Theory and practice of phase unwrapping

Rob Mellors, Eric Lindsey, Xiaohua (Eric) Xu, Kurt Feigl

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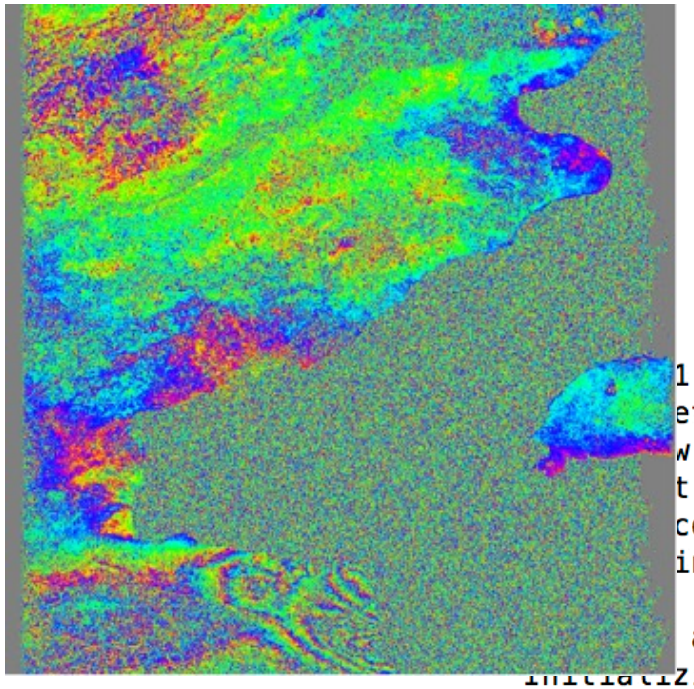

- What is phase unwrapping?
- Ways to unwrap
- Using SNAPHU in GMTSAR
- Reduce or avoid the problem

```
1.4.2
eters input from file /Users/mellors1/src/GMT5SAR/share/
wrapped phase from file phase.in
t file specified.  Assuming uniform weights
correlation data from file corr.in
ing smooth-solution cost parameters
 range cost arrays
 azimuth cost arrays
initializing flows with MST algorithm
Running approximate minimum spanning tree solver
Running nonlinear network flow optimizer
Maximum flow on network: 25
Number of nodes in network: 21423926
Flow increment: 1  (Total improvements: 0)
Treesize: 12512201   Pivots: 788730692   Improvements: 885795
```

LLNL-PRES-XXXX
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overview of unwrapping

Given an interferogram(s)

- usually need to convert phase to useful units
- we should know radar wavelength and geometry

