Least-squares fit of range history for each point in DEM provides both the accurate position in range azimuth $\left[R_o, s_o\right]$ space and the Dopper centroid and rate parameters needed to focus the image. This analysis only needs to be applied to the master image.

phase history of point reflector

$$C(s) = \exp \left\{ i \frac{4\pi}{\lambda} \left[ R(s) \right] \right\}$$

parabolic approximation to range history

$$R(s) = R_o + \dot{R}_o (s - s_o) + \frac{\ddot{R}_o}{2} (s - s_o)^2 + \ldots$$

$$f_{DC} = \frac{-2\ddot{R}}{\lambda} \quad f_R = \frac{2\dot{R}}{\lambda}$$
http://topex.ucsd.edu/gmtsar

Scripps Institution of Oceanography Technical Report

**GMTSAR: An InSAR Processing System Based on Generic Mapping Tools**

David Sandwell\(^{(1)}\), Rob Mellors\(^{(2)}\), Xiaopeng Tong\(^{(1)}\), Matt Wei\(^{(3)}\), and Paul Wessel\(^{(4)}\)

May 1, 2011

\(^{(1)}\) Scripps Institution of Oceanography, University of California, San Diego, La Jolla, CA

\(^{(2)}\) Lawrence Livermore National Laboratory, Livermore, CA

\(^{(3)}\) Woods Hole Oceanographic Institution, Woods Hole, MA

\(^{(4)}\) Department of Geology and Geophysics, University of Hawaii at Manoa, HI
Table of Contents:

1. Abstract
   1.1 Objectives and limitations of GMTSAR
   1.2 Algorithms: SAR, InSAR and the need for precise orbits
      1.2.1 Proper focus
      1.2.2 Transformation from geographic to radar coordinates
      1.2.3 Image alignment
      1.2.4 Flattening interferogram - no trend removal
2. Software
   2.1 Standard products
   2.2 Software design
3. Processing Examples
   3.1 Two-pass processing
   3.2 Stacking and time series
   3.3 ScanSAR Interferometry
4. References
5. Problems

Appendix A. Principles of Synthetic Aperture Radar
Appendix B. SAR Image Formation
Appendix C. InSAR
Appendix D. ScanSAR processor and interferometry
Appendix E. Geolocation accuracy for Pineon corner reflectors
Appendix F. Installation of GMTSAR
amplitude and phase

step 1 - SAR (amplitude)

step 2 - InSAR (phase difference)
coherence and pixel matching
The illumination pattern on the screen is shown in the following diagram.

The first zero crossing, or angular resolution \( \theta_c \), of the sinc function occurs when the argument is \( \pi \) so \( \sin \theta_c = \frac{\lambda}{L} \) and for small angles \( \theta_c \approx \lambda/L \) and \( \tan \theta_c \approx \sin \theta_c \). Note that
resolution: optical vs. microwave

\[ D_s = 2H \sin \theta_r = 2H \frac{\lambda}{L} \]

\[ H = 800 \text{km}. \]

**Optical:**

\[ L = 1m \]
\[ \lambda = 0.5 \mu m \]
\[ D_s = 0.8m \]

**Microwave:**

\[ L = 10m \]
\[ \lambda = 0.23m \]
\[ D_s = 46,000m!!!!!! \]
2-D Aperture

\[
P(\theta_x, \theta_y) = \int_{-W/2}^{W/2} \int_{-L/2}^{L/2} A(x, y) \exp \left[ i \frac{2\pi}{\lambda} (x \sin \theta_x + y \sin \theta_y) \right] dx dy
\]

\[
P(\theta_x, \theta_y) = LW \ \text{sinc} \left( \frac{\pi W \sin \theta_x}{\lambda} \right) \ \text{sinc} \left( \frac{\pi L \sin \theta_y}{\lambda} \right)
\]
range resolution

\[ R_r = \frac{C \tau}{2 \sin \theta} \]

