# Combination of GPS-observed vertical motion with absolute gravity changes constrain the tie between reference frame origin and Earth center of mass

H.-P. Plag*, C. Kreemer, W. Hammond
Nevada Bureau for Mines and Geology and Seismological Laboratory
University of Nevada, Reno, Mail Stop 178, Reno, NV 89557, *email: hpplag@unr.edu

## Introduction

It has long been suspected that the *Reference Frame Origin* (RFO) of the *International Terrestrial Reference Frame* (ITRF) exhibits a secular trend with respect to the *Center of Mass of the whole Earth system* (CM). Even the most recent versions of ITRF, namely ITRF2000 (Altamimi et al., 2002) and ITRF2005 (Altamini et al., 2006) are expected to have a secular motion of the RFO with respect to CM of the order of 1 mm/yr or more (e.g. Ray et al., 2004; Morel & Willis, 2005; Plag, 2006). Such a secular trend would cause a global bias of vertical rates with a spherical harmonic degree of two. This bias hampers the interpretation of vertical rates in terms of geodynamic processes. In particular, the apparent generally upward vertical motion of GPS sites in the Basin and Range Province with respect to ITRF2000 is contrary to the expectation based on a province extending owing to gravitational collapse.

Whereas geometric observations with techniques like the *Global Positioning System* (GPS), *Satellite Laser Ranging* (SLR), and *Very Long Baseline Interferometry* (VLBI) determine the height changes with respect to a reference frame defined through a global polyhedron of tracking stations (such as the ITRF), gravity observations measure the changes in gravity caused by changes in mass distribution and station height. Thus, gravity measurements are directly related to the CM, while the geometric RFO may not be perfectly connected to the CM. In fact, the latter is more likely to be a *Center of Figure* (CF) of the solid Earth frame (Blewitt, 2003). The question how well the geometric RFO is tied to the CM is very relevant to global and regional studies (e.g., Blewitt et al., 2006). For the most accurate global reference frame, the ITRF, the RFO is today constrained to the CM mainly by use of SLR.

Wahr et al. (1995) showed that the combination of secular gravity and height changes can be used to isolate the contribution to the vertical motion induced by present-day mass changes. According to their results, model predictions of *Postglacial Rebound* (PGR) show a constant ratio of gravity to height changes. Therefore, they concluded, that deviations of this ratio from the predicted value can be assumed to be caused by nearby concurrent mass changes. Here we show that combining absolute gravity measurements with geometric observations also helps to constrain the tie between RFO and CM.

## The Data

We have collected published values of gravity trends for a globally distributed station network of sites, for which also GPS observations were publicly available. The spatial distribution of these stations is far from ideal (Figure 1): most stations are found in North America and Europe.

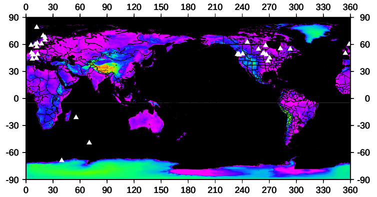

Figure 1: Absolute gravity stations used to constrain the tie between RFO and CM.

All the GPS data were homogeneously processed with the GIPSY-OASIS II (GIPSY) software package of *Jet Propulsion Laboratory* (JPL) using the *Precise Point Positioning* (PPP) method to determine daily coordinates. A full description of the analysis is given in Plag et al. (2007). All time series were determined in ITRF2000. The time series of daily station coordinates were then used to determine vertical rates taking into account semi-annual and annual harmonic constituents. The uncertainties of the vertical rates were estimated taking into account the general presence of colored noise in addition to white noise in these time series. The colored noise was approximated as flicker noise.

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## Theory

Following Wahr et al. (1995), the total gravity anomaly $\delta g(t) = g(t) - g(t_0)$ measured by a gravimeter ($g(t)$: gravity measured at time $t$; $t_0$: arbitrary reference time), can be written as

$$\delta g = \delta g_u + \delta g_m \,, \tag{1}$$

where $\delta g_u$ and $\delta g_m$ are the anomalies due to the vertical displacement $u(t) = h(t) - h(t_0)$ of the instrument through the unperturbed gravity field and the actual mass effect caused by concurrent mass redistribution, respectively. $h$ is the vertical position of the gravimeter. $\delta g_u$ is related to the vertical displacement by $\delta g_u = -2ug/a = \beta u$, ($a$: Earth's radius; $\beta \approx -3.086$ nms$^{-2}$/mm on global average).

