixel} + \text{pixel} \rightarrow \text{receiver}$$

Radar is one such coherent imaging system

Moving on to radar fundamentals

- We have used simple variants of Young's interferometer to <u>motivate</u> the use of radar as a coherent imaging system.

- Next section of talk will focus on fundamentals of radar imaging.

- Besides being a coherent imaging system, One of the main benefits of using radar: <u>Remote Sensing</u>

What is Remote Sensing?

- In modern usage, **Remote sensing** generally refers to the use of aerial [or satellite] sensor technologies to detect and classify objects on Earth [or other planets] (both on the surface, and in the atmosphere and oceans) by means of propagated signals (e.g. electromagnetic radiation emitted from aircraft or satellites).

- Active
  - Radar, sonar, lidar…

- Passive
  - Multi/hyperspectral, photometers, radiometers, gravity sensors, field detectors, seismometers…

- Technique and sensor choice depends on what information is desired and the required accuracy and resolution for that information.

What do we want to measure?

- Topography*
- Geography*
- Chemistry
  - Composition
  - Phase
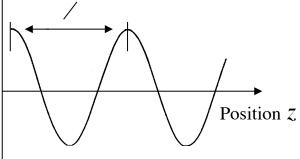- Dynamics
  - Thermo-
  - Hydro*-
  - Geo*-
  - Bio*-

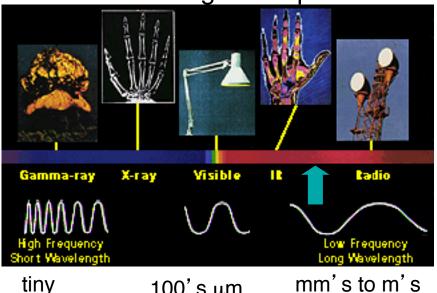*Radar and Radar Interferometry can be useful

Radar and Light Waves

Millimeters to meters wavelength

Position z

$$\lambda = \frac{c}{f}$$

- Radars operate at microwave frequencies, an invisible part of the electromagnetic spectrum

- Microwaves have wavelengths in the millimeter to meter range

- Like lasers, radars are coherent and nearly a pure tone

Common Radar Frequency Bands

Band	Ka	Ku	X	C	S	L	P
Wavelength (cm)	1	2	3	6	12	24	75
Frequency (G-cycles/s)	30	15	10	5	2.5	1.2	0.4

The Electromagnetic Spectrum

Gamma-ray   X-ray   Visible   IR   Radio

High Frequency Short Wavelength   Low Frequency Long Wavelength

tiny   100's μm   mm's to m's

The Radar Concept

Transmitter/ Receiver

At light speed, c

Object scatters energy back to radar
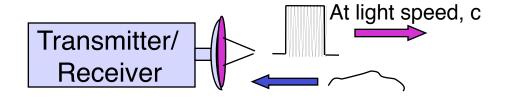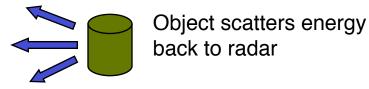
- Much like sound waves, radar waves carry information that echoes from distant objects

- The time delay of the echo measures the distance to the object

- The changes of the message in the echo determines the object characteristics
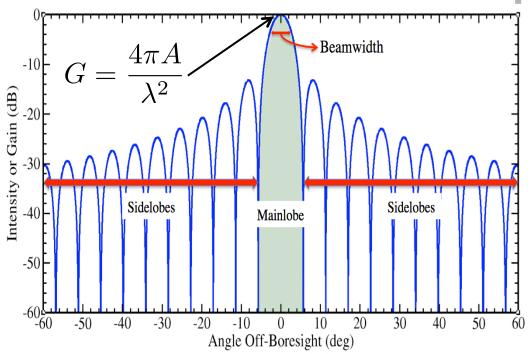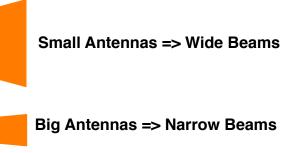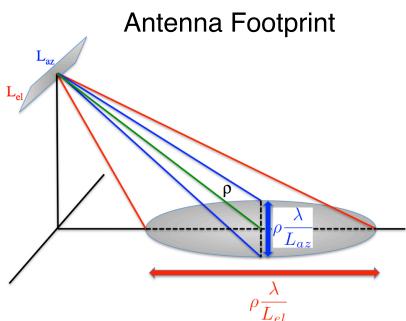
A Note on Antennas