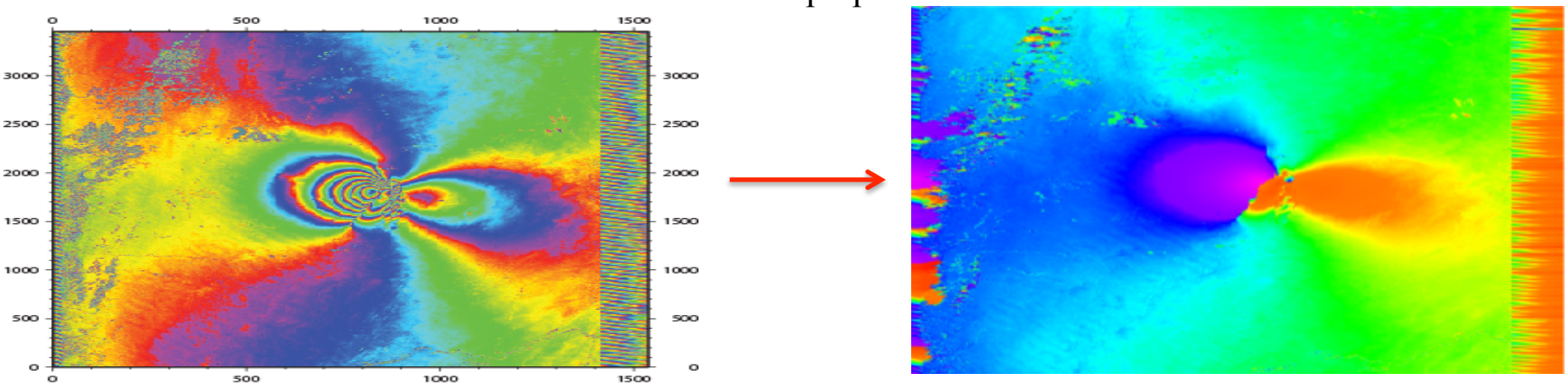
Usually requires unwrapping

- unwrapping not always easy or fast

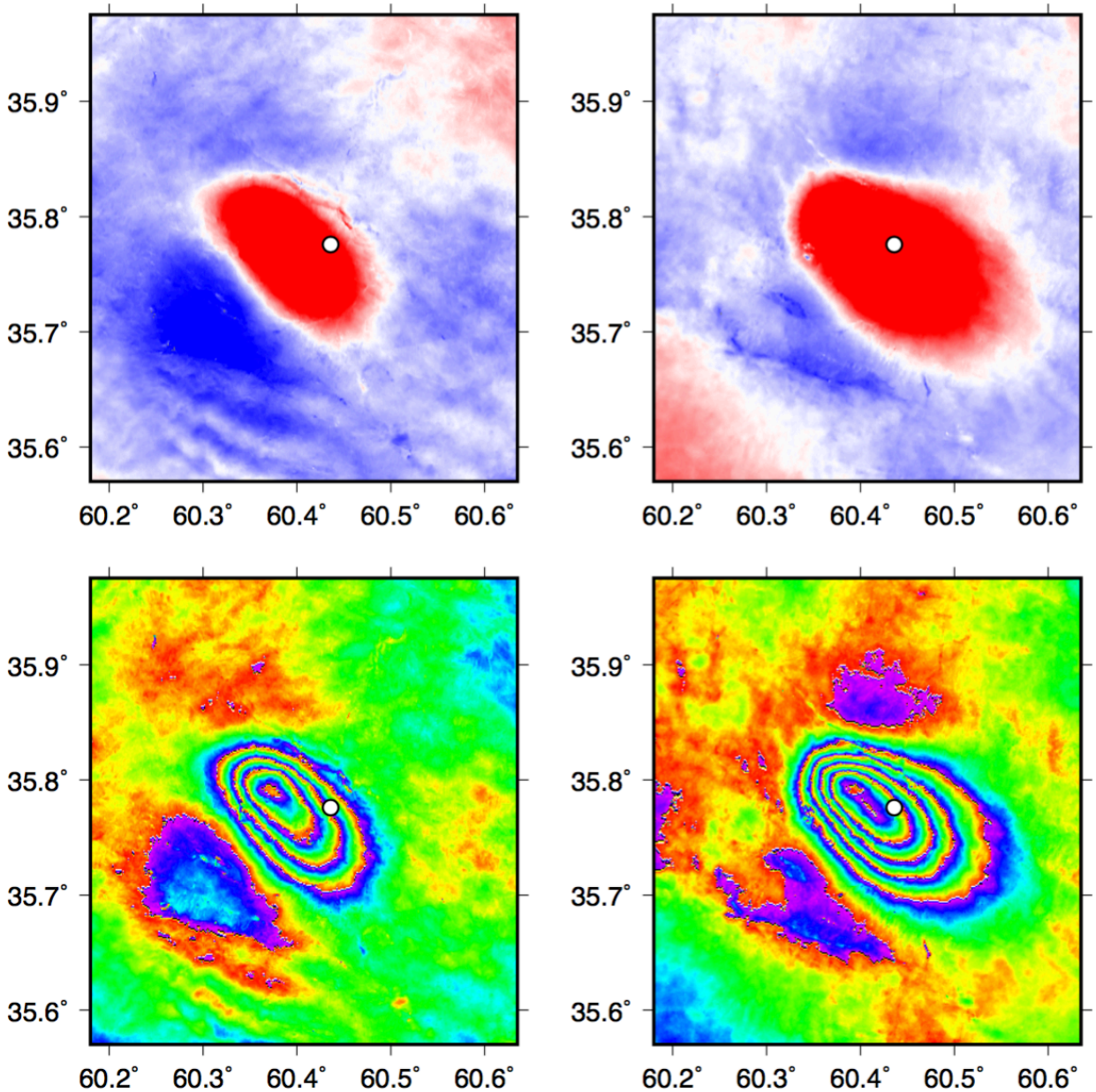
Two popular algorithms

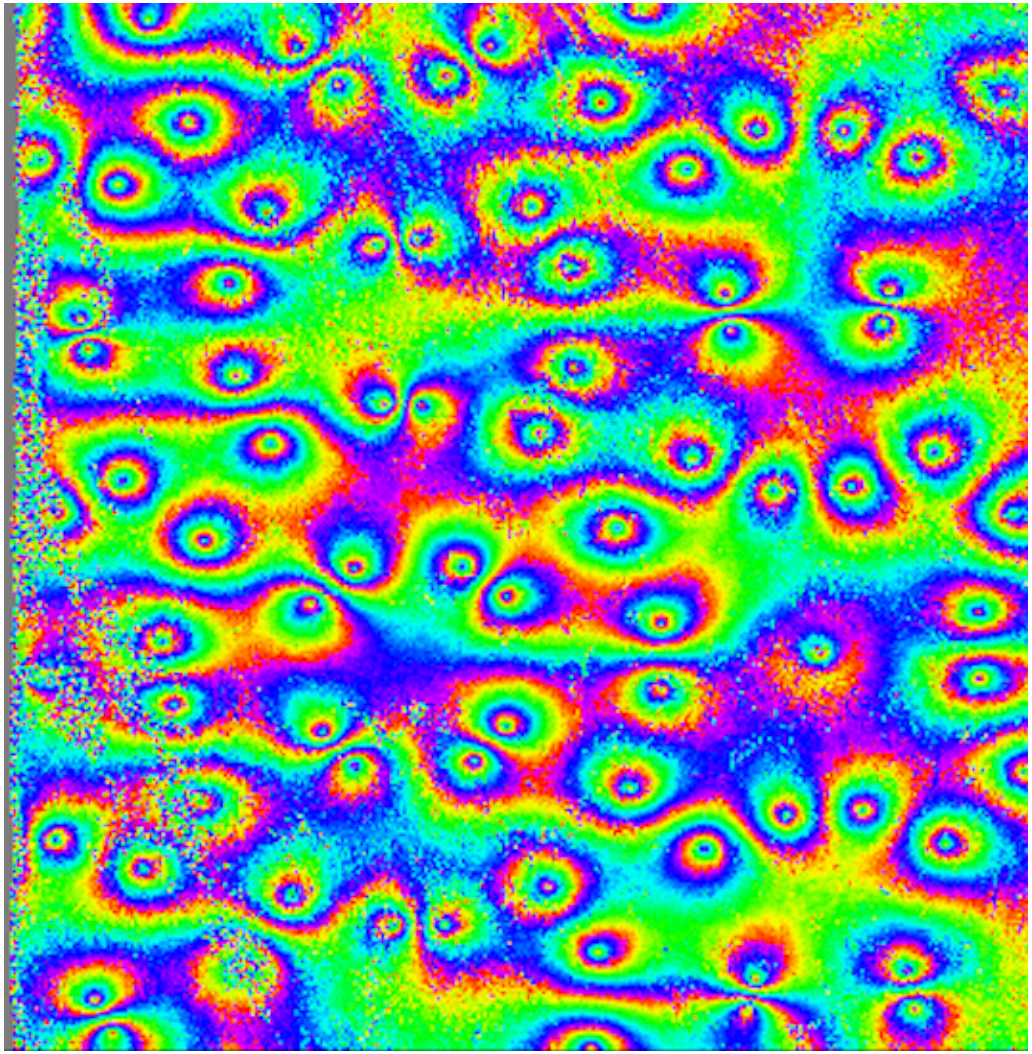
- Goldstein
- SNAPHU (Chen and Zebker, 2000)
- others exist! (global, gradient, etc)

Saudi 7/1/08-8/19/08 ALOS Bperp 20 m



Sentinel, ascending & descending





An example of poor unwrapping

extracting the phase

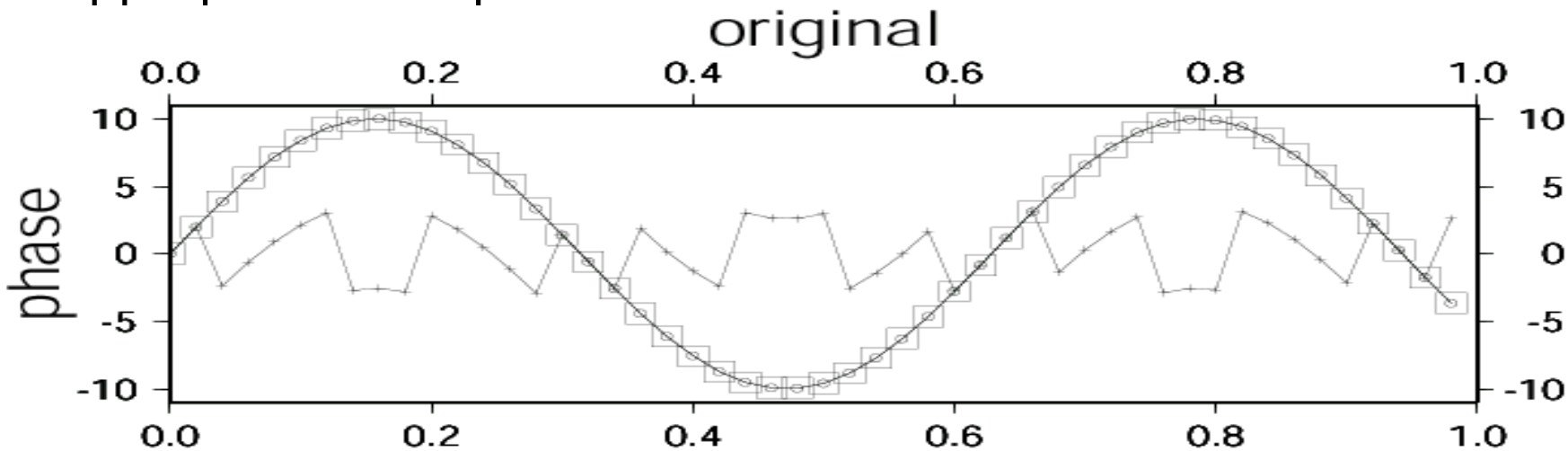
We can get only the wrapped phase*

$$\Phi(t) = \arctan(I(s(t)),R(s(t)))$$

where $-\pi < \Phi(t) \leq \pi$

We would like the continuous phase.

This appears simple. Look for 2π jumps and then add the appropriate multiple of 2π.