- \( \theta \): look angle
- \( H \): spacecraft height
- \( \tau \): pulse length
- \( C \): speed of light (sound)
azimuth resolution

$L$ - length of radar antenna
$\rho$ - nominal slant range $H/\cos\theta$
$\lambda$ - wavelength of radar

unfocussed

$$R_a = \rho \sin \theta_r = \rho \lambda / L$$

focussed

$$R_a' = \frac{\lambda H}{2R_a \cos \theta} = \frac{L}{2}$$
**PRF - upper bound**

The PRF cannot be too large or the return pulses from the near range and far range will overlap in time as shown in Figure A9.

![Figure A9. End view of the distance to the near range and far range of the radar illumination pattern.](image)

\[
PRF < \frac{1}{t_2 - t_1}
\]

\[
t_1 = \frac{2H}{C \cos \theta_1} \quad t_2 = \frac{2H}{C \cos \theta_2}
\]

\[
\Delta t = \frac{2H}{C} \left( \sec \theta_2 - \sec \theta_1 \right)
\]

\[
\frac{2V}{L} < PRF < \frac{C}{2H} \left( \sec \theta_2 - \sec \theta_1 \right)^{-1}
\]

For ERS the look angles to the rear range and far range are 18° and 24°, respectively. Thus the maximum PRF is 4777 Hz. The actual PRF of 1680 is safely below this value.
Problems

1) What is the illumination pattern for an aperture with a sign reversal at its center? What is \( P(0) \)? Is the function real or imaginary? Is the function symmetric or asymmetric?

The aperture is

\[
A(y) = \begin{cases} 
0 & |y| \geq \frac{L}{2} \\
1 & 0 < y \leq \frac{L}{2} \\
-1 & -\frac{L}{2} \leq y < 0
\end{cases}
\]

3) What is the theoretical azimuth resolution of a spotlight-mode SAR that can illuminate the target over a \( 10^\circ \) angle as shown in the diagram below.
5) What is the ground-range resolution of side-looking radar with a pulse length of $6 \times 10^{-8}$ s and a look angle of $45^\circ$?

7) (a) What is the period for a satellite in a circular orbit about the moon where the radius of the orbit is $1.9 \times 10^6$ m? The mass of the moon is $7.34 \times 10^{22}$ kg.
(b) You are developing a SAR mission for the moon. The length of your SAR antenna is 10 m. What minimum pulse repetition frequency is needed to form a complete aperture? The circumference of the moon is $1.1 \times 10^7$ m. You will need the orbital period from problem (a).

8) Derive equation (A10)
Table of Contents:

Abstract

1. Introduction
   1.1 Objectives and limitations of GMTSAR
   1.2 Algorithms: SAR, InSAR and the need for precise orbits
      1.2.1 Proper focus
      1.2.2 Transformation from geographic to radar coordinates
      1.2.3 Image alignment
      1.2.4 Flattening interferogram - no trend removal

2. Software
   2.1 Standard products
   2.2 Software design

3. Processing Examples
   3.1 Two-pass processing
   3.2 Stacking and time series
   3.3 ScanSAR Interferometry

4. References

5. Problems

Appendix A. Principles of Synthetic Aperture Radar
Appendix B. SAR Image Formation
Appendix C. InSAR
Appendix D. ScanSAR processor and interferometry
Appendix E. Geolocation accuracy for Pinon corner reflectors
Appendix F. Installation of GMTSAR
How is the image focused in the computer?