The mass contribution to the observed gravity anomaly can be split up into an elastic part due to concurrent mass changes (both the Newtonian attraction of the surface mass being redistributed and the incremental contribution caused by load-induced mass redistribution in the solid Earth) and a viscous part resulting from past mass changes (only the viscous mass redistribution in the solid Earth) giving $\delta g_m = \delta g_m^e + \delta g_m^v$. Similarly, we can separate $u = u^e + u^v$. PGR model studies show that

$$u^v = \alpha \delta g_m^v \,, \tag{2}$$

with $\alpha \approx 0.65$ mm/(nms$^{-2}$), independent of ice history and mantle rheology (e.g. Wahr et al., 1995; Fang & Hager, 2001; Peltier, 2004). Noting that

$$\begin{aligned} \tilde{\Delta} &= u - u^v \qquad (3) \\ &= u - \alpha \delta g_m^v \\ &= u - \alpha(\delta g - \delta g_u - \delta g_m^e) \\ &= (1 + \alpha\beta)u - \alpha(\delta g - \delta g_m^e) \,, \end{aligned}$$

we define

$$\begin{aligned} \Delta &= \tilde{\Delta} - \alpha \delta g_m^e \qquad (4) \\ &= (1 + \alpha\beta)u - \alpha \delta g \,, \end{aligned}$$

which, in the absence of other disturbing factors, should depend only on the Earth's elastic response caused by present-day changes in mass load. We now consider secular changes $\gamma$ and $\xi$ in gravity and height, respectively, $\nu$ denotes the secular change in $\Delta$. In order to use observed $\gamma$ and $\xi$ to compute $\nu$, we rewrite (4) in terms of secular changes, and using (2) we get

$$\nu = (1 + \alpha\beta)\xi - \alpha\gamma \,. \tag{5}$$

With $\xi_{CM} = \xi + \eta$ we introduce a bias $\eta$ of the vertical rates determined from GPS caused by a secular translation with rate $\vec{d}$ of the RFO with respect to the CM. Assuming a spherical Earth,

$$\begin{aligned} \eta &= < \vec{d}, \vec{\hat{r}} > \qquad (6) \\ &= d_x \sin\theta \cos\phi + d_y \sin\theta \sin\phi + d_z \cos\theta \,, \end{aligned}$$

where $\vec{\hat{r}}$ is the unit radial vector, $< .,. >$ denotes the scalar vector product, and $\phi$ and $\theta$ are the geographical longitude and co-latitude, respectively. Replace in (5) $\xi$ by $\xi + \eta$, gives

$$\nu = (1 + \alpha\beta)(\xi + \eta) - \alpha\gamma \tag{7}$$

In the absence of ongoing mass relocation, $\nu = 0$, and we can write

$$\gamma = (1/\alpha + \beta)(\xi + \eta) \,. \tag{8}$$

Here, $\gamma$, $\xi$, $\eta$, and $\beta$ depend on the geographical location of the observing point. In principle, both $\alpha$ and $\vec{d}$ are unknown, and (8) together with (6) can be used to determine both $\alpha$ and the three components of $\vec{d}$ in a fit of the observed height and gravity rates. For regional studies, $\eta$ can be considered as constant. Then, for $\gamma = m\xi + d$, we find (by comparison to (8))

$$\begin{aligned} m &= (1/\alpha + \beta) \qquad (9) \\ d &= (1/\alpha + \beta)\eta \,, \end{aligned}$$

i.e. $\alpha = 1/(m - \beta)$ and $\eta = d/m$, which can be used to determine $\alpha$ and $\gamma$ once $m$ and $d$ have been determined for a number of points in the region with observed secular gravity and height changes.

## Conclusions

The collocation of absolute gravity and geometric sites can provide a valuable constraint on the tie between RFO and CM. Despite a poor geographical station distribution, published gravity trends combined with homogeneously determined vertical rates indicate a translation of ITRF2000 with respect to the CM along the Z axis of the order of -2 mm/year. This is comparable to the difference between ITRF2000 and ITRF2005, indicating that ITRF2005 is better constrained to the CM. Consequently, the reference frame of choice for studies of vertical motion is ITRF2005.

