Principles and Theory of Radar Interferometry

Eric J Fielding

Jet Propulsion Laboratory

August 1, 2016
UNAVCO Short Course

Disclaimer: Almost all of this talk has been derived from UNAVCO's ROI_PAC short course (2013) lecture by Piyush Agram and (2012) lecture by Paul Rosen, JPL.

Outline of Tutorial

I. Properties of radar images

II. Geometric Aspects of Interferometry and Interferometric Phase

- A. Interferometry for Topographic Mapping
- B. Interferometry for Deformation Mapping

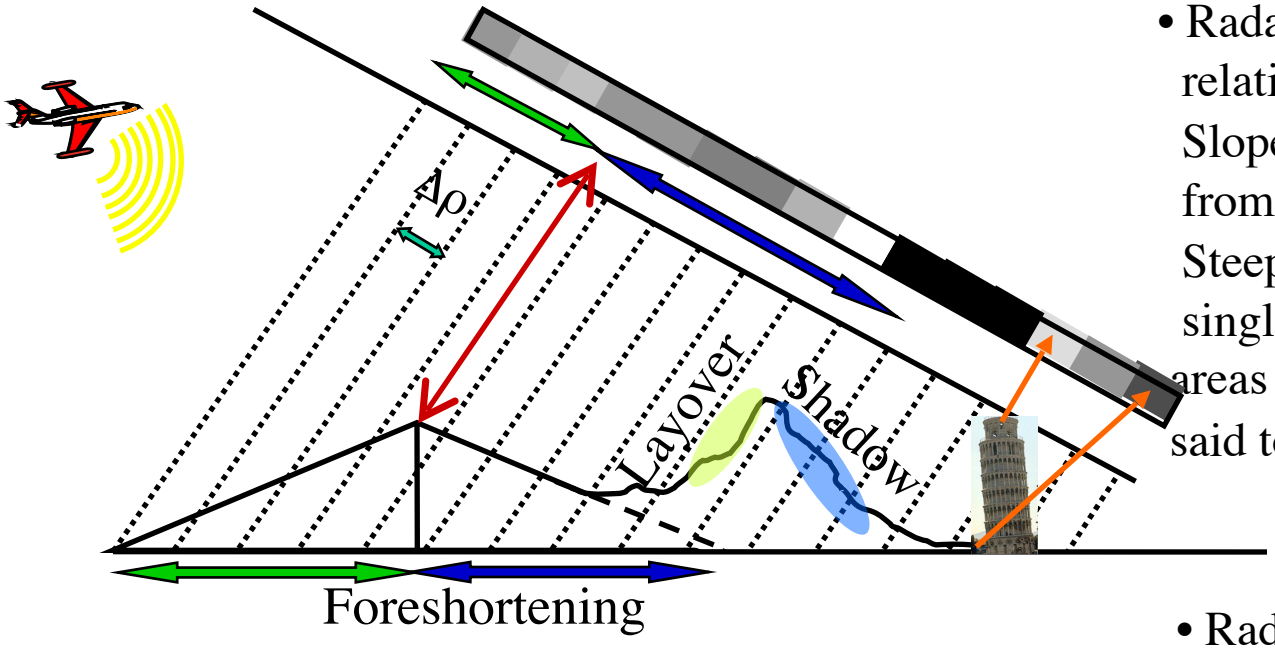
III. Interferometric Correlation

- A. SNR and Interferometric Correlation
- B. Geometric, Temporal and Volumetric Decorrelation
- C. Other Error Sources

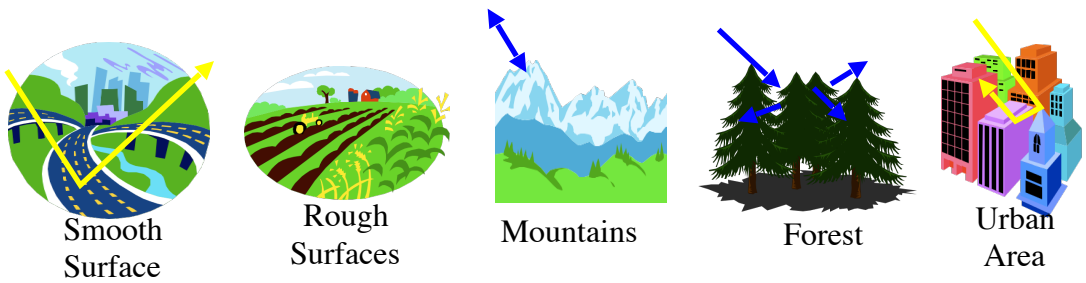
What Radar Can Tell Us

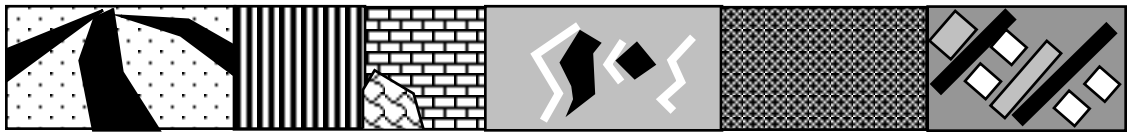
- Transmitted radar signals have known characteristics
  - Amplitude
  - Polarization (Details will follow)
  - Phase and Time Reference
  - Wavelength, or Frequency
- A distant object that scatters the radar signal back toward the receiver alters the amplitude, polarization, and phase, differently for different wavelengths
- Comparison of the received signal characteristics to the transmitted signal allows us to understand the properties of the object.

Radar Imaging Properties



- Radar images are distorted relative to a planimetric view. Slopes facing toward or away from the radar appear foreshortened. Steep slopes are collapsed into a single range cell called layover and areas occulted by other areas are said to be shadowed.





- Radar is primarily sensitive to the structure of objects being imaged, whereas optical images are primarily sensitive to chemistry.
- The scale of objects relative to the radar wavelength determines how smooth an object appears to the radar and how bright or dark it is in the imagery.

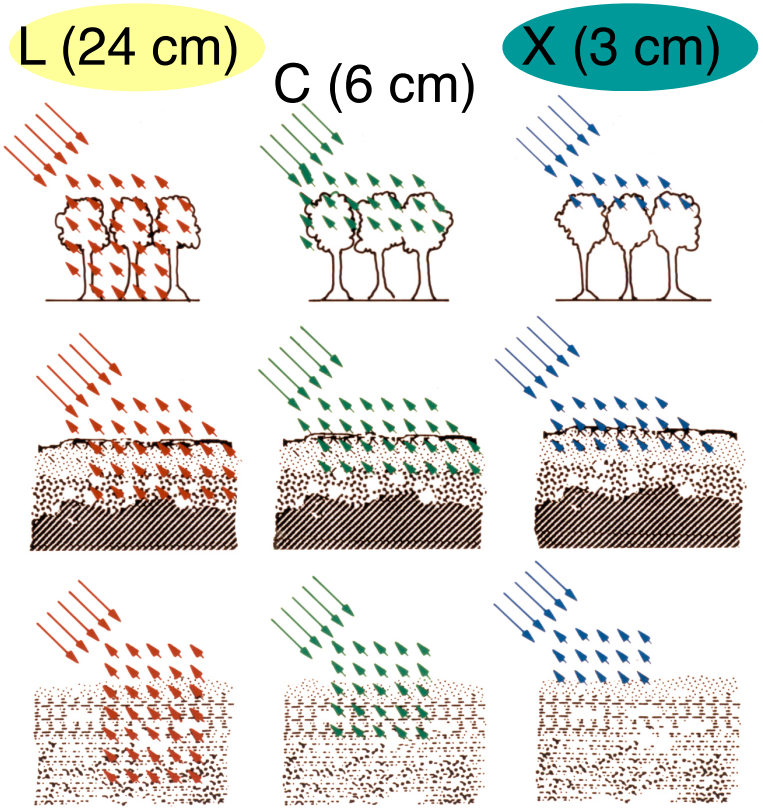
Wavelength - A Measure of Surface Scale

Light interacts most strongly with objects on the size of the wavelength

Forest: Leaves reflect X-band wavelengths but not L-band

Dry soils: Surface looks rough to X-band but not L-band

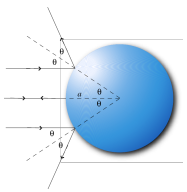
Ice: Surface and layering look rough to X-band but not L-band