- SAR Antenna
- Satellite trajectory
- Look Angle
- Radar Pulses
- Sub-satellite ground track
- Swath
- Footprint
SAR processor

Digital SAR processing overview

1. Range compression performed on each radar echo

2. Patch processing of range-compressed echoes

3. Range migration performed on each pixel of range-compressed data whose columns have been Fourier transformed

4. Azimuth compression performed on each column of range-compressed, range-migrated data

Single Look Complex (SLC) image data file
Table B1. Example Parameter (PRM) file (e2_10001_2925.PRM)

<table>
<thead>
<tr>
<th>file information/format</th>
<th>value</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>input_file</td>
<td>e2_10001_2925.fix</td>
<td>name of raw data file</td>
</tr>
<tr>
<td>SC_identity</td>
<td>2</td>
<td>(1) – ERS-1 or (2) – ERS-2</td>
</tr>
<tr>
<td>bytes_per_line</td>
<td>11644</td>
<td>number of bytes in row of raw data</td>
</tr>
<tr>
<td>first_sample</td>
<td>206</td>
<td>412 bytes of timing information to skip over</td>
</tr>
<tr>
<td>fd1</td>
<td>248.115</td>
<td>Doppler centroid estimated from data $f_{DC}$</td>
</tr>
<tr>
<td>I_mean</td>
<td>15.5</td>
<td>mean value of real numbers</td>
</tr>
<tr>
<td>Q_mean</td>
<td>15.5</td>
<td>mean value of imaginary numbers</td>
</tr>
<tr>
<td>icu_start</td>
<td>2576039268.848</td>
<td>spacecraft clock for first sample</td>
</tr>
<tr>
<td>SC_clock_start</td>
<td>1997078.767881851</td>
<td>UTC for first sample YYYDDD.DDDDDDDDD</td>
</tr>
<tr>
<td>SC_clock_stop</td>
<td>1997078.768074699</td>
<td>UTC for last sample YYYDDD.DDDDDDD</td>
</tr>
</tbody>
</table>

| radar characteristics | | |
|-----------------------|------------------|
| PRF | 1679.902394 | pulse repetition frequency |
| range_sampling_rate | 1.89625e+07 | |
| chirp_slope | 4.17788e+11 | |
| pulse_duration | 3.712e-05 | |
| radar_wavelength | 0.056666 | |

| orbital information | | |
|---------------------|------------------|
| near_range | 829924.365777 | distance to first range bin |
| earth_radius | 637146.4379 | earth radius ~1/2 way into image |
| SC_height | 787955.52 | spacecraft height above the earth_radius |
| SC_vel | 7125.0330 | effective speed |

| processing information | | |
|------------------------|------------------|
| num_valid_azi | 2800 | size of patch to process |
| num_patches | 10 | number of patches to process |
| first_line | 1 | |
| deskew | n | deskew (yes or no) |
| st_rng | 1 | start processing at row number |
| num_rng_bins | 6144 | number of range bins in output file |
| chirp_ext | 614 | extend the chirp to process outside swath |
| Flip发明专利 | n | exchange real and imaginary numbers (yes or no) |
| nlooks | 1 | number of azimuth echoes to average |

| image alignment | | |
|-----------------|------------------|
| rshift | 15.1 | range shift to align image to master |
| ashift | 483.2 | azimuth shift to align image to master |
| stretch_r | 0.014569 | range stretch versus range |
| stretch_a | -0.0019436 | azimuth stretch versus range |
| a_stretch_r | 0.0 | range stretch versus azimuth |
| a_stretch_a | 0.0 | azimuth stretch versus azimuth |
412 bytes

11232 bytes – 5161 complex numbers

timing

raw signal data

range

azimuth

patch

incomplete aperture

complete aperture

incomplete aperture
Digital SAR processing overview

(1) Range compression performed on each radar echo

(2) Patch processing of range-compressed echoes

(3) Range migration performed on each pixel of range-compressed data whose columns have been fourier transformed

(4) Azimuth compression performed on each column of range-compressed, range-migrated data

Single Look Complex (SLC) image data file
\[ s(t) = e^{j \omega t} \quad |t| \leq \tau_p / 2. \]

where

- \( k \) - chirp slope (4.17788 \times 10^{11} \text{ s}^{-2})
- \( \tau_p \) - pulse duration (3.712 \times 10^{-5} \text{ s or } \sim 11 \text{ km long})
- \( f_s \) - range sampling rate (1.89625 \times 10^3 \text{ s}^{-1}).