- Antennas direct the radiation into a desired angular region that depends on the size of the antenna and its wavelength.

$$\theta_{3dB} \approx k\frac{\lambda}{L}$$

Small Antennas => Wide Beams

Big Antennas => Narrow Beams

Antenna Gain Pattern

$$G = \frac{4\pi A}{\lambda^2}$$

Beamwidth

Intensity or Gain (dB)

Sidelobes   Mainlobe   Sidelobes

Angle Off-Boresight (deg)

Antenna Footprint

L_{az}

L_{el}

ρ

$\rho\frac{\lambda}{L_{az}}$

$\rho\frac{\lambda}{L_{el}}$

Radar on a Moving Platform

Small antenna - big beam

Big antenna -small beam

Radar antenna

Radar flight path

radar pulse in air

Azimuth direction

Radar swath

Range direction

continuous stream of pulses
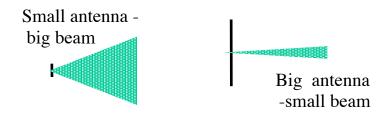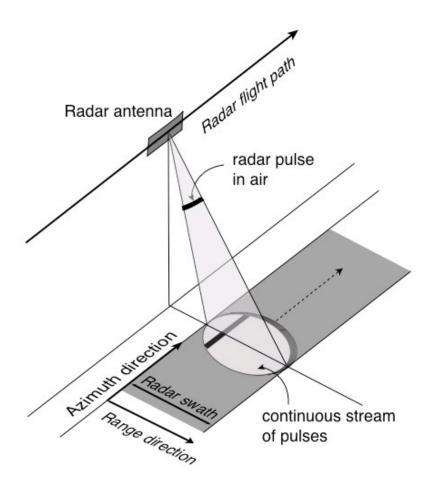
- Pulses are transmitted from the radar platform as it moves along its flight path

- Each pulse has finite extent in time, illuminating a narrow strip of ground as it sweeps through the antenna beam

- Some of the energy from the ground is scattered back to the radar instrument

Antenna Beamwidth

$$\theta_{3dB} = \frac{\lambda}{L}$$

Footprint Size on Ground

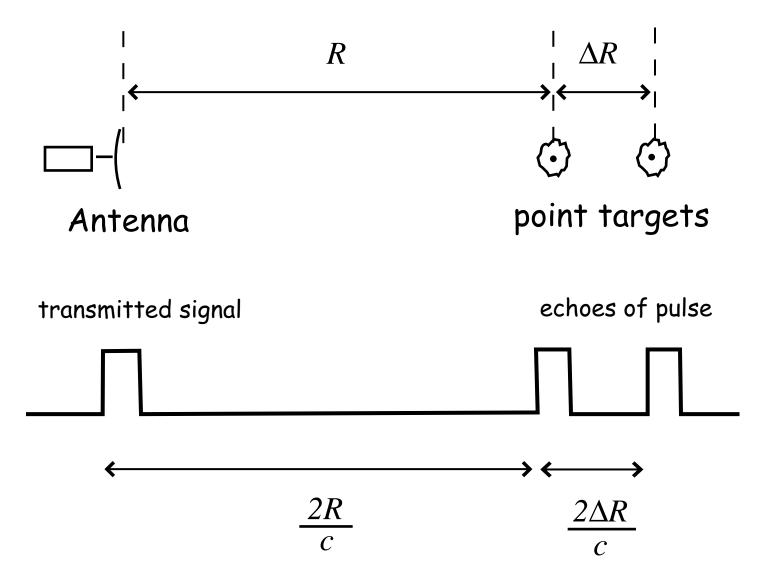$$\Delta X_{az} \approx \rho\theta_{3dB_{az}} = \frac{\rho\lambda}{L_{az}}$$