If the data are good, phase unwrapping is straightforward

- We could take derivative in complex version (e.g. Sandwell & Price)

phase unwrapping

Assume that we have two signals taken at different times:

$g_1 = a_1 \exp(i4\pi R_1/\lambda)$

$g_2 = a_2 \exp(i4\pi R_2/\lambda)$

a1,a2 = amplitude

R_1, R_2 is range from antenna to surface

λ = wavelength

At a given point, assume $a_1 = a_2 = a$

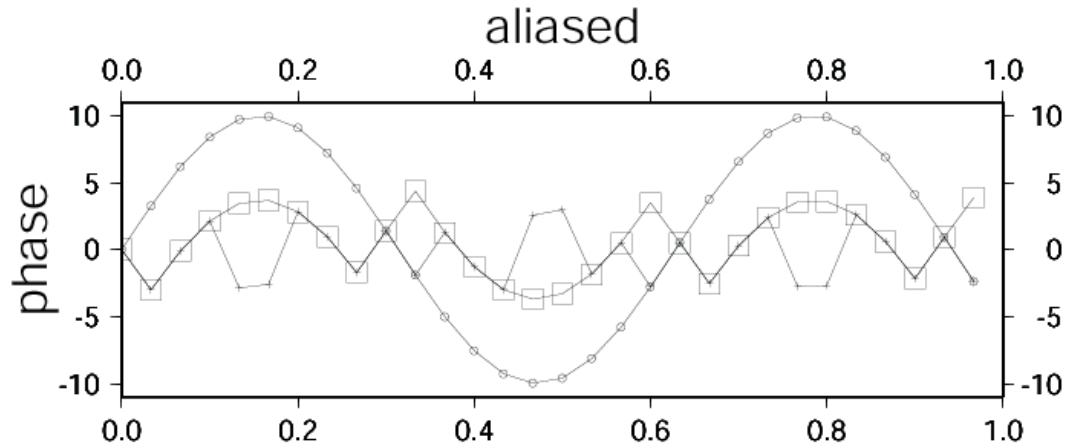
$(g1)(g2^*) = (a^2)\exp[i4\pi(R_1-R_2)] = s(t)$

The phase of this function is proportional to the effective difference in range, which in turn depends on satellite geometry, topography, atmosphere, soil moisture, or maybe even *deformation*.

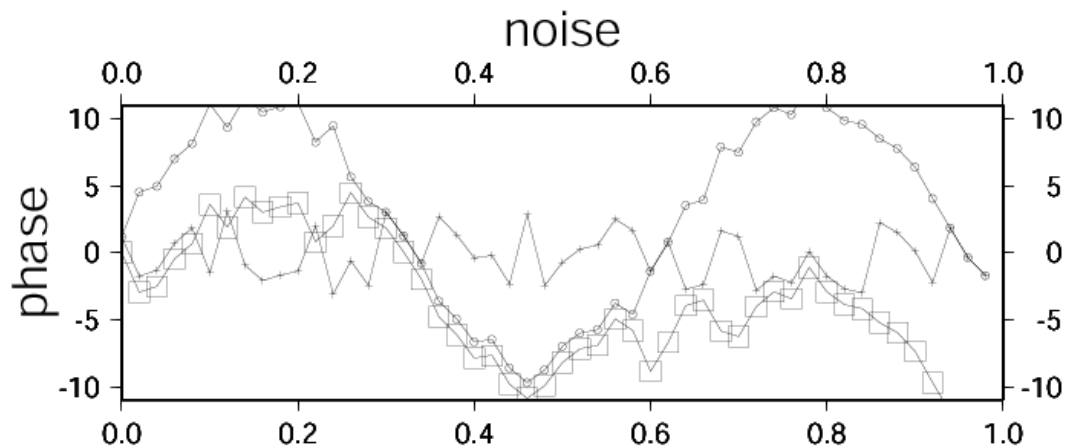
Want to find the function, that, when wrapped, matches the data

problems: aliasing and/or noise

Aliasing
True phase changes by more than 1 cycle (2π radians) between samples.
Caused by large baselines, steep topography or large deformation (steep phase gradient)

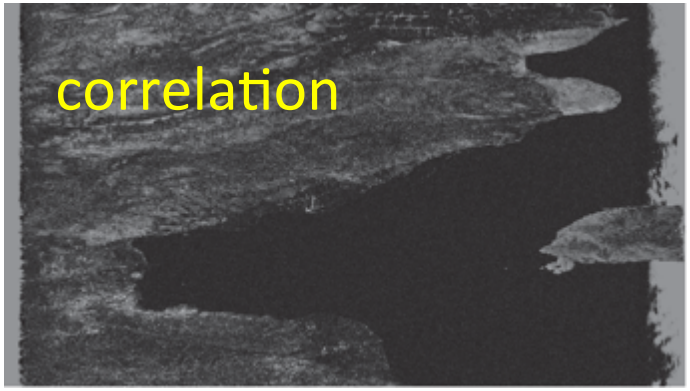


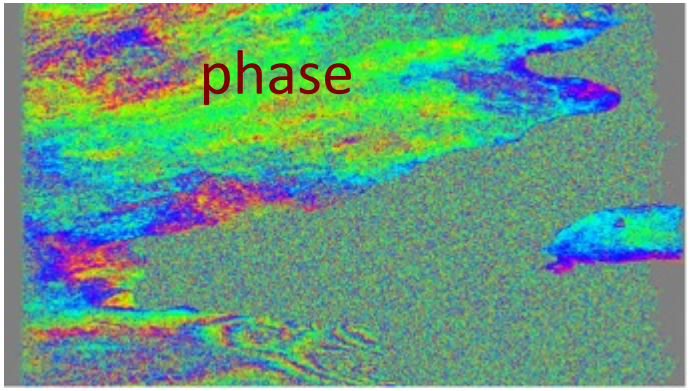
Noise or data gaps
Changes on the surface (e.g., vegetation, snow, erosion)
may cause the two images to de-correlate, introducing noise



identifying and resolving problems

1) Mask out areas of poor data.
    1) Use correlation as a guide (mask out all pixels with correlation less than a threshold value)
    2) Identify points of inconsistent data (residues)
2) Filter to reduce problems.
3) Use model to subtract expected phase.





filtering

objective: improve signal-to-noise of fringes prior to unwrapping

static
- usually lowpass
- convolve with set of filter coefficients (boxcar, Gaussian, etc)
- detrend (*detrend_before_unwrap.csh*)

adaptive
-*Goldstein and Werner* [1998] spectral filter.
- effective but "can significantly change the structure of the interferogram"[*Baran et al*, 2003]

adaptive filter

- filter parameter vary depending of properties of each patch

$$H(u,v) = (S\{|Z(u,v)|\}^a)(Z(u,v))$$

Z(u,v) Fourier spectrum of small 2D patch of complex interferogram (perhaps 32 by 32 pixels)

S smoothing (e.g. 3 by 3 pixels)
H output
a exponent (a = 0, no filter, a > 0 filters)

-take Fourier of small patch (32 by 32 pixels)
-raise spectrum to power
- inverse FFT
-results depends on noise and phase!
- *Baran et al* [2003] use $a = 1 - \gamma$ where γ is the coherence

(filter also developed by Goldstein)

adaptive filter in GMTSAR

phasefilt

- reads in real and imaginary files (imag.grd and real.grd)
- alpha = exponent for filter (default is 0.5; between 0.0 and 1.5)
- psize = patch size (usually 16 or 32)
- Can define alpha with correlation also

Run by filter.csh
Filter.csh run by p2p*.csh scripts
creates phasefilt.grd
(phasefilt makes filtphase.grd which is flipped with grdmath to create phasefilt.grd)

```
phasefilt -imag imagfilt.grd -real realfilt.grd -amp1 amp1.grd -amp2 amp2.grd -psize 16
```