An example of a portion of the chirp for the ERS- radar as well as its power spectrum and impulse response is shown below.

![Graphs](image)

(a) real part of the frequency-modulated chirp for ERS.
(b) power spectrum of FM chirp has a time-bandwidth product of 570 and is well approximated by a boxcar function.
(c) deconvolved chirp has a resolution of about 27 m and prominent sidelobes.

Figure B4 Chirp for ERS-1

The bandwidth of the chirp, \( B = k \tau_p \), has a value of 15.5 MHz for the ERS radar. A matched filter is used to deconvolve the chirp from the data. In this case, the matched filter is simply the complex conjugate of the chirp or \( s^*(t) = e^{-j \omega t} \). The convolution of \( s^*(t) \) and \( s(t) \) is shown in Figure B4b. The effective range resolution of the radar is about 27 m.
Figure B5. Raw ERS signal data (left) and range compressed data (right) for data over Pinon Flat, California where two radar corner reflectors are installed (lower).
Azimuth compression

Azimuth compression or azimuth focusing involves coherent summation of echoes at a constant range from the point reflector. The geometry of the strip-mode acquisition is shown in Figure B6

Figure B6. Geometry of radar passing over a point reflector where

- $V$ – the effective speed which is about equal to the ground track speed
- $s$ – slow time along the satellite track
- $s_o$ – time when the center of the radar echo passes over the point reflector
- $R_o = R_{near} + n^* (C / fs)$ minimum range from the spacecraft to the target
- $R_{near}$ – near range to first data sample in the swath
The range to the point reflector evolves with time as

\[ R^2(s) = R_o^2 + V^2(s - s_o)^2. \]

The complex phase of the return echo is

\[ C(s) = \exp\left[-i \frac{4\pi}{\lambda} R(s) \right]. \]

The range versus slow time is approximately a hyperbola but for mathematical convenience we’ll approximate this using a parabola

\[ R(s) = R_o + \dot{R}_o(s - s_o) + \frac{\ddot{R}_o}{2}(s - s_o)^2 + ... \]

where the dot indicates derivative with respect to slow time, \( s \). *Curlander and McDonough [1991]* discuss the accuracy of this polynomial approximation and it is also discussed below in terms of the ALOS SAR. The approximation is good enough for strip-mode SAR but may be inadequate for the much longer apertures associated with spotlight-mode SAR. Now we can write the phase of the return signal as a function of range, range rate, and range acceleration.

\[ C(s) = \exp\left\{-i \frac{4\pi}{\lambda} \left[R_o + \dot{R}_o(s - s_o) + \frac{\ddot{R}_o}{2}(s - s_o)^2 / 2 \right] \right\}. \]

It is more common to describe the parameters for focusing the SAR image as the Doppler centroid \( f_{dc} \) and the Doppler frequency rate \( f_R \). The relationships are:

\[ f_{dc} = \frac{-2 \dot{R}}{\lambda} \quad \text{and} \quad f_R = \frac{-2 \ddot{R}}{\lambda}. \]
Figure B12. (upper) range versus slow time for ALOS for a 6 second aperture. The actual aperture is 4.2 seconds (Table B3). (lower) deviation from a parabolic fit to this range vs. time function.
**Length of synthetic aperture** - The length of the synthetic aperture $L_a$ depends on the length of the radar ground footprint in the azimuth direction, which is approximately

\[
L_a = R_o \frac{\lambda}{L}
\]  
(1)

where $L$ is the physical length of the antenna, $\lambda$ is the wavelength and $R_o$ is the slant range given above. The length of the aperture in terms of radar echos is given by

\[
n_a = \frac{L_a \cdot PRF}{V}
\]

This is simply the length of the synthetic aperture divided by the along-track sampling distance. The following table provides these quantities for SARs of interest for interferometry today.