$$\Delta X_{\rho} \approx \rho\theta_{3dB_{\rho}} = \frac{\rho\lambda}{L_{\rho}}$$
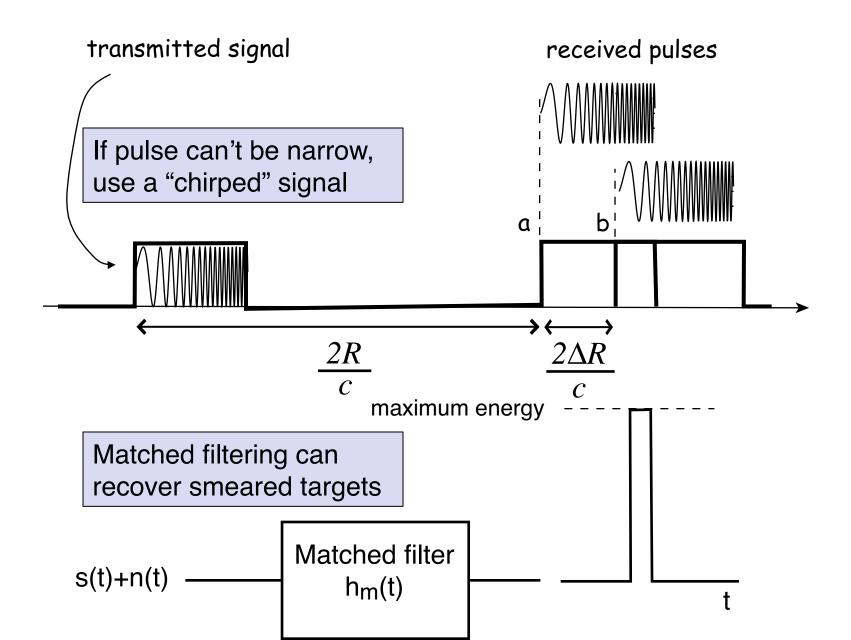
Achieving resolution in range

R ΔR

Antenna   point targets

transmitted signal   echoes of pulse

$$\frac{2R}{c}$$

$$\frac{2\Delta R}{c}$$

If possible, use a narrow pulse to resolve close targets

Achieving resolution in range

transmitted signal

received pulses

If pulse can't be narrow, use a "chirped" signal

a   b

$$\frac{2R}{c}$$

$$\frac{2\Delta R}{c}$$

maximum energy

Matched filtering can recover smeared targets

s(t)+n(t) —— Matched filter $h_m(t)$

t

Range and Range Resolution

$$\rho = \frac{ct_{rt}}{2}$$
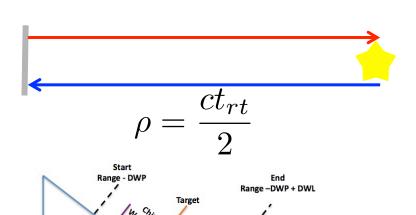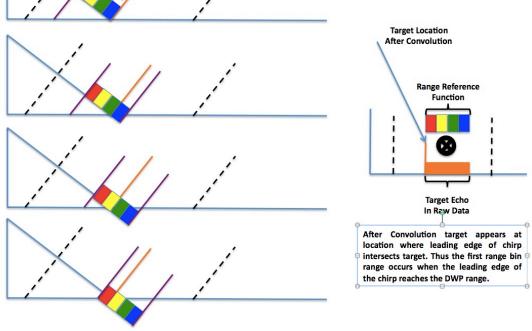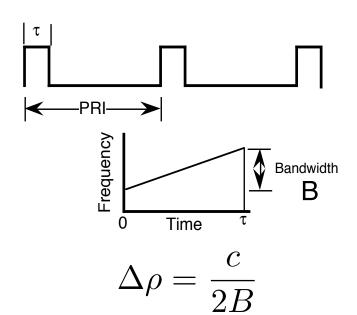
- Range is the distance between the radar and an object

  - Range resolution would normally be limited by the length of the emitted pulse.
  - Better resolution is achieved by transmitting coded waveforms. Most typical encoding is a chirp.