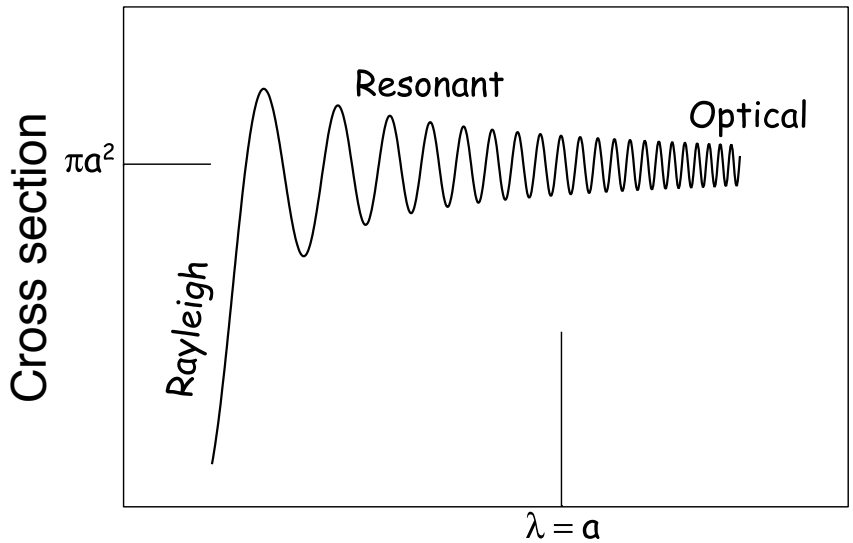


Surface and Volume Scattering Models for Radar

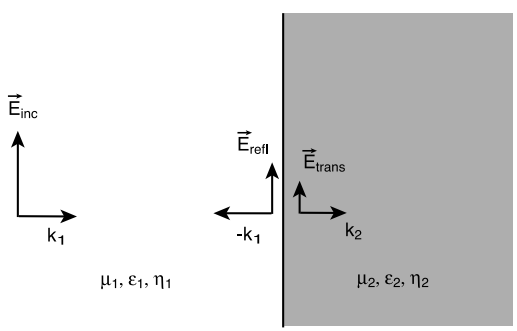
We can model simple scattering from particles or surfaces…

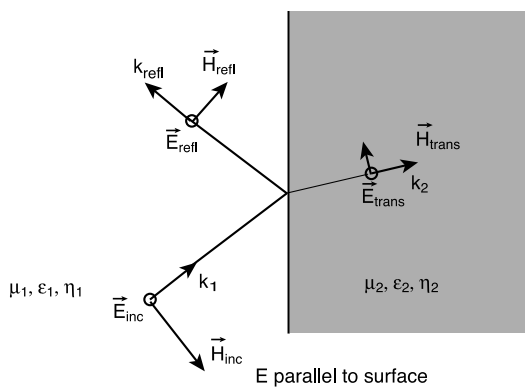
Cross section of a large sphere is its projected area



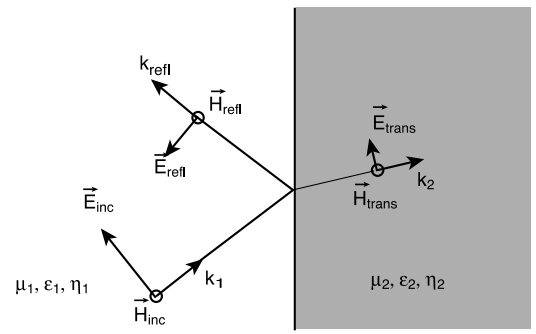


Cross section of a large flat facet goes as area squared



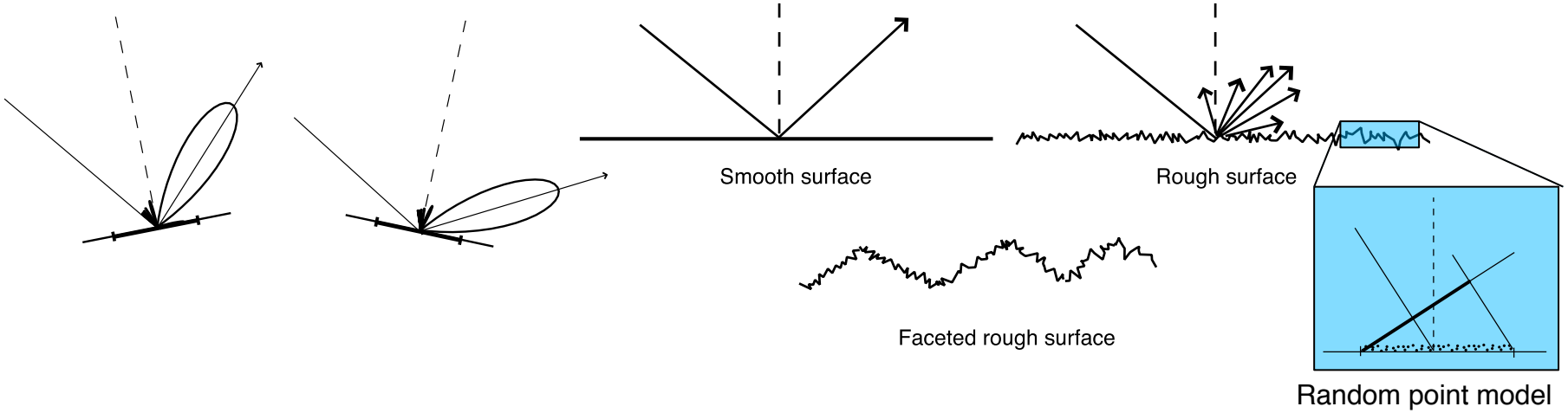


E parallel to surface



H parallel to surface

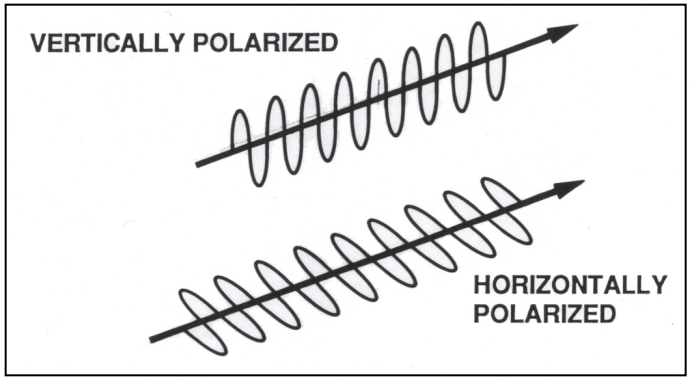
Surface and Volume Scattering Models for Radar

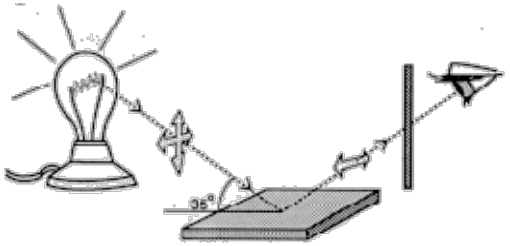


And we can develop intuition from combinations of these elements…

Polarization - A Measure of Surface Orientations and Properties

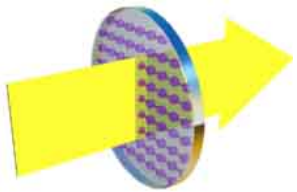
Wave Polarization



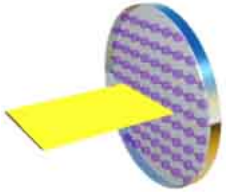


Mostly horizontal polarization is reflected from a flat surface.

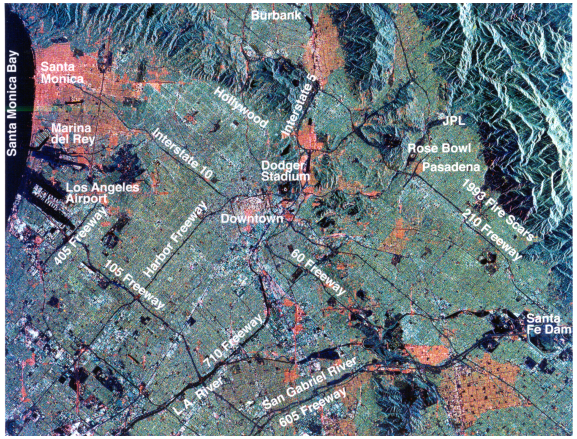
Polarization Filters

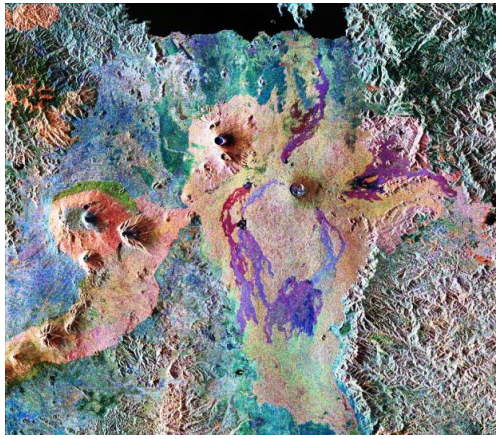


Vertical polarization passes through horizontally arranged absorbers.



Horizontal polarization does not pass through horizontally arranged absorbers.