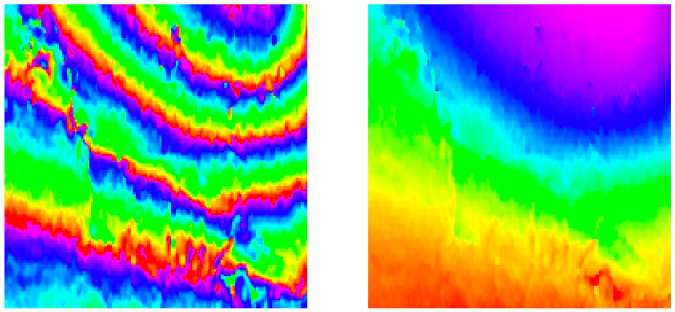
effects of adaptive filter

M 6.1 2017-04-05 06:09:12
UTC 35.776°N 60.436°E

13.0 km depth

Sentinel
P13, F112
P93, F472

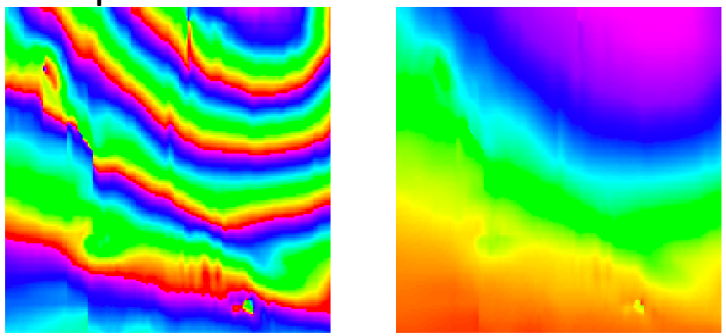
No filter



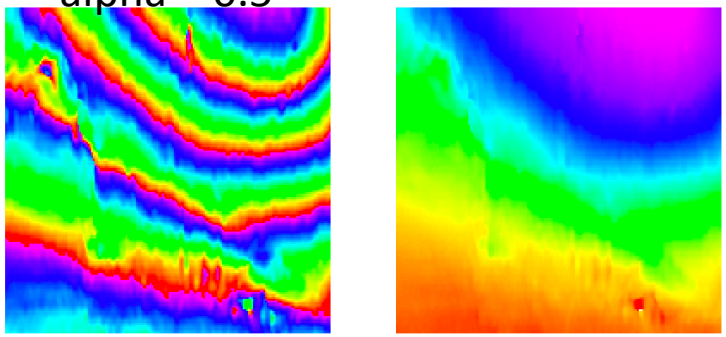
wrapped unwrapped

phasefilter, psize=32

alpha = 1.5



alpha = 0.5



wrapped unwrapped

local unwrapping (path following)

Similar to the 1D approach

1.) Calculate the differences of the wrapped phase.

2.) Wrap the differences.

3.) Set the value of the first value.

3.) Integrate along all values.

Do this along a line throughout 2D area (in a zigzag back and forth along the rows, for example)

Works great if there is no noise.

With noise:

1) An error near the start of the path propagates along the whole path.

2) Answer may vary with path.

3) Need to identify bad pixels. How?

identifying inconsistent data

Phase should reflect the topography.
We know that topographic surfaces are conservative

- Any points that violate this rule should be avoided.
- These points are known as residues.
- Any integration path that circles a residue will contain errors

=> need to make “branch cuts”

- A residue is a property of phase differences, not a single pixel.
- can be positive or negative
- Can be identified by examining groups of pixels

wrapped differences should sum to zero around a loop.
If sum is not zero, then identify the loop as a residue.

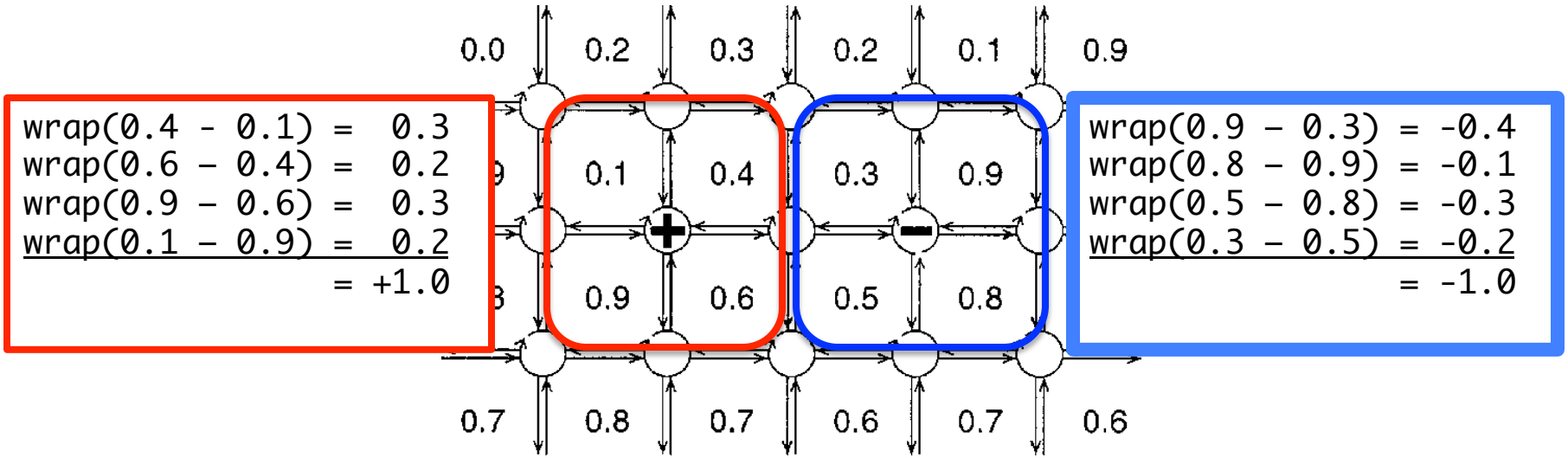
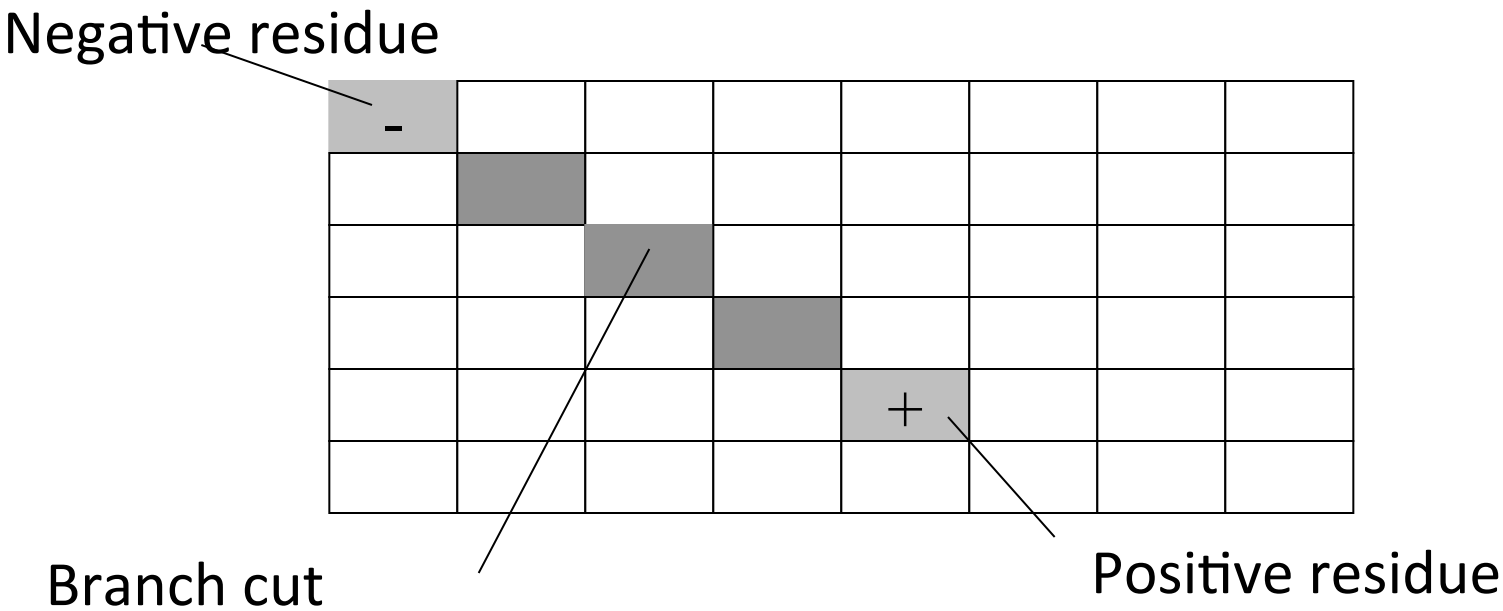