Table B3. Length of synthetic aperture for three satellites.

<table>
<thead>
<tr>
<th></th>
<th>$V$ (m/s)</th>
<th>$\lambda$ (m)</th>
<th>$PRF$ (Hz)</th>
<th>$R_o$ (km)</th>
<th>$L$ (m)</th>
<th>$L_a$ (m)</th>
<th>$n_a$ theory</th>
<th>$n_a$ actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>ERS-1/2</td>
<td>7125</td>
<td>0.057</td>
<td>1679</td>
<td>~850</td>
<td>10</td>
<td>4850</td>
<td>1142</td>
<td>1164</td>
</tr>
<tr>
<td>Envisat</td>
<td>7125</td>
<td>0.057</td>
<td>2067</td>
<td>~1020</td>
<td>10</td>
<td>5830</td>
<td>1690</td>
<td>1740</td>
</tr>
<tr>
<td>ALOS</td>
<td>7125</td>
<td>0.236</td>
<td>2159</td>
<td>~1020</td>
<td>9</td>
<td>26,830</td>
<td>8128</td>
<td>9216</td>
</tr>
</tbody>
</table>
Least-squares fit of range history for each point in DEM provides both the accurate position in range azimuth \([R_o, s_o]\) space and the Dopper centroid and rate parameters needed to focus the image. This analysis only needs to be applied to the master image.
(a) real part of the azimuth phase function for a doppler centroid of 200 Hz.

(b) power spectrum of the azimuth phase function is nearly a boxcar function that is shifted in frequency by the doppler centroid.

(c) deconvolved azimuth chirp has a resolution of about 8 m.
Figure B8. Range compressed (left) and fully focussed image (right). The two bright reflectors are 3-m corner cubes at Pinon Flat observatory. These were deployed in 1996 and therefore provide stable calibration points for ERS-1, ERS-2, Envisat, ALOS and all future satellites.
SAR processor