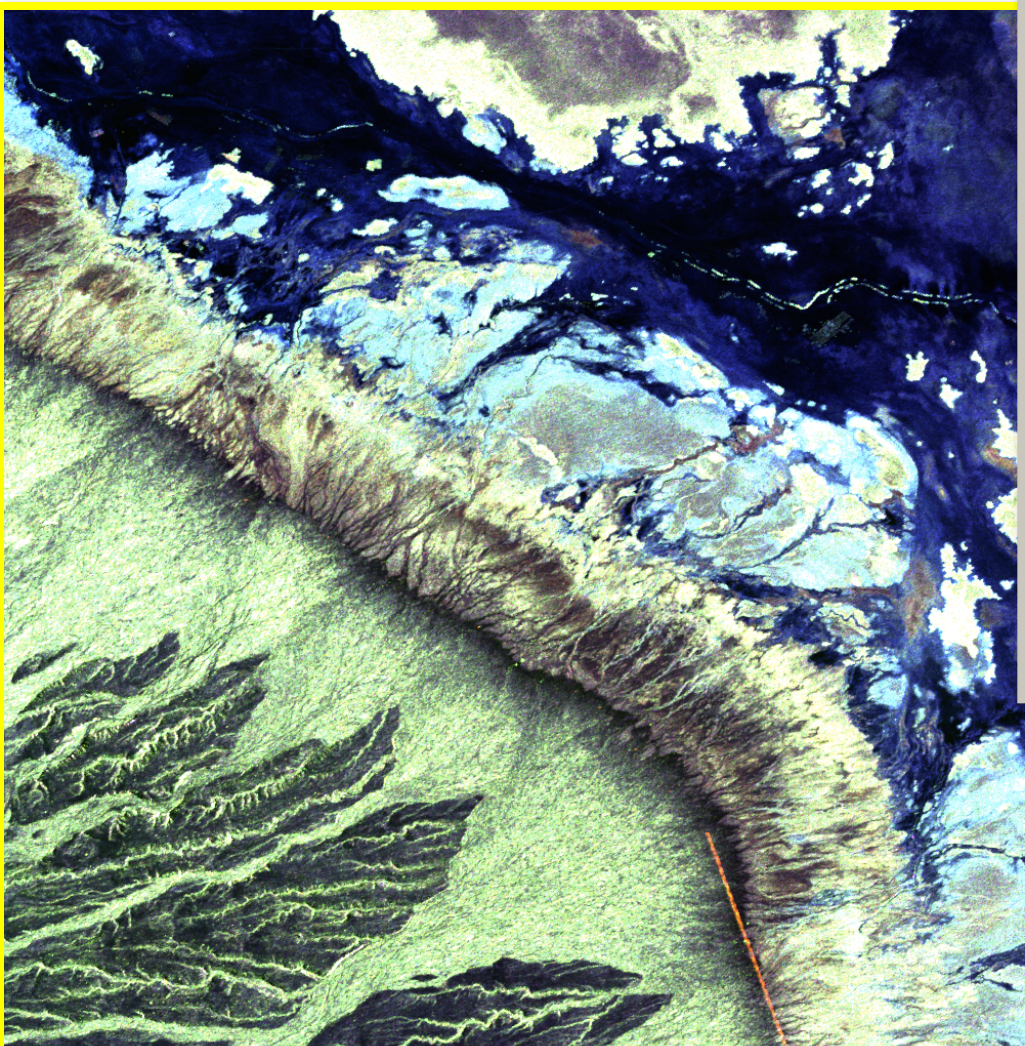


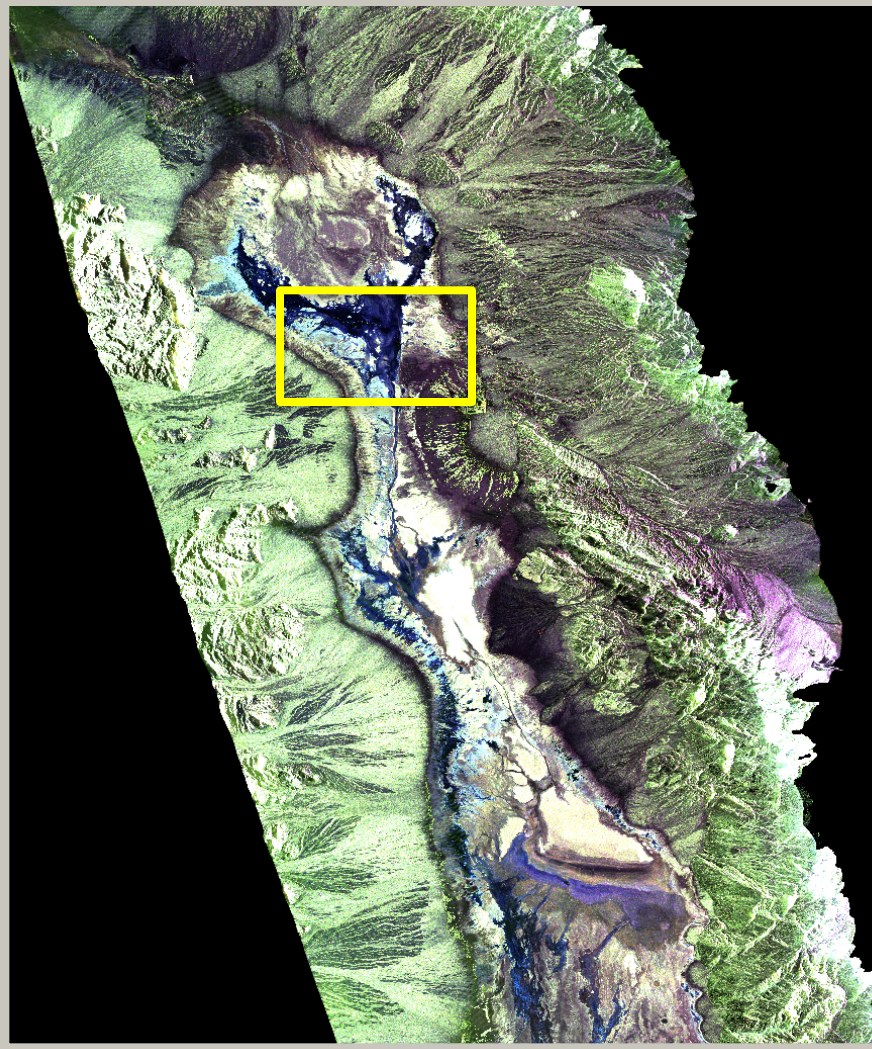


Color figures from www.colorado.edu/physics/2000

Death Valley

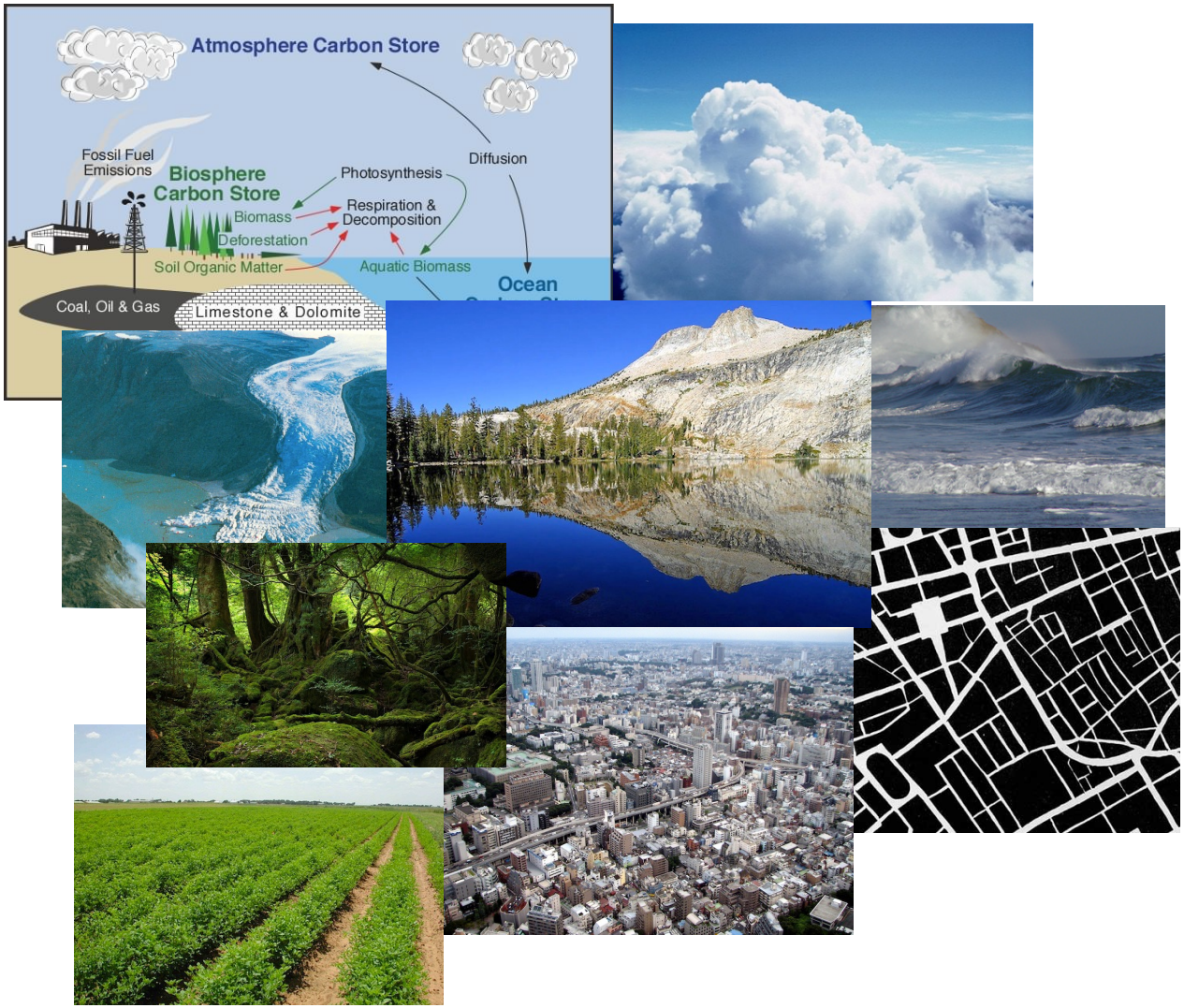
(HH-Red, HV – Green, VV – Blue)