Start
Range - DWP

End
Range –DWP + DWL

Target

Chirp Waveform

Target Location
After Convolution

Range Reference
Function

⊗

Target Echo
In Raw Data

After Convolution target appears at location where leading edge of chirp intersects target. Thus the first range bin range occurs when the leading edge of the chirp reaches the DWP range.

τ

PRI

Frequency

0   Time τ

Bandwidth
B

$$\Delta\rho = \frac{c}{2B}$$

Quick detour: intro2radar.com

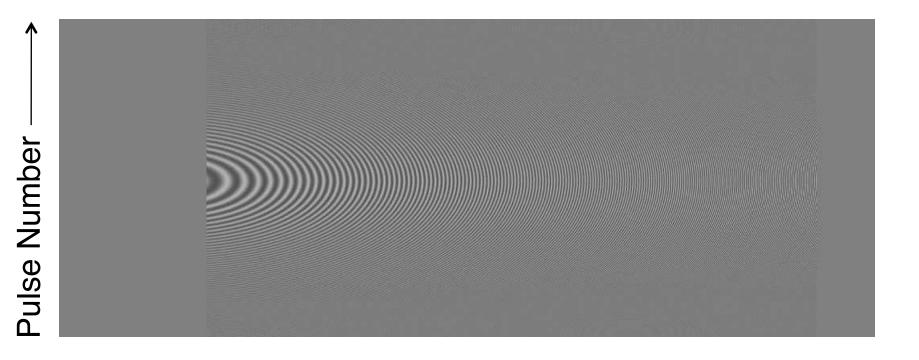A VISUAL INTRODUCTION TO RADAR REMOTE SENSING

By Iain H Woodhouse

Raw Signal of a Point Target
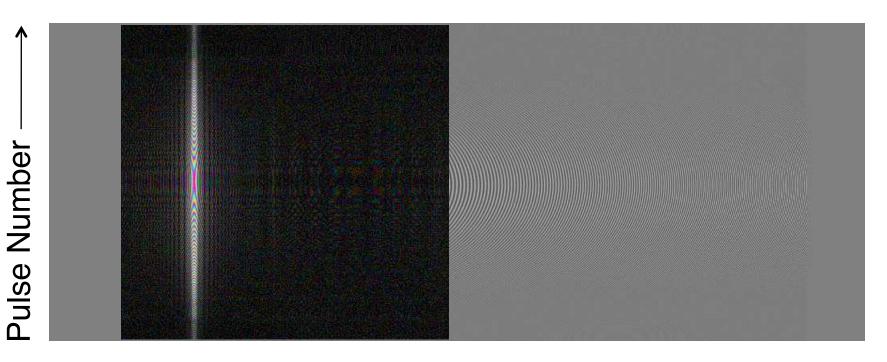
Range →

Pulse Number →

This "image" shows a sequence of simulated pulse echoes from a single point target

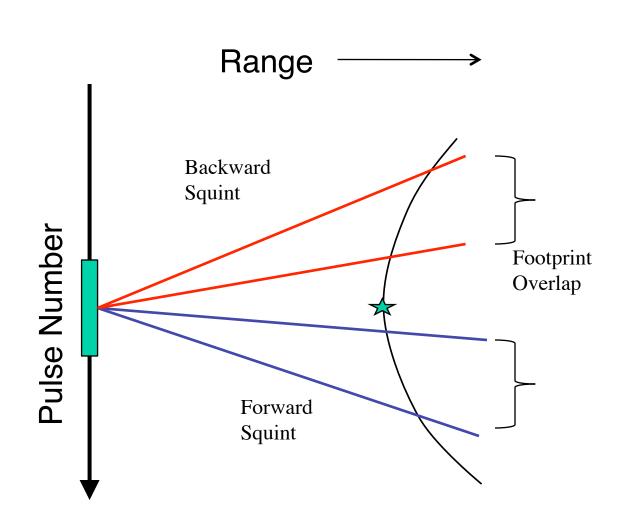# Range Compression of a Point Target; after Matched filtering

Range ⟶

Pulse Number ⟶

This "image" shows the simulated pulse echoes after range compression matched filtering

- Squint determines which part of the range curve is sampled by the footprint.