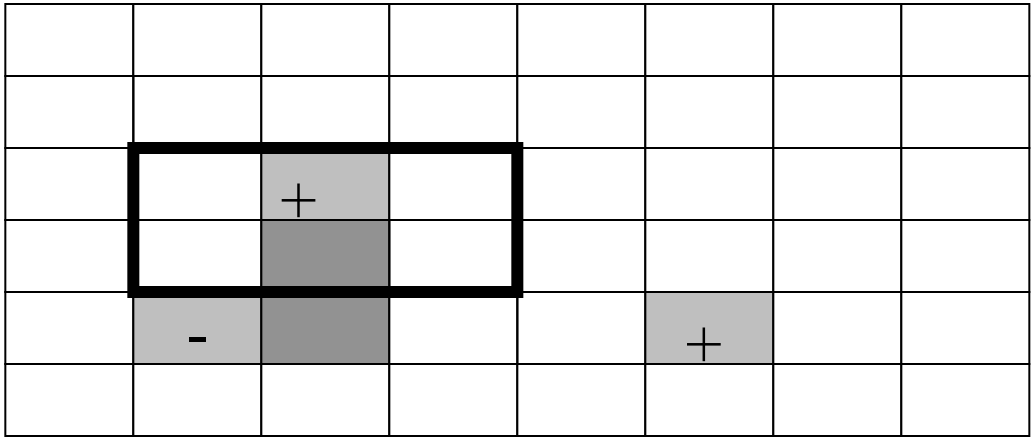
Fig. 1. Example network equivalent of the phase unwrapping problem. The numbers represent the 2-D array of phase samples (normalized to one cycle). Each 2×2 clockwise loop integral of wrapped phase gradients is a node in the network, and positive and negative residues result in supply and demand nodes. Neighboring nodes are connected by arcs, or possible flow paths. The amount of flow on an arc represents the difference (in cycles) between the unwrapped and the wrapped phase gradients associated with that arc. The net amount of flow out of a node must be equal to the node's surplus.

Chen, C. W., and H. A. Zebker (2000), Network approaches to two-dimensional phase unwrapping: intractability and two new algorithms, *J. Opt. Soc. Am. A, 17*, 401-414.

path following with masking and residues

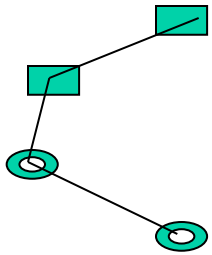
1.) Calculate correlation for phase data.
2.) Mask out all areas with correlation less than a certain threshold value.
3.) Go through all pixels and identify residue locations (upper left of 4 pixels).
4) Start with first residue, look for nearest residue. Draw a "line" of marked pixels between the two.
- if residues cancel, go to next residue and start new "tree"
- otherwise, look for next nearest and draw line
- can also "cancel" by connecting to edge.
- connected lines are called a tree.
5) path-integrate along remaining pixels.

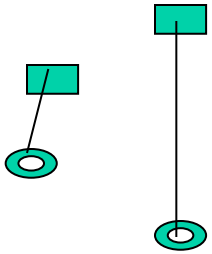


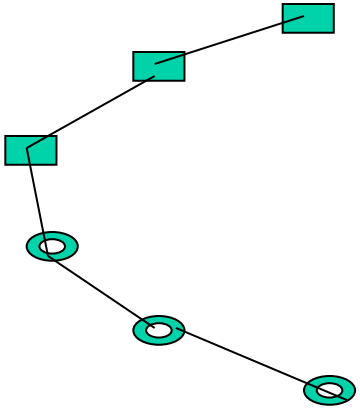


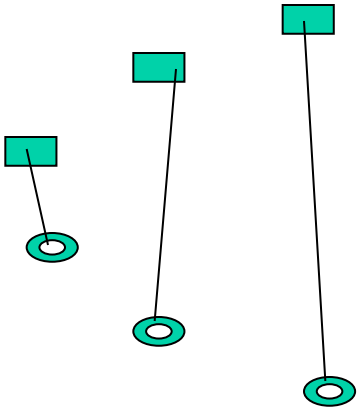
1) Create branches (and trees)
2) Unwrap without crossing

best way to connect residues is not always obvious….









Good – L0

Bad – L1

Negative residue

Positive residue

Residues caused by topographic layover with illumination from one side

Chen and Zebker's SNAPHU

1) Uses a statistical estimate of 'cost'
2) Other algorithms may solve with different norms (e.g. L^1 or L^2, see *Ghiglia and Pratt,* 1998).

- C & Z (2000) claim low norms are best.
- Three statistical models: topography, deformation, or smooth)
- L^0 unwrapping can be slow (NP hard).

GMT5SAR mostly uses SNAPHU, which is C&Z's algorithm

https://web.stanford.edu/group/radar/softwareandlinks/sw/snaphu/

Chen and Zebker, 2000, 2001, and 2002

using SNAPHU in GMTSAR

GMTSAR: snaphu.csh

- Makes correlation mask
- Convert grd to binary for snaphu
- Parameters: *threshold_snaphu* and *defomax* in configuration file – see comments
- Deformation mode is default; defomax = 0 means smooth phase (will detrend) while defomax > 0 sets maximum phase jump
- Can also use landmask or deformation mask
- Extracts parameters from *snaphu.conf.brief* in *$sharedir/snaphu/config*
- *snaphu.csh* run by *p2p*csh scripts*
- Creates unwrap.grd

Usage: snaphu.csh correlation threshold maximum_discontinuity [<r0>/<r1>/<a0>/<a1>]"
Example: snaphu.csh .12 40 1000/3000/24000/27000"

phase unwrapping for the impatient

In general GMTSAR masks out low correlation, which usually masks out areas with random phase.

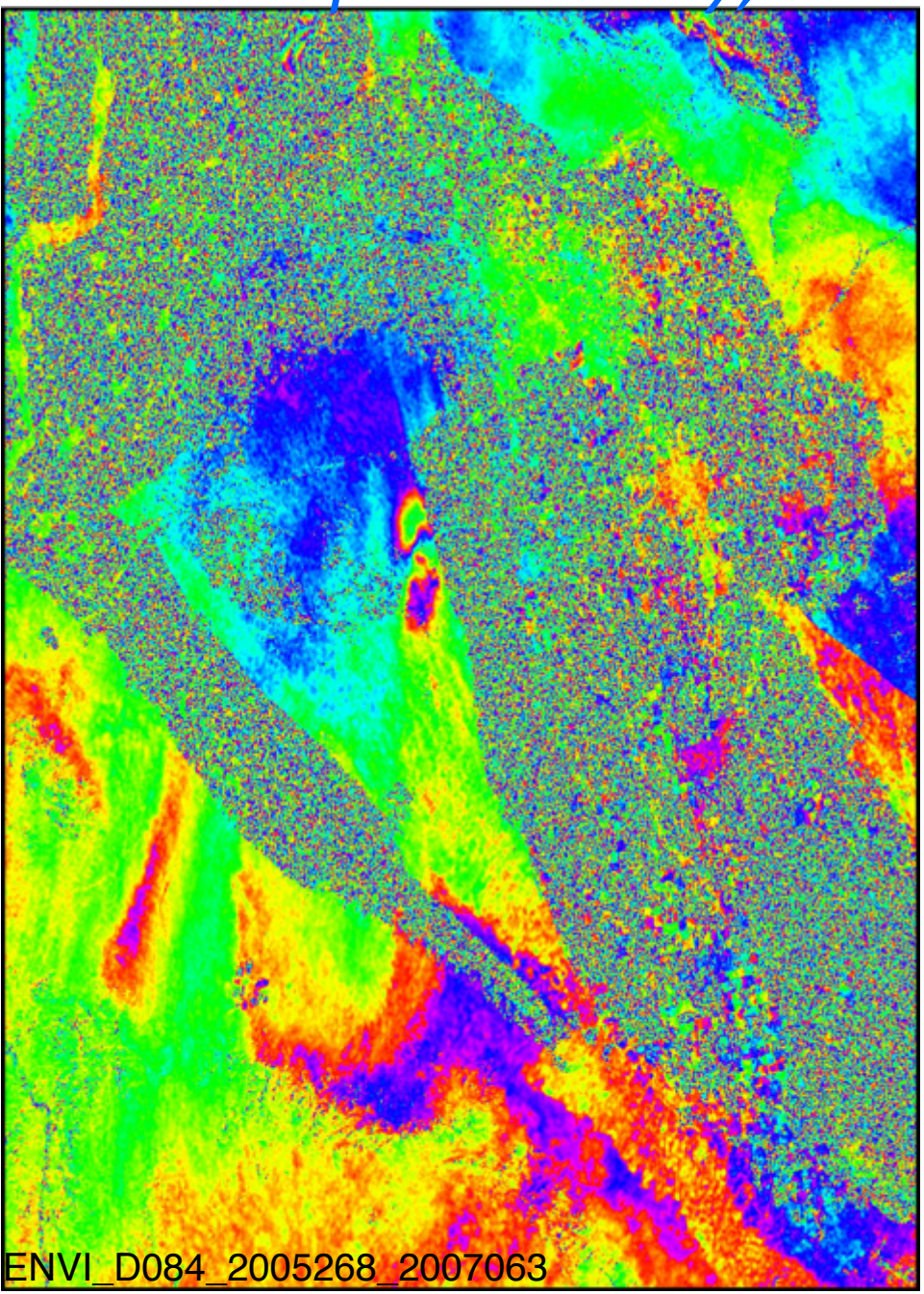
Sometimes a scene possesses large decorrelated areas.
In these cases, SNAPHU can take ~forever.