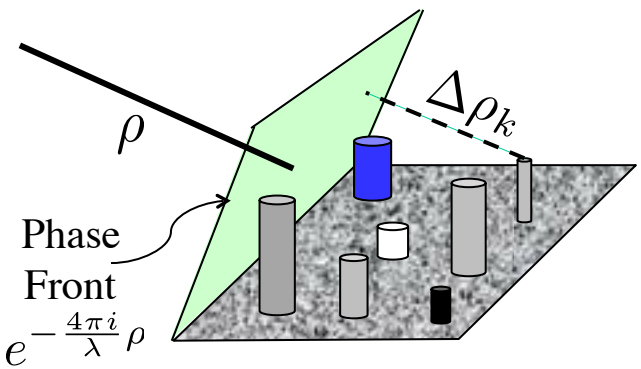
The Real World is Complicated

- Interpretation of physical signal is often accomplished through empirical relationships between backscatter and the signal of interest



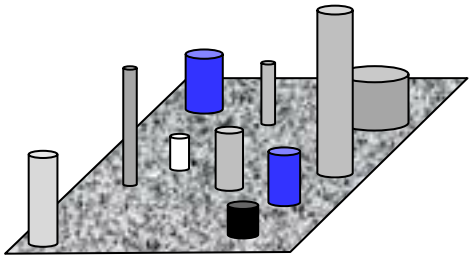
SAR Imagery and Speckle

- Full resolution SAR imagery has a grainy appearance called speckle, which is a phenomena due to the coherent nature of SAR imaging.



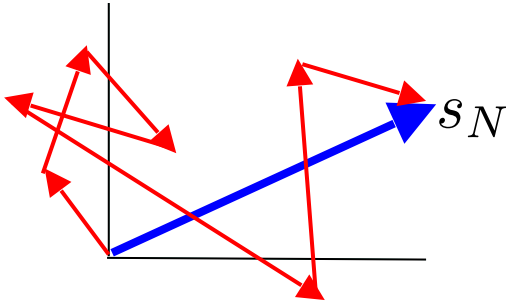
Pixel N

Number and arrangement of scattering elements within resolution cell varies from pixel to pixel.

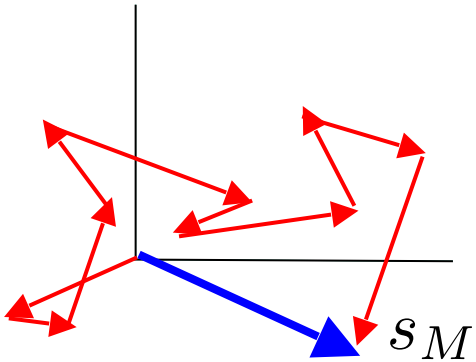


Pixel M

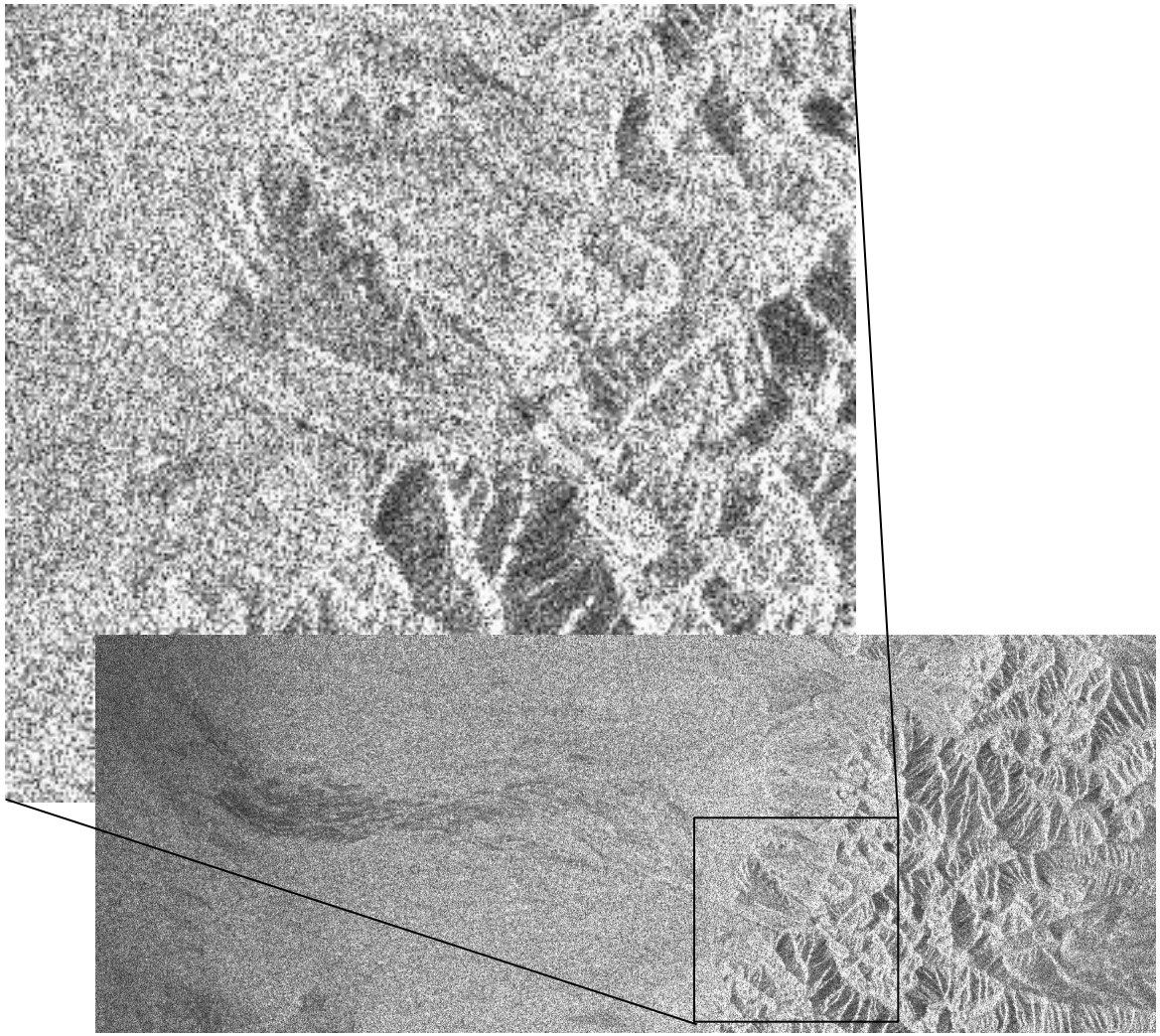
Returned signal is a coherent combination of the returns from the scattering elements.



$$s = A \underbrace{e^{-\frac{4\pi i}{\lambda}\rho}}_{\text{Range Phase}} \underbrace{\sum_{k=1}^{N} a_k e^{-\frac{4\pi i}{\lambda}\Delta\rho_k}}_{\text{Scatterer Contribution}}$$