Applying the translation to secular trends determined for a network of North American GPS sites, the vertical trends in that region in both ITRF2005 and in the absolute gravity frame are on average increased compared to the ITRF2000 trends by approximately 1.1 mm/yr. For the Basin and Range province, a general uplift with respect to the CM is confirmed, consistent with a gain in gravitational energy of the lithosphere, contrary to the expectation that the province is collapsing owing to excess gravitational potential.

## References (continued)

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## Global Results

Since all points with reliable vertical rates are at relatively high latitude on the northern hemisphere, the station distribution does not allow to determine significant translations in the X and Y components. For a translation in the Z component only, the bias is a function of $\cos(\theta)$. As a zero order approach, we considered the bias as constant, and perform a regression of the gravity and vertical rates (Figure 2). For this regression, we have eliminated four stations with either large $\nu$ (indicating present-day mass changes) or uncertain gravity or vertical rates. For the remaining 26 data points, the correlation coefficient between gravity and vertical rates is $-0.92 < -0.83 < -0.65$ with the lower and upper values being the 99% uncertainties. The unweighted regression line is $\gamma = -2.179 \pm 0.469 - 1.226 \pm 0.168 \cdot \xi$ (for $\gamma$ in nms$^{-2}$/year and $\xi$ in mm/year). From the regression coefficient and offset we deduce $\alpha = 0.538$ mm/(nms$^{-2}$) and translation in the Z component $d_z = 1.78$ mm/yr. The value for $\alpha$ is about 20% lower than the value deduced from PGR model studies. The translation in the Z component is very close to the value for the translation from ITRF2000 to ITRF2005 of $1.8$ mm/year (Figure 3).

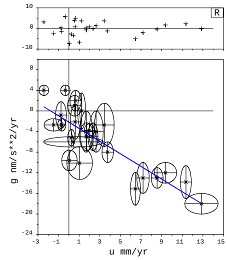

Figure 2: Result of a regression of gravity on vertical rates.

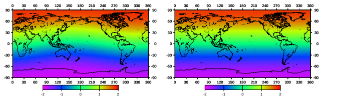

Figure 3: Vertical rate differences for ITRF2005 - ITRF2000 (left) and absolute gravity frame - ITRF2000 (right). Differences are in mm/yr.

## Basin and Range

The extensional tectonics, contemporary areal growth and generally high *gravitational potential energy* (GPE) (Jones et al., 1996; Flesch et al., 2000; Humphreys & Coblentz, 2007) of the Basin and Range Province suggest that gravity plays an active role in driving and/or guiding the modern Pacific/North America plate boundary deformation. However, the degree to which intraplate forces, for example those attributable to GPE variations, in the lithosphere rule over plate boundary or sub-lithospheric forces is still in dispute.

If active extension is driven by GPE variations, then potential energy must be converted into work associated with continental deformation. Thus GPE must be reduced during the extension process. However, after correction for drift of the RFO with respect to the CM, the observed vertical rates are on average positive across the Province (Figure 4). This seems to contradict the theory that GPE is driving extension.

Resolving this puzzle will require making a careful comparison between 1) the spatial variations in vertical rates to 2) present variations in GPE, and 3) rates of contemporary extension (as observed with GPS at many new PBO and other GPS sites in the Province) and extension history of the recent geologic past. If the observation holds, then there may be an unrecognized process that is increasing the GPE available to deform the Basin and Range Province. However, we must rule out contributions from other long wavelength geophysical processes (e.g. PGR) before we can make such an assertion.

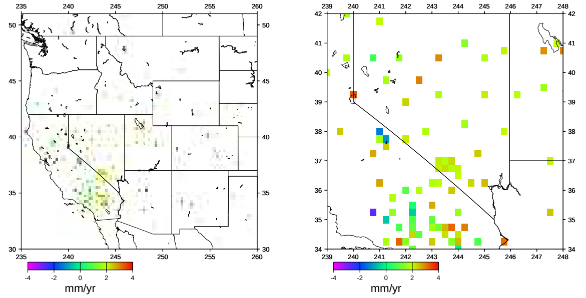

Figure 4: Vertical secular motion in the Western U.S.A. Left: For the whole Western U.S., right: the Basin and Range Province. Rates are in mm/yr and in the absolute gravity frame.

## Acknowledgments

This work has been conducted with support from NASA Solid Earth and Natural Hazards, NASA Interdisciplinary Science, and by DOE contract FC-08-98NV12081.