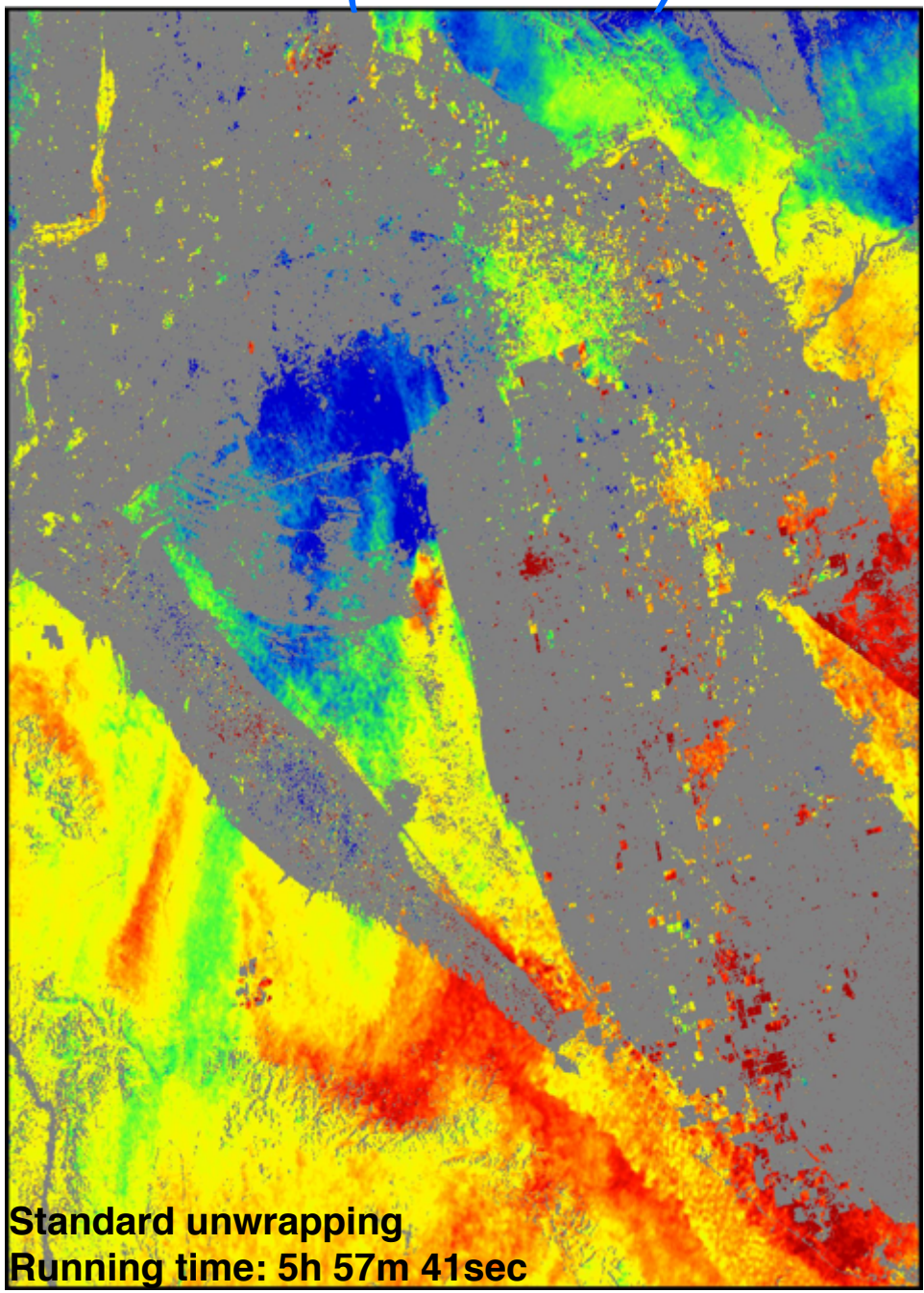
- Increase threshold (not always a good idea).
- Interpolate [in GMTSAR, *snaphu_interpolate.csh*]
- Or buy a better computer.

- Nearest neighbor interpolation preserves the topology of any loops containing residues
- Unwrapped, masked result should be the same, whether or not we interpolate first

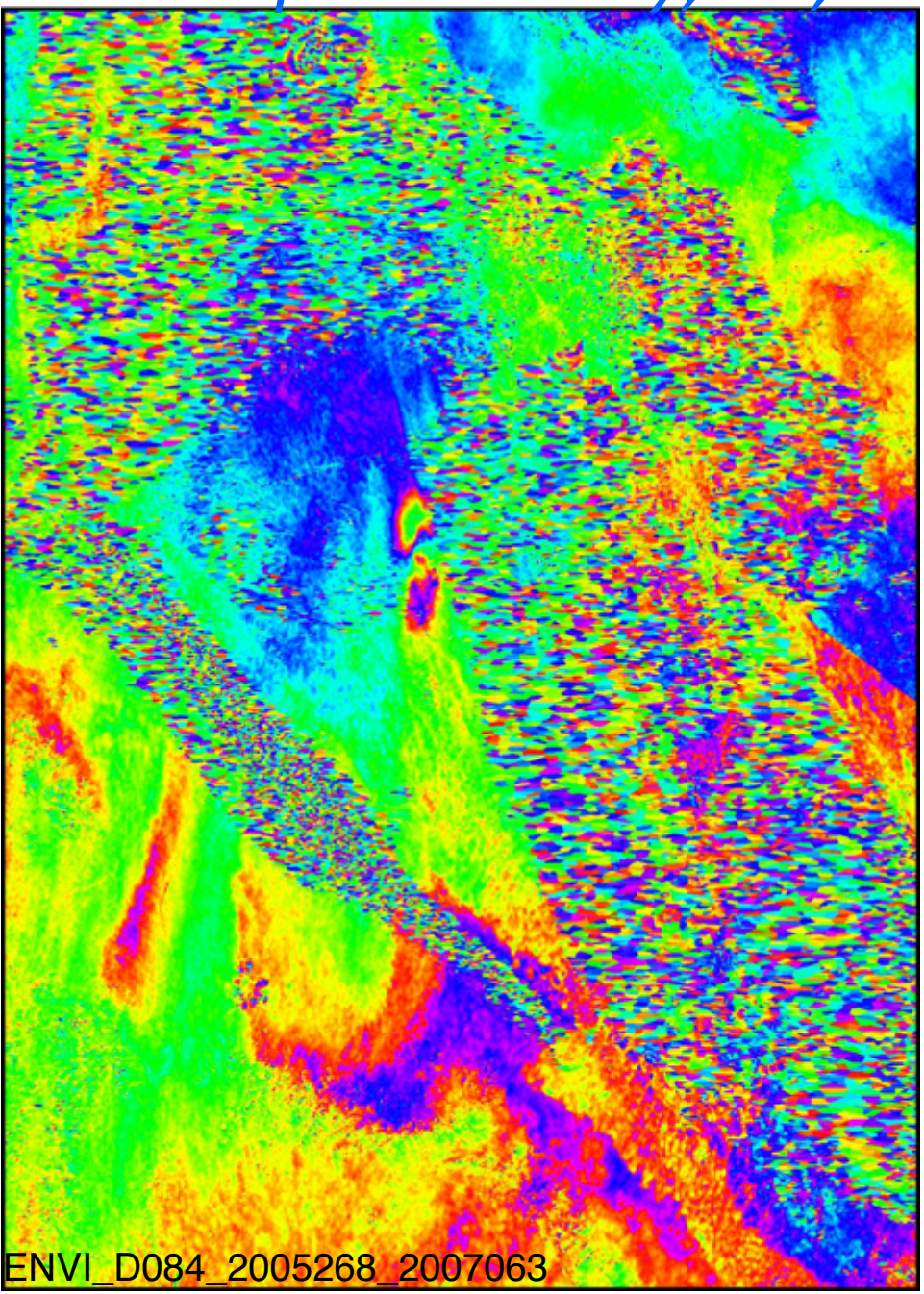
Agram and Zebker, Sparse two-dimensional phase unwrapping using regular grid methods, IEEE Geosci. Rem. Sens., 2009