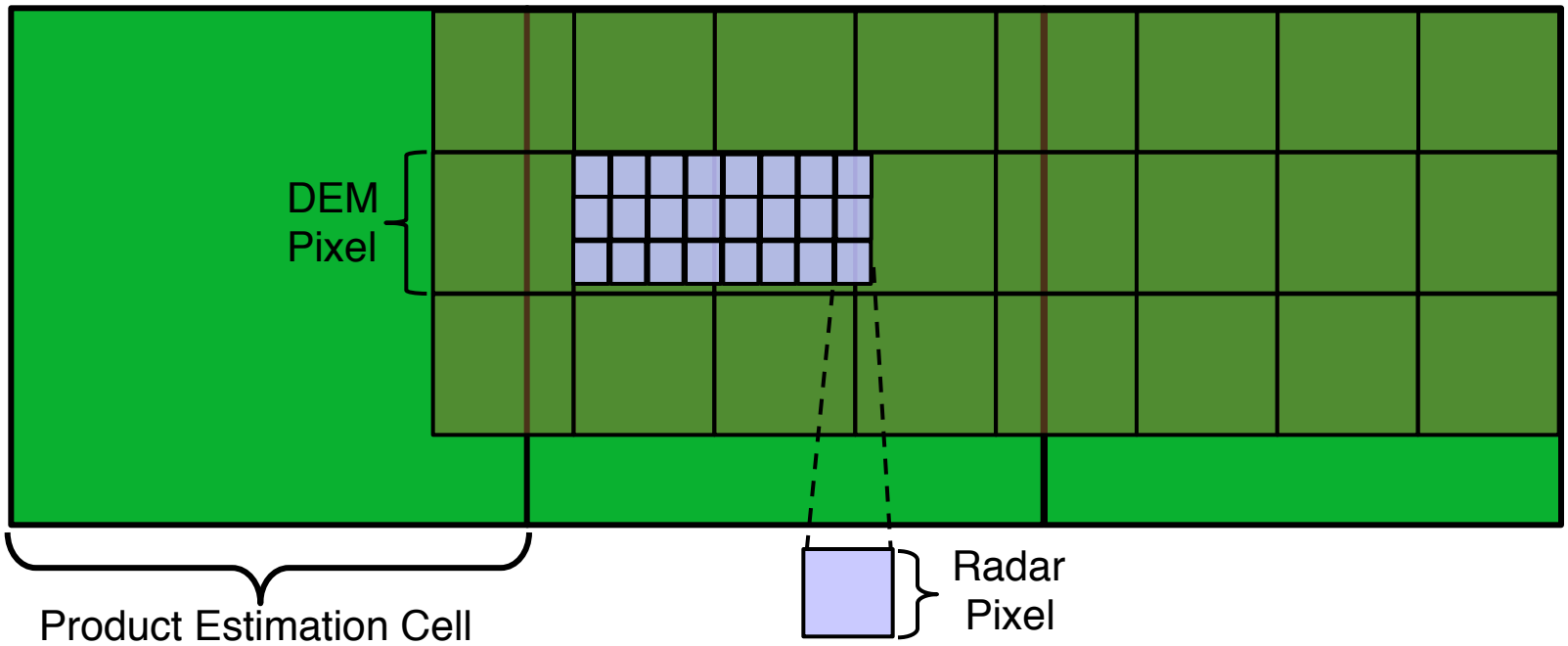
Speckle and Radiometric Accuracy



Natural speckle masks intrinsic radar backscatter which contains surface information, independent of SNR!

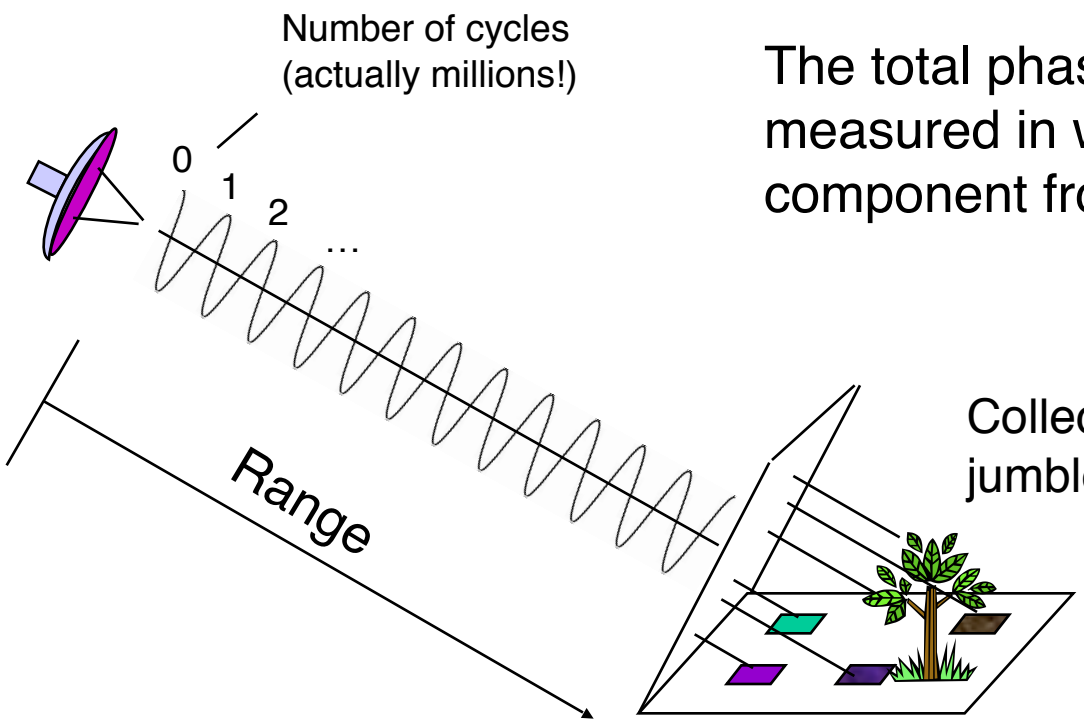
Spatial averaging (looks) reduces the speckle and draws out the natural backscatter reflectivity

Spatial Averaging or Looks



Phase - A Measure of the Range and Surface Complexity

The phase of the radar signal is the number of *cycles of oscillation* that the wave executes between the radar and the surface and back again.



The total phase is two-way range measured in wave cycles + random component from the surface

Collection of random path lengths jumbles the phase of the echo

Only *interferometry* can sort it out!

Simplistic view

$$\phi_1 = \frac{4\pi}{\lambda} \cdot \rho_1 + \text{ other constants } + n_1$$

$$\phi_2 = \frac{4\pi}{\lambda} \cdot \rho_2 + \text{ other constants } + n_2$$

1. The "other constants" cannot be directly determined.

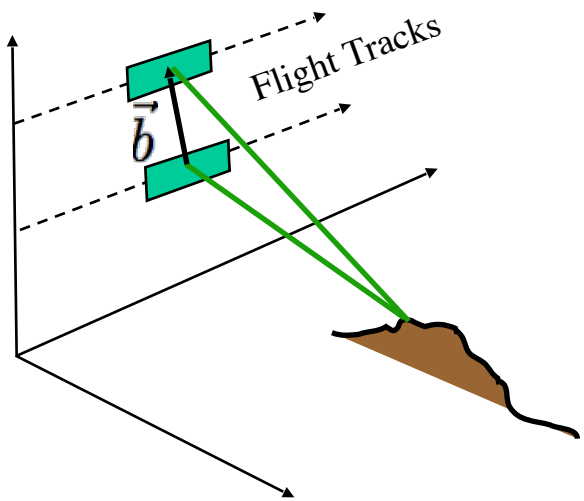
2. "Other constants" depends on scatterer distribution in the resolution cell, which is unknown and varies from cell to cell.

3. Only way of observing the range change is through interferometry (cancellation of "other constants").

Types of Radar Interferometry

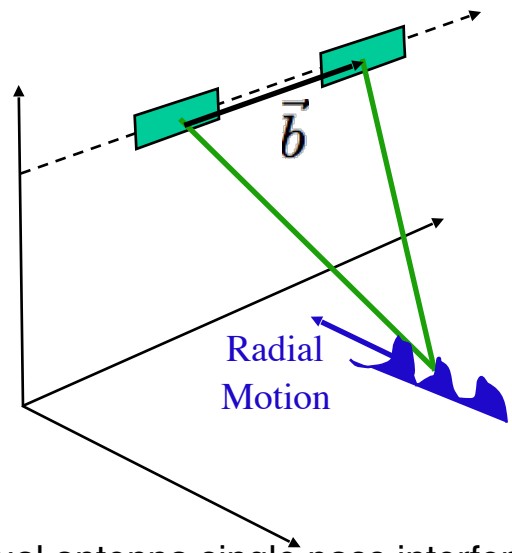
- Two main classes of interferometric radars are separated based on the geometric configuration of the baseline vector:
  - Interferometers are used for topographic measurements when the antennas are separated in the cross-track direction.
  - Interferometers are used to measure line-of-sight motion when the antennas are separated in the along-track direction.
  - A single antenna repeating its path can form an interferometer to measure long-term deformation

Cross-Track Interferometer



- Dual antenna single pass interferometers
- Single antenna repeat pass interferometers
  ==> Topography and Deformation

Along-Track Interferometer



- Dual antenna single pass interferometer
- Along-track separation
  ==> Radial velocity