Digital SAR processing overview

1. Range compression performed on each radar echo

2. Patch processing of range-compressed echoes

3. Range migration performed on each pixel of range-compressed data whose columns have been Fourier transformed

4. Azimuth compression performed on each column of range-compressed, range-migrated data

Single Look Complex (SLC) image data file
<table>
<thead>
<tr>
<th>File Information/Format</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>file name</td>
<td><code>e2_10001_2925.fix</code></td>
</tr>
<tr>
<td>file name</td>
<td>name of raw data file</td>
</tr>
<tr>
<td>SC_id</td>
<td>2</td>
</tr>
<tr>
<td>bytes_per_line</td>
<td>11644</td>
</tr>
<tr>
<td>first_sample</td>
<td>206</td>
</tr>
<tr>
<td>rd1</td>
<td>248.115</td>
</tr>
<tr>
<td>I_mean</td>
<td>15.5</td>
</tr>
<tr>
<td>Q_mean</td>
<td>15.5</td>
</tr>
<tr>
<td>ardu_star</td>
<td>2576039268.848</td>
</tr>
<tr>
<td>SC_clock_start</td>
<td>1997078.767881851</td>
</tr>
<tr>
<td>SC_clock_stop</td>
<td>1997078.768074699</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Radar Characteristics</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRF</td>
<td>1679.902394</td>
</tr>
<tr>
<td>range_sampling_rate</td>
<td>1.89625e+07</td>
</tr>
<tr>
<td>chirp_slope</td>
<td>4.17788e+11</td>
</tr>
<tr>
<td>pulse_dur</td>
<td>3.712e-05</td>
</tr>
<tr>
<td>radar_wavelength</td>
<td>0.056666</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Orbital Information</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>near_range</td>
<td>829924.365777</td>
</tr>
<tr>
<td>earth_radius</td>
<td>637146.4379</td>
</tr>
<tr>
<td>SC_height</td>
<td>787955.52</td>
</tr>
<tr>
<td>SC_vel</td>
<td>7125.0330</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Processing Information</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>num_valid_aq</td>
<td>2800</td>
</tr>
<tr>
<td>num_patches</td>
<td>10</td>
</tr>
<tr>
<td>first_aq</td>
<td>1</td>
</tr>
<tr>
<td>deskew</td>
<td>n</td>
</tr>
<tr>
<td>st_range</td>
<td>1</td>
</tr>
<tr>
<td>num_range</td>
<td>6144</td>
</tr>
<tr>
<td>chirp_ext</td>
<td>614</td>
</tr>
<tr>
<td>flip</td>
<td>n</td>
</tr>
<tr>
<td>nlooks</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Image Alignment</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>rshift</td>
<td>15.1</td>
</tr>
<tr>
<td>ashift</td>
<td>483.2</td>
</tr>
<tr>
<td>stretch_r</td>
<td>0.0014569</td>
</tr>
<tr>
<td>stretch_a</td>
<td>-0.0019436</td>
</tr>
<tr>
<td>a_stretch_r</td>
<td>0.0</td>
</tr>
<tr>
<td>a_stretch_a</td>
<td>0.0</td>
</tr>
</tbody>
</table>
1) This is an image of radar backscatter from a stack of ERS SAR data. The flight path is top to bottom and the radar looks from the right. The area is the Salton Sea and Cochella Valley, and the tic marks are spaced at 10 km. The satellite is 7159717m from the center of the Earth, the local Earth radius is 6371593 m, and the range to the center of the image is 850148 m. Calculate the look angle to the center of the image. Identify areas of layover. What is the minimum mountain slope in the areas of layover? Why is the Salton Sea dark?
Problems

1) Explain why the raw signal data are provided as complex numbers. Why are the numbers restricted to the range 0-31?

2) Why is the SAR processing done in patches rather than all at once? What is the minimum possible patch size?

3) Most SAR instruments emit a frequency modulated chirp rather than a short pulse. Why? Write a Matlab program to deconvolve the FM chirp for ERS and reproduce the impulse response function shown in Figure B4.

\[ C(s) = \exp \left\{ -i \frac{4\pi}{\lambda} \left[ R_o + \dot{R}_o (s-s_o) + \ddot{R}_o (s-s_o)^2 / 2 \right] \right\} \]

4) Make a plot of the real and imaginary parts of the function given in (B5) for a time interval of -2 to 2 seconds. Use \( R_o = 850 \text{ km} \), \( \dot{R}_o = 0 \), \( \ddot{R}_o = \frac{V^2}{R_o} \) where \( V = 7125 \text{ ms}^{-1} \).

5) Derive equation (B12)

\[ V = \frac{V_t}{\left( 1 + \frac{H}{R_o} \right)^{1/2}} \]

6) Derive equation (B13). (You may need to look back at Appendix A.)

\[ L_a = R_o \frac{\lambda}{L} \]
Table of Contents:

Abstract
1. Introduction
   1.1 Objectives and limitations of GMTSAR
   1.2 Algorithms: SAR, InSAR and the need for precise orbits
       1.2.1 Proper focus
       1.2.2 Transformation from geographic to radar coordinates
       1.2.3 Image alignment
       1.2.4 Flattening interferogram - no trend removal
2. Software
   2.1 Standard products
   2.2 Software design
3. Processing Examples
   3.1 Two-pass processing
   3.2 Stacking and time series
   3.3 ScanSAR Interferometry
4. References
5. Problems

Appendix A. Principles of Synthetic Aperture Radar
Appendix B. SAR Image Formation
Appendix C. InSAR
Appendix D. ScanSAR processor and interferometry
Appendix E. Geolocation accuracy for Pinon corner reflectors
Appendix F. Installation of GMTSAR