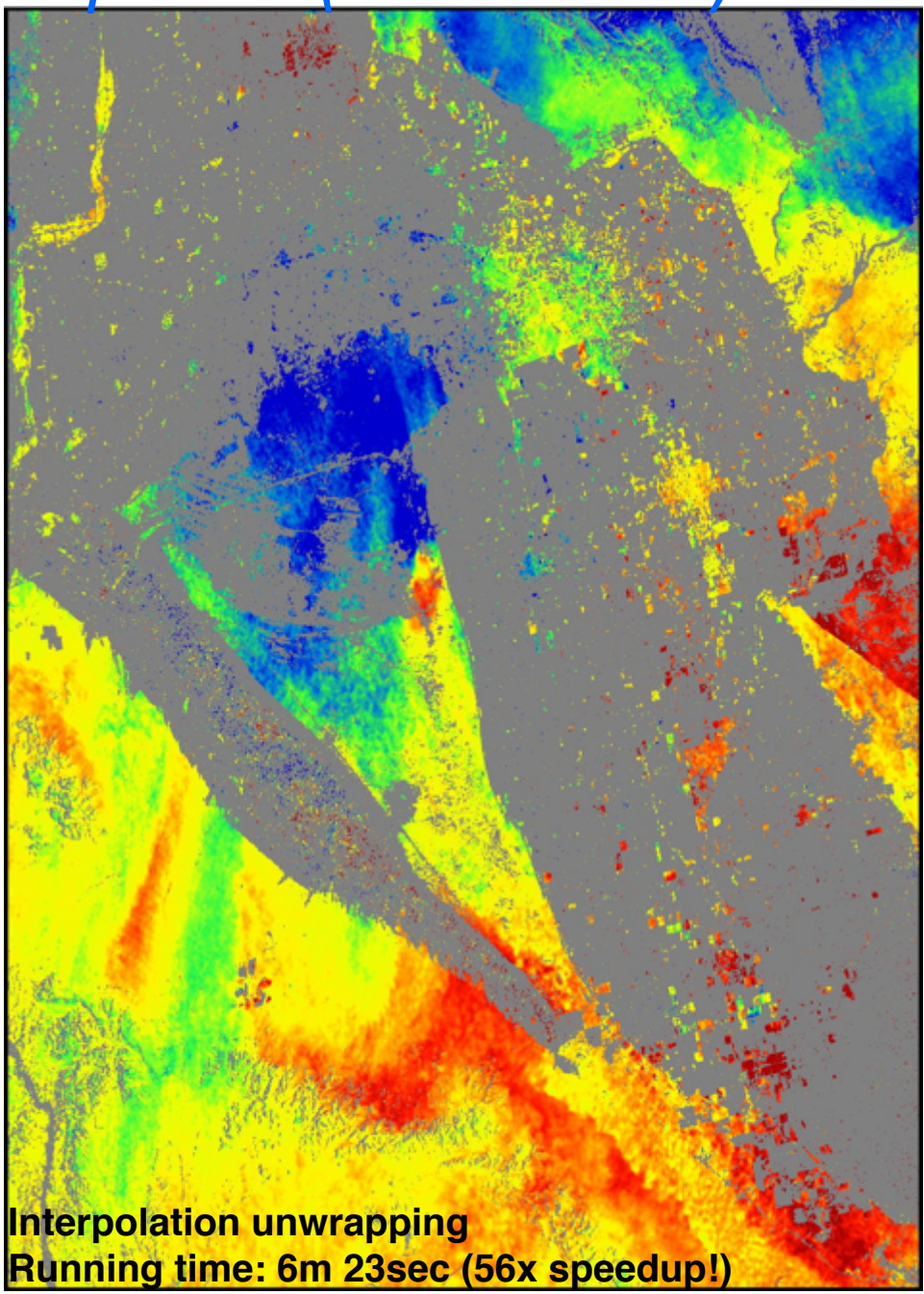
Imperial Valley, CA – standard (~6 hours)





Imperial Valley, CA, interpolate (~6 minutes)





Evaluating model and phase unwrapping

- How do we evaluate?
  - Look at it – does it look okay?
  - May be impractical for large datasets and automatic processing.
  - Subtract out estimated model of deformation before unwrapping.
  - Can be done iteratively
  - After unwrapping, add in to regain original signal.
  - Can bias results with assumptions in model