Interferometry Applications

- Mapping/Cartography
  - Radar Interferometry from airborne platforms is routinely used to produce topographic maps as digital elevation models (DEMs).
    - 2-5 meter circular position accuracy
    - 5-10 m post spacing and resolution
    - 10 km by 80 km DEMs produced in 1 hr on mini-supercomputer
  - Radar imagery is automatically geocoded, becoming easily combined with other (multispectral) data sets.
  - Applications of topography enabled by interferometric rapid mapping
    - Land use management, classification, hazard assessment, intelligence, urban planning, short and long time scale geology, hydrology
- Deformation Mapping and Change Detection
  - Repeat Pass Radar Interferometry from spaceborne platforms is routinely used to produce topographic *change* maps as digital displacement models (DDMs).
    - 0.3-1 centimeter relative displacement accuracy
    - 10-100 m post spacing and resolution
    - 100 km by 100 km DDMs produced rapidly once data is available
  - Applications include
    - Earthquake and volcano monitoring and modeling, landslides and subsidence
    - Glacier and ice sheet dynamics
    - Deforestation, change detection, disaster monitoring

Interferometry for Topography

Measured phase difference:

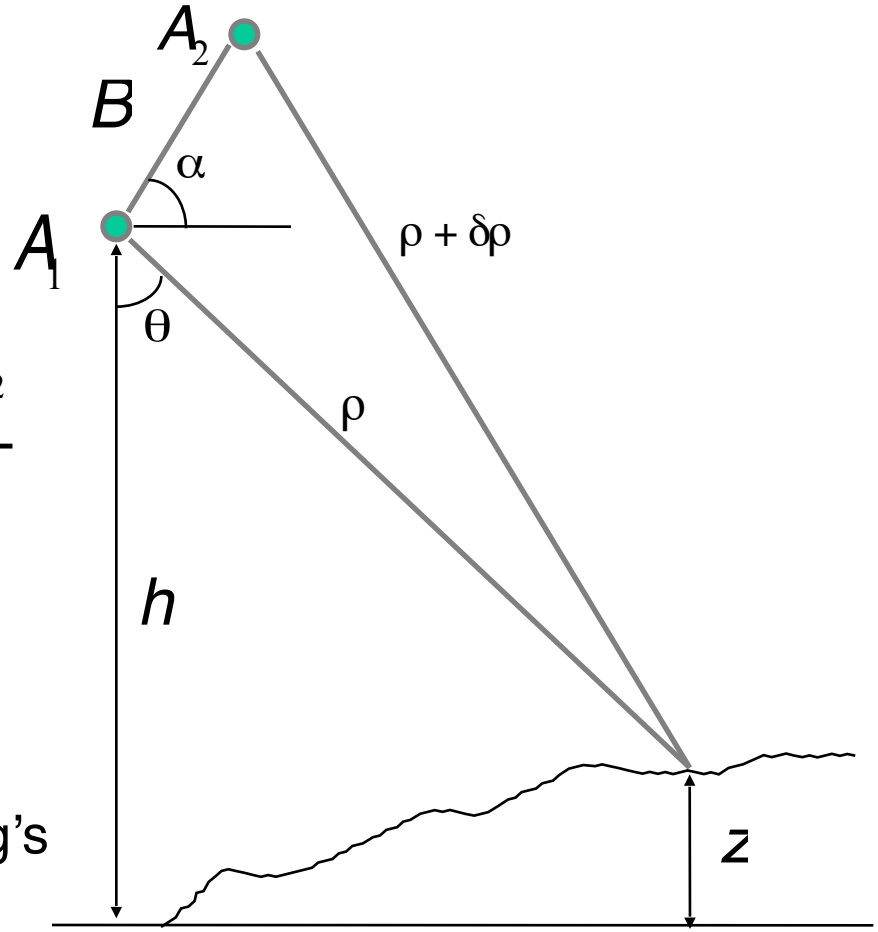
$$\Delta\phi = -\frac{2\pi}{\lambda}\delta\rho$$

Triangulation:

$$\sin(\theta - \alpha) = \frac{(\rho + \delta\rho)^2 - \rho^2 - B^2}{2\rho B}$$

$$z = h - \rho\cos\theta$$

Critical Interferometer Knowledge:
- Baseline, (B, α) to mm's
- System phase differences, to deg's



Data Collection Options

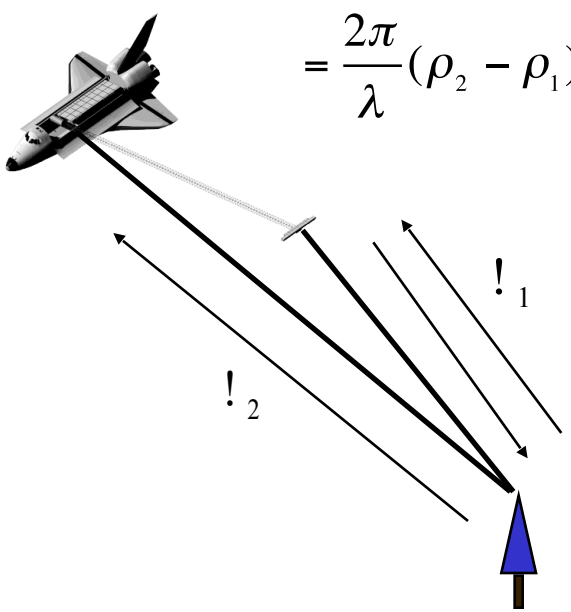
For single pass interferometry (SPI) both antennas are located on the same platform. Two modes of data collection are common:

- *single-antenna-transmit mode* - one antenna transmits and both receive
- *ping-pong mode* - each antenna transmits and receives its own echoes effectively doubling the physical baseline.

Classic

$$\Delta\phi = \frac{2\pi}{\lambda}(\rho_2 + \rho_1) - \frac{2\pi}{\lambda}(\rho_1 + \rho_1)$$

$$= \frac{2\pi}{\lambda}(\rho_2 - \rho_1)$$



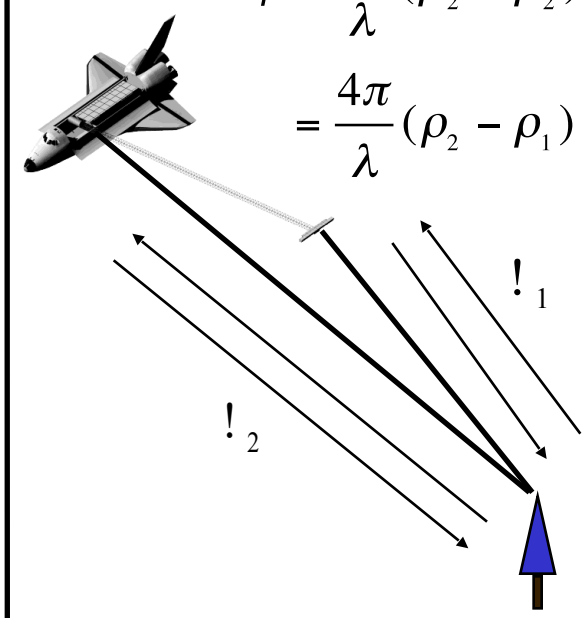
$$\Delta\phi = \frac{2\pi p}{\lambda}\delta\rho,$$

$$p = 1$$

Ping-Pong

$$\Delta\phi = \frac{2\pi}{\lambda}(\rho_2 + \rho_2) - \frac{2\pi}{\lambda}(\rho_1 + \rho_1)$$

$$= \frac{4\pi}{\lambda}(\rho_2 - \rho_1)$$



$$\Delta\phi = \frac{2\pi p}{\lambda}\delta\rho,$$

$$p = 2$$

Height Reconstruction

- Interferometric height reconstruction is the determination of a target's position vector from known platform ephemeris information, baseline information, and the interferometric phase.