- Doppler centroid is a measure of squint.

Range

Pulse Number

Backward Squint

Forward Squint

Footprint Overlap

Doppler Shift

- Objects moving relative to a radar experience a frequency shift called the Doppler shift.

- Objects moving toward the radar have higher frequencies

- Objects moving away from the radar have lower frequencies.

$$f_d = \frac{2\langle \vec{v}, \hat{\ell} \rangle}{\lambda} = \frac{2v \cos\theta_{sq}}{\lambda}$$

Doppler in SAR Imaging

$$R(x) = \sqrt{R_0^2 + (x - x_0)^2}$$

$$\approx R_0 \left(1 + \frac{(x - x_0)^2}{2R_0^2}\right)$$

$$\approx R_0 + \frac{(x - x_0)^2}{2R_0}$$

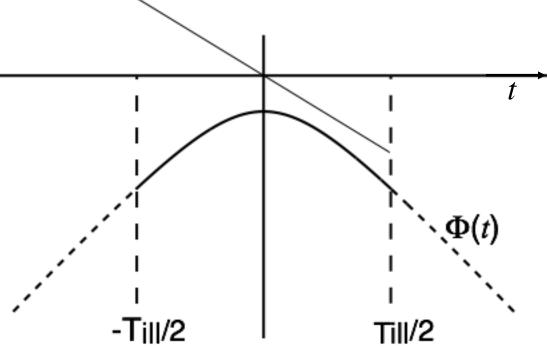Target at (x_0, R_0)