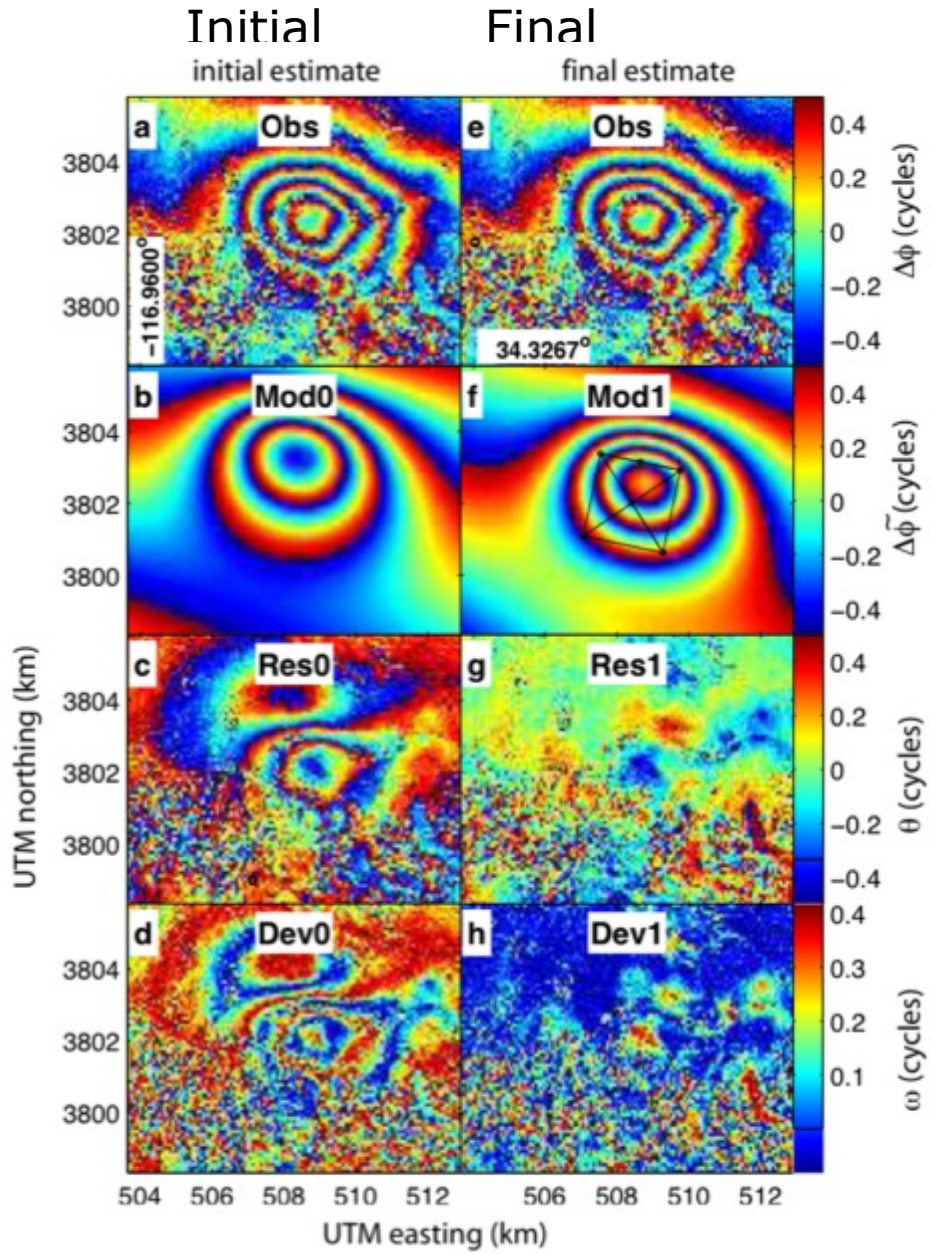
example

Observed

Modeled

Residual

Deviations



cycles

0.2796 cy

0.1483 cy

removing deformation prior to unwrapping

If an accurate deformation model is available, then many of these problems can be alleviated during calculation of the interferogram.

- Reduce need for unwrapping.
- Can be done iteratively.
- Deformation model is optional input to intf.csh (intf.csh run by p2p*csh scripts)

```
intf.csh ref.PRM rep.PRM [-topo topogrd] [-model modelgrd]
```

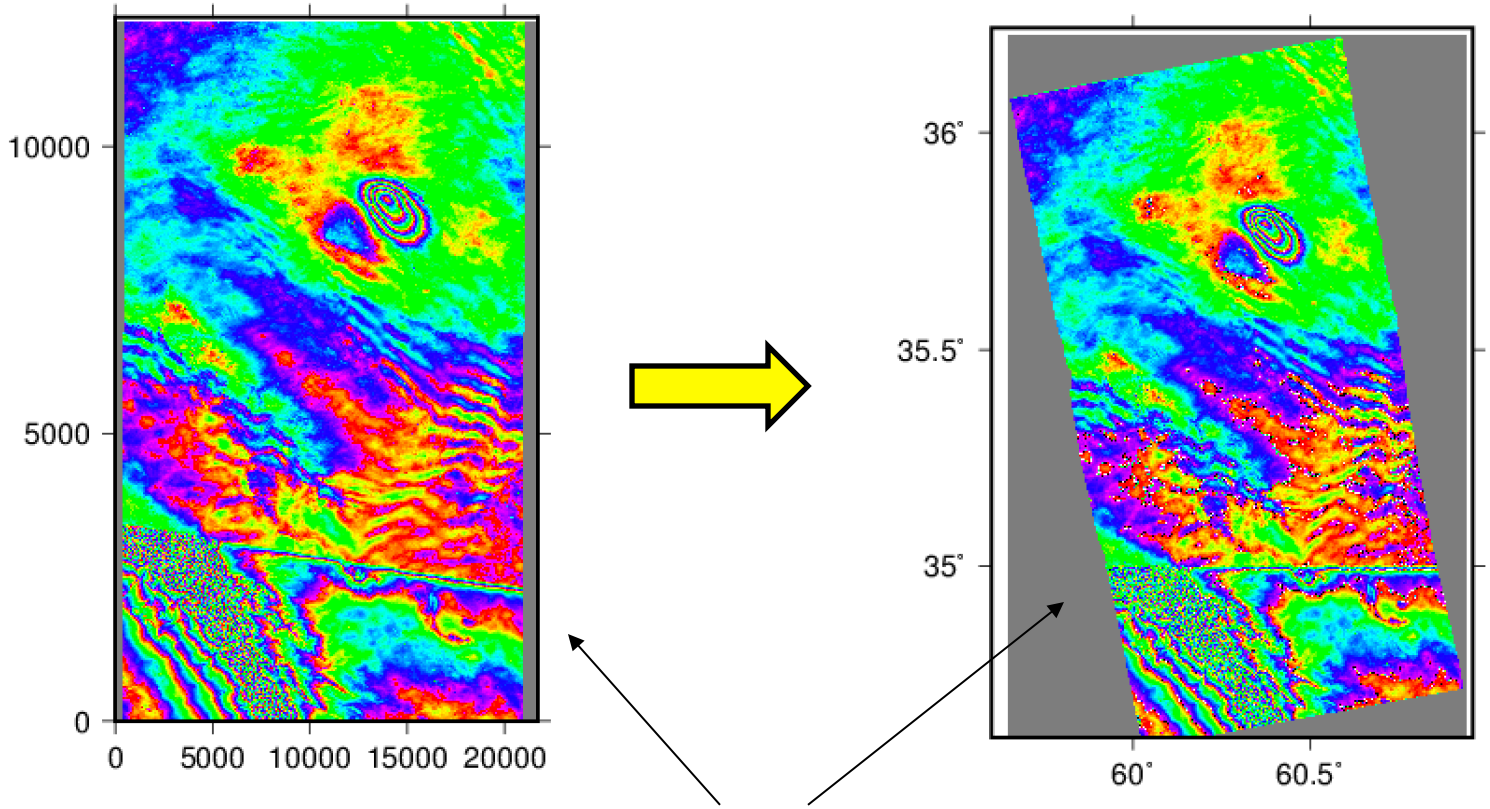
persistent scatterers: 3D unwrapping

- The PS technique leads to widely spaced pixels. Phase relationships between these pixels may be challenging to define.

- If we have a time series of interferograms, phase unwrapping becomes a 3d problem.

geocoding

- Go from radar coordinates (range, azimuth) to ground coordinates (latitude, longitude, or possibly UTM).



Bonus question: what happened here?

proj_ra2ll.csh

Usage: proj_ra2ll.csh trans.dat phase.grd phase_ll.grd

trans.dat - file generated by llt_grid2rat (r a topo lon lat)

phase_ra.grd - a GRD file of phase or anything

phase_ll.grd - output file in lon/lat-coordinates

```
proj_ra2ll.csh trans.dat phasefilt.grd phasefilt_ll.grd
proj_ra2ll.csh trans.dat corr.grd corr_ll.grd
```

trans.dat generated by llt_grid2rat (range azi topo lon lat)

Some references

Chen, C.W, and H. A. Zebker, 2000, Network approaches to two-dimensional phase unwrapping: intractability and two new algorithms, Journal of the Optical Society of America A, vol. 17, pp. 401-414.

Chen, C. W., and H. A. Zebker, 2001, Two-dimensional phase unwrapping with use of statistical models for cost functions in nonlinear optimization, Journal of the Optical Society of America A, vol. 18, pp. 338-351.

Chen, C. W. and H. A. Zebker, 2002, Phase unwrapping for large SAR interferograms: Statistical segmentation and generalized network models,'IEEE Transactions on Geoscience and Remote Sensing, vol. 40, pp. 1709-1719/

Ghiglia, D.C. and M. D. Pratt, 1998, Two-dimensional phase unwrapping, 493 p, John Wiley & Sons.

Goldstein, R. M. and C. Werner, 1998, Radar interferogram filtering for geophysical applications, Geophys. Res. Lets. 25, 21, 4035-4038, 1998.

Goldstein, R. M., H. A. Zebker, and C. L. Werner, Satellite radar interferometry: Two dimensional phase unwrapping, Radio Science, July-August, 713-720.