$\vec{P}$ = platform position vector

ρ = range to target

$\hat{\ell}$ = unit vector pointing from platform to target

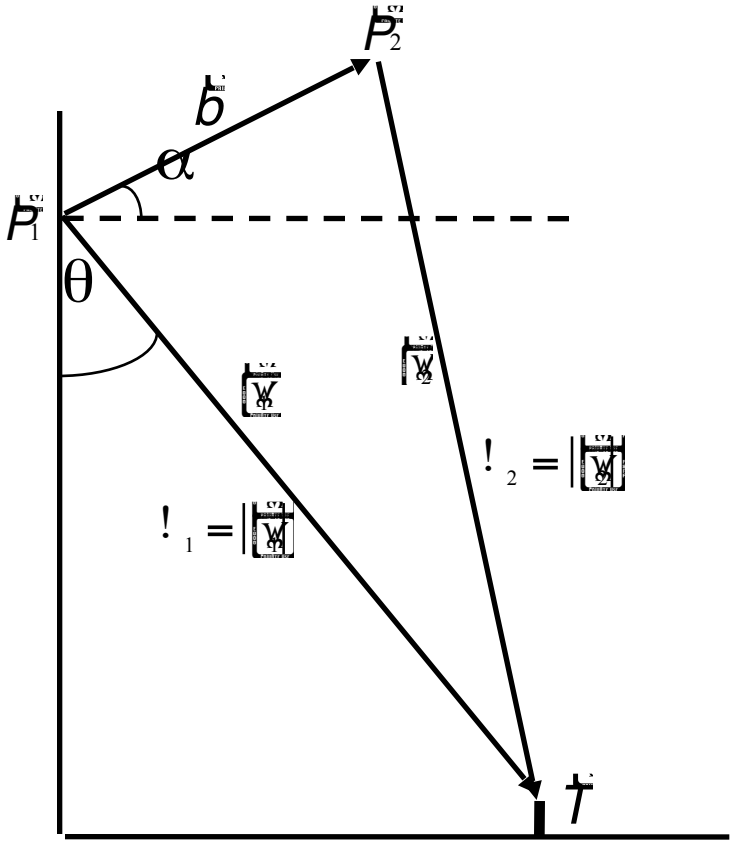
$\vec{T}$ = target location vector

BASIC EQUATION

$$\vec{T} = \vec{P} + \rho\,\hat{\ell}$$

- Interferometry provides a means of determining $\hat{\ell}$

Interferometric Geometry



$$\vec{b} = \vec{P}_2 - \vec{P}_1 = \vec{\ell}_2 - \vec{\ell}_1$$

$$\vec{\ell}_2 = \vec{\ell}_1 + \vec{b}$$

$$b = |\vec{b}| = \langle \vec{b}, \vec{b} \rangle^{\frac{1}{2}}, \quad \hat{\ell}_1 = \frac{\vec{\ell}_1}{|\vec{\ell}_1|} = \frac{\vec{\ell}_1}{\ell_1}$$

$$\Delta\phi = \frac{2\pi p}{\lambda}(\rho_2 - \rho_1) = \frac{2\pi p}{\lambda}\left(|\vec{\ell}_2| - |\vec{\ell}_1|\right)$$

$$= \frac{2\pi p}{\lambda}\left(\langle \vec{\ell}_2, \vec{\ell}_2 \rangle^{\frac{1}{2}} - \rho_1\right)$$

$$= \frac{2\pi p}{\lambda}\left(\langle \vec{\ell}_1 - \vec{b}, \vec{\ell}_1 - \vec{b} \rangle^{\frac{1}{2}} - \rho_1\right)$$

$$= \frac{2\pi p}{\lambda}\left(\left(\rho^2{}_1 - 2\langle \vec{\ell}_1, \vec{b} \rangle + b^2\right)^{\frac{1}{2}} - \rho_1\right)$$

$$= \frac{2\pi p}{\lambda}\rho_1\left(\left(1 - \frac{2\langle \hat{\ell}_1, \vec{b} \rangle}{\rho_1} + \left(\frac{b}{\rho_1}\right)^2\right)^{\frac{1}{2}} - 1\right)$$

- p equals 1 or 2 depending on system

2-D Height Reconstruction - Flat Earth

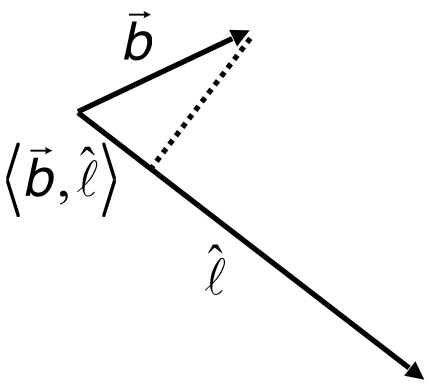
- Before considering the general 3-D height reconstruction it is instructive to first solve the two dimensional problem.

Assume that b << ρ and let $\hat{\ell}_1 = \frac{\vec{\ell}_1}{|\vec{\ell}_1|} = \frac{\vec{\ell}_1}{\rho}$. Taking a first order Taylor's expansion of

$$\Delta\phi = \frac{2\pi p}{\lambda}\rho_1\left(\left(1 - \frac{2\langle\hat{\ell}_1, \vec{b}\rangle}{\rho_1} + \left(\frac{b}{\rho_1}\right)^2\right)^{\frac{1}{2}} - 1\right)$$

the interferometric phase can be approximated as

$$\boxed{\Delta\phi \approx -\frac{2\pi p}{\lambda}\langle\hat{\ell}_1, \vec{b}\rangle}$$



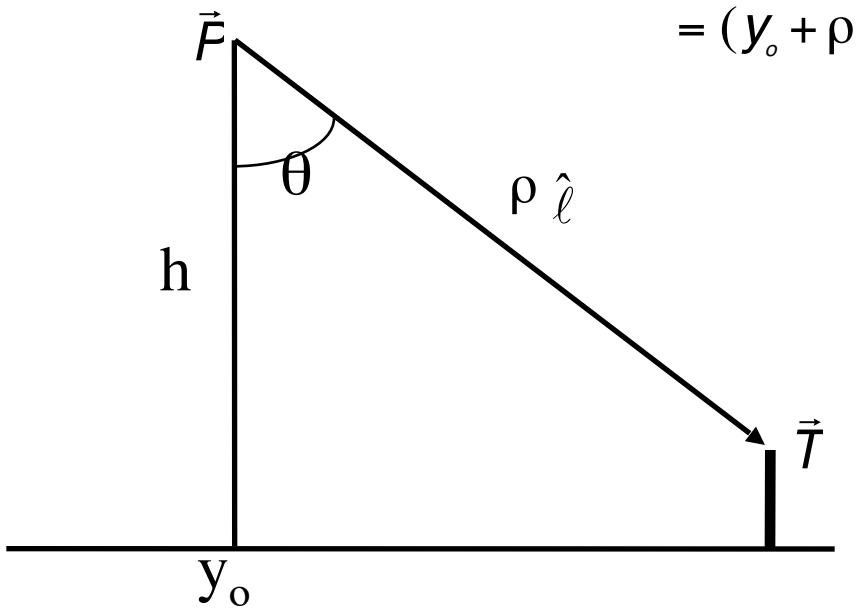
With $\vec{b} = (b\cos(\alpha), b\sin(\alpha))$ and $\hat{\ell} = (\sin(\theta), -\cos(\theta))$ then

$$\Delta\phi = -\frac{2\pi p}{\lambda} b\sin(\theta - \alpha)$$

2-D Height Reconstruction - Flat Earth II

- Let $\vec{P} = (y_o, h)$ be the platform position vector, then

$$\vec{T} = \vec{P} + \rho\,\hat{\ell}$$
$$= (y_o, h) + \rho\,(\sin(\theta), -\cos(\theta))$$
$$= (y_o + \rho\,\sin(\theta), h - \rho\,\cos(\theta))$$



- Solving for θ in terms of the interferometric phase, Δϕ, yields

$$\theta = \sin^{-1}\left(\frac{-\lambda\Delta\phi}{2\pi p b}\right) + \alpha$$

Flattened Phase

- Often when looking at interferograms or prior to unwrapping it is desirable to remove the flat earth fringes so that the resulting fringes will follow the local topography. This process is call flattening and the resulting phase is called the flattened phase.
- The flattened phase is given by

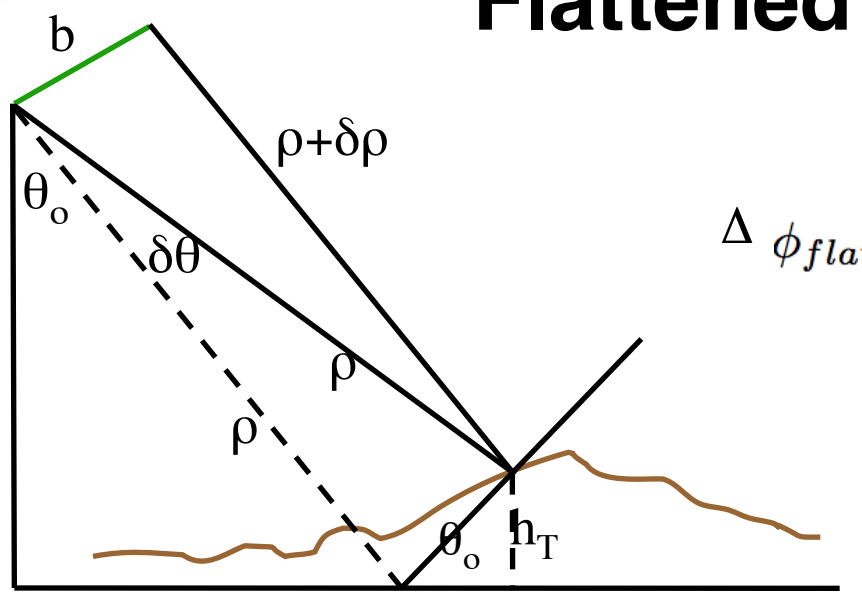
$$\Delta\,\phi_{flat} = -\frac{4\pi}{\lambda}\left(\langle\hat{\ell},\vec{b}\rangle - \langle\hat{\ell}_o,\vec{b}\rangle\right)$$

where $\hat{\ell}$ is the look vector to a point and $\hat{\ell}_o$ is the corresponding look vector to the flat surface at the same range.