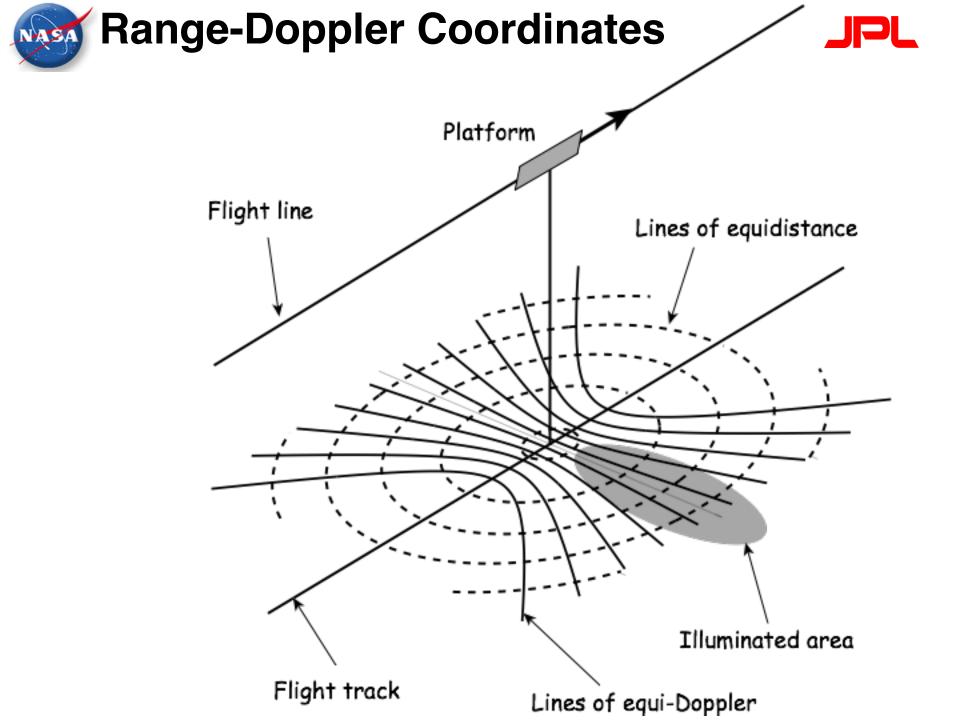x is satellite position

$f_D(t)$

Dropping the constant term

$$\phi(t) = -\frac{\pi}{F^2} v^2 (t - t_0)^2$$

$$\omega(t) = -2\pi \frac{v^2}{F^2} (t - t_0)$$

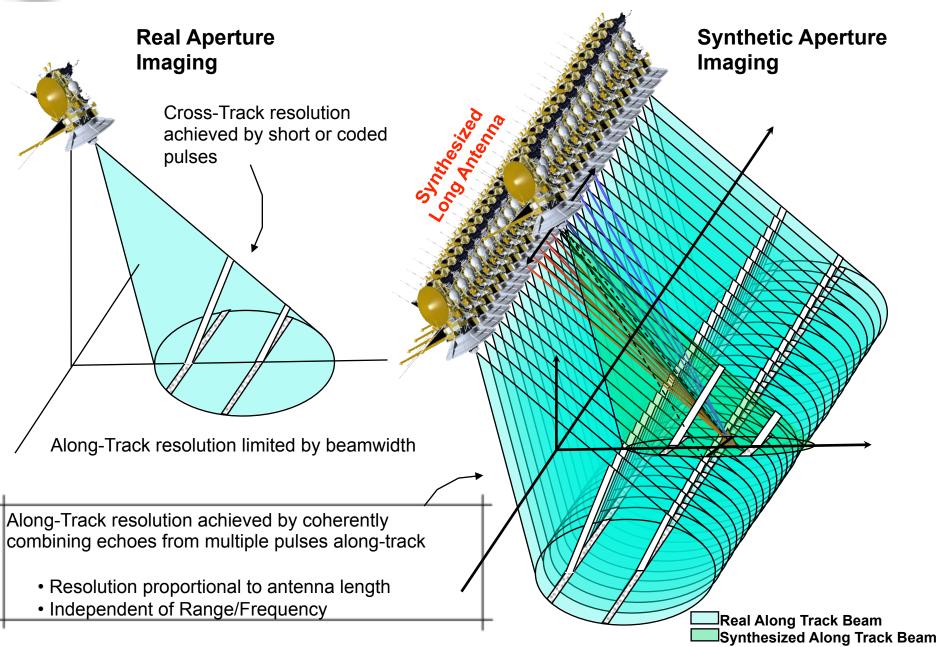$$f_D(t) = -\frac{v^2}{F^2} (t - t_0)$$

$$F^2 = \lambda R_0 / 2.$$

t

$\Phi(t)$

$-T_{ill}/2$ $T_{ill}/2$

Platform

Flight line

Lines of equidistance

Illuminated area

Flight track

Lines of equi-Doppler

SAR Imaging Concept

Real Aperture Imaging

Cross-Track resolution achieved by short or coded pulses

Along-Track resolution limited by beamwidth

Along-Track resolution achieved by coherently combining echoes from multiple pulses along-track

- Resolution proportional to antenna length
- Independent of Range/Frequency

Synthetic Aperture Imaging

Synthesized Long Antenna

Real Along Track Beam
Synthesized Along Track Beam

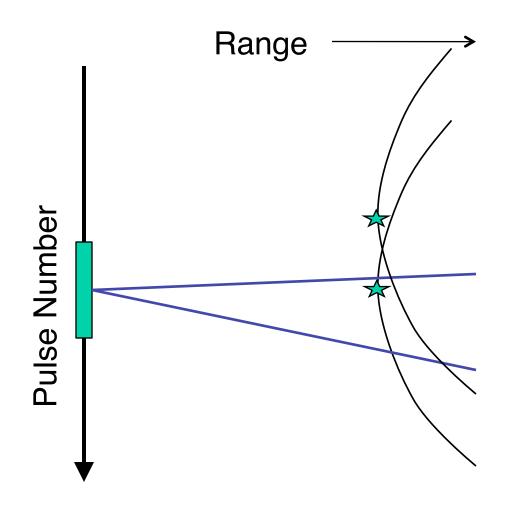# Targets after range compression

Two targets at same range have different range curves.

Combining information from multiple pulses allows use to resolve the targets.

Nyquist Sampling Theorem

- A consequence of the Nyquist Sampling Theorem is the the radar PRF must be twice the bandwidth of the received signal in order to avoid spectral aliasing of the signal.

- Using the expression from the previous viewgraph for the azimuth bandwidth we have that

$$PRF \geq \frac{2v}{L_{az}} \cos \theta_{az}$$

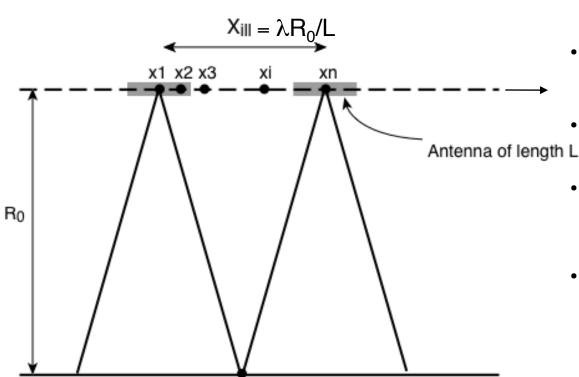which for small squint angles, θ_{az} smaller than 25°, is often simplified to

$$PRF \geq \frac{2v}{L_{az}}$$

- Note that the required PRF is independent of wavelength and range and only depends on the platform velocity and the azimuth antenna dimension.

Courtesy Scott Hensley

Azimuth Resolution from Aperture Synthesis

$X_{ill} = \lambda R_0/L$

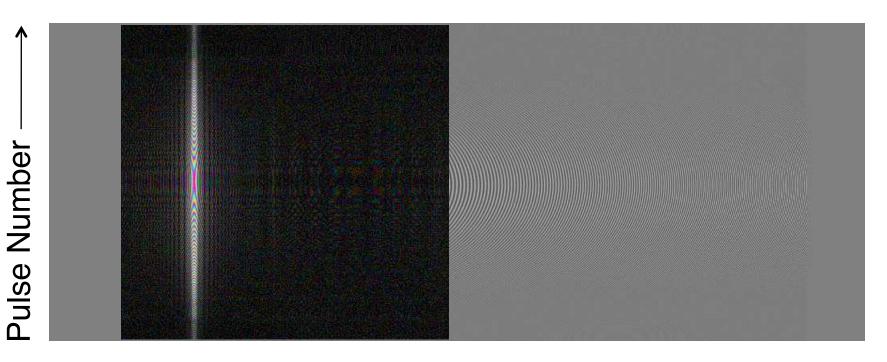x1 x2 x3   xi   xn

Antenna of length L

R_0

- Azimuth Footprint, X_{ill} = $\lambda R_0/L$
- Target is visible for
  2*Footprint at R_0
- Thus, length of synthetic aperture is 2 x Footprint.
- Equivalent azimuth resolution = $(\lambda R_0/(2*X_{ill})=$ (L/2) is constant
- Aperture synthesis processing is very similar to matched filtering in range

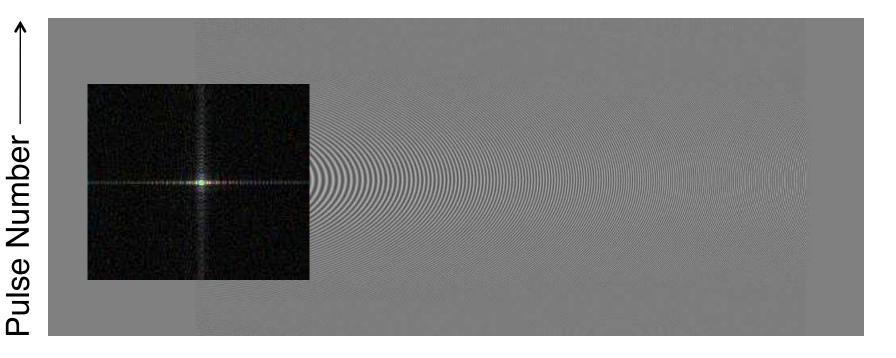# Range Compression of a Point Target; after Matched filtering

Range →

Pulse Number →

This "image" shows the simulated pulse echoes after range compression matched filtering

Point Target after Azimuth Compression Matched filtering

Range ⟶

Pulse Number ⟶

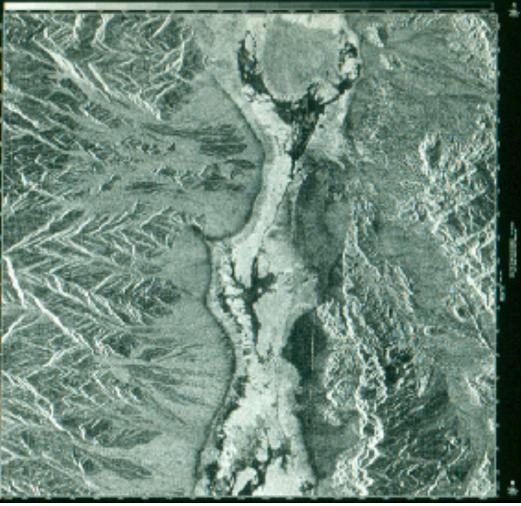This "image" shows the simulated pulse echoes after range compression matched filtering

SEASAT – SAR in Space in 1978
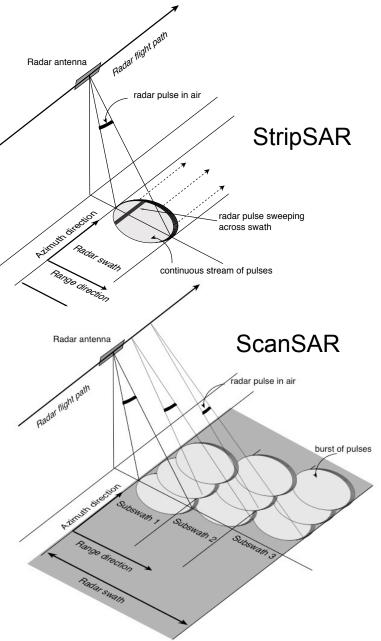

SEASAT SAR image of Death Valley

Conventional SAR modes

- ✦ Strip-mode SAR
  - ❏ Standard SAR mode
  - ❏ Send a pulse of energy; receive echo; repeat
  - ❏ One pulse transmit and receive at a time
  - ❏ Swath width limited by radar ambiguities
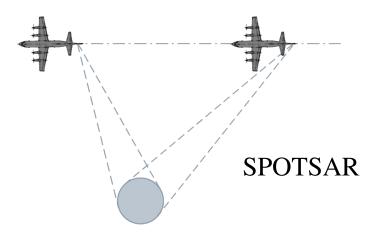
- ✦ ScanSAR
  - ❏ Wider swath low resolution SAR mode
  - ❏ Execute sequence as follows:
    - ◆ Send a pulse of energy; receive echo; repeat 50-100 times
    - ◆ Repoint the beam across-track to position 2 electrically (almost instantaneous)
    - ◆ Send a pulse of energy; receive echo; repeat 50-100 times
    - ◆ Repoint the beam across-track to position 3 electrically (almost instantaneous)
    - ◆ Send a pulse of energy; receive echo; repeat 50-100 times
  - ❏ Again, one pulse transmit and receive at a time
  - ❏ ScanSAR trades resolution (in along-track dimension) for swath: low impact on data rate
  - ❏ Generally poorer ambiguity and radiometric performance than Strip SAR

StripSAR

ScanSAR

Conventional SAR modes

SPOTSAR

Copyright: radartutorial.eu

TOPSSAR

Copyright: DLR

✦ Spotlight SAR

❑ Steers beam to focus on smaller area

❑ Higher resolution than Stripmap

❑ Doppler variation along track

✦ TOPS SAR

❑ Wider swath low resolution SAR mode

❑ Steer the beam to cover a larger swath

❑ Trades resolution (in along-track dimension) for swath.

❑ Sentinel-I later this year

In the afternoon session:

I. Properties of radar images

II. Geometric Aspects of Interferometry and Interferometric Phase

III. Interferometric Correlation

IV. Interferometric Processing Workflow