- Making the usual 2 dimensional simplifications

$$\begin{aligned}\Delta\,\phi_{flat} &= \frac{4\pi}{\lambda}b(\sin(\theta-\alpha)-\sin(\theta_o-\alpha)) \\ &\approx \frac{4\pi}{\lambda}b\cos(\theta-\alpha)\delta\theta\end{aligned}$$

Flattened Phase II



$$\Delta\,\phi_{flat} = \frac{4\pi}{\lambda} b(\sin(\theta-\alpha) - \sin(\theta_o - \alpha))$$
$$\approx \frac{4\pi}{\lambda} b\cos(\theta-\alpha)\delta\theta$$

From the figure we have

$$\delta\theta \approx \frac{h_T}{\rho \sin\theta_o}$$

which gives

$$\Delta\,\phi_{flat} = \frac{4\pi}{\lambda} b_\perp \delta\theta$$
$$= \frac{4\pi}{\lambda} b_\perp \frac{h_T}{\rho\sin\theta_o}$$

Thus the topographic portion of the interferometric phase is a function of the perpendicular baseline length.

Sensitivity of Height with Respect to Phase

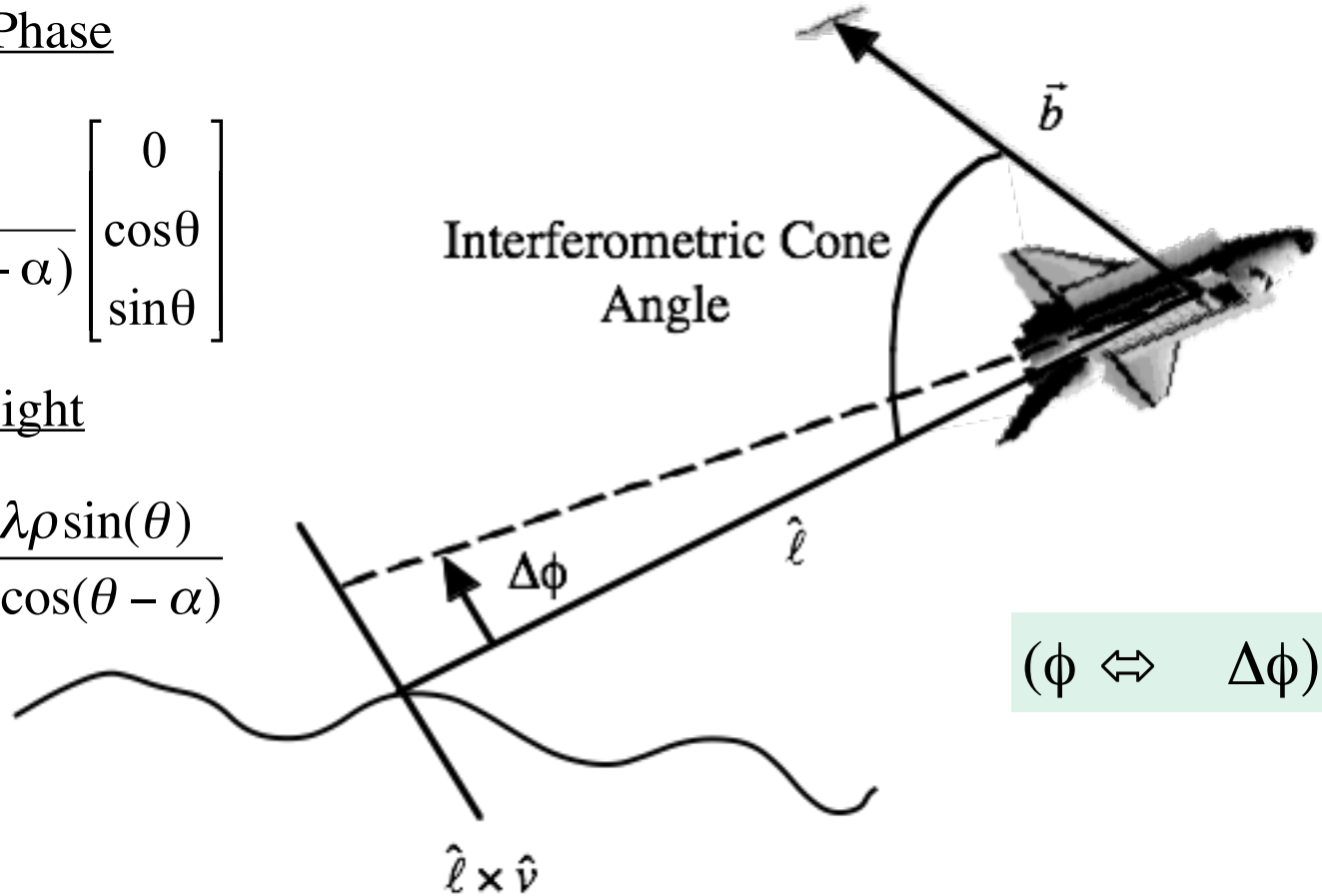
Sensitivity to Phase

$$\frac{\partial \vec{T}}{\partial \phi} = \frac{-\lambda\rho}{2\pi p b \cos(\theta - \alpha)} \begin{bmatrix} 0 \\ \cos\theta \\ \sin\theta \end{bmatrix}$$

Ambiguity Height

$$h_a = 2\pi \frac{\partial T_z}{\partial \phi} = \frac{-\lambda\rho \sin(\theta)}{pb\cos(\theta - \alpha)}$$

$p=1,2$

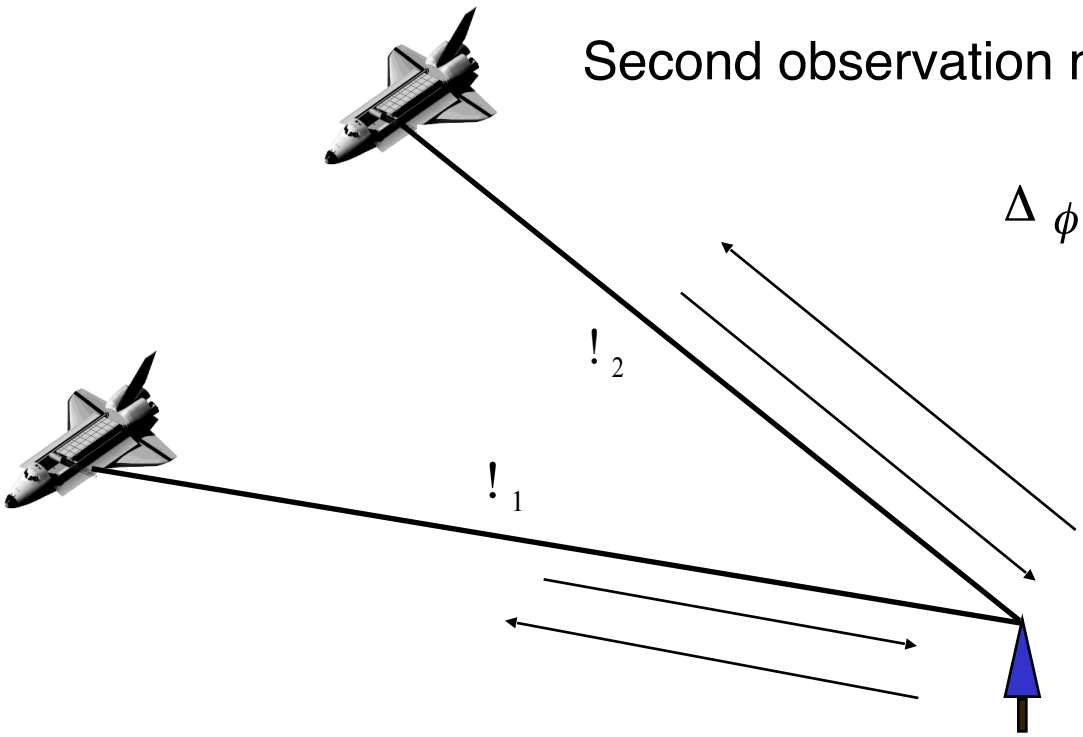


$(\phi \Leftrightarrow \Delta\phi)$

- Observe that $\frac{\partial \vec{T}}{\partial \phi}$ is parallel to $\hat{\ell} \times \hat{v}$.

Data Collection Options II

Interferometric data can also be collected in the repeat pass mode (RPI). In this mode two spatially close radar observations of the same scene are made separated in time. The time interval may range from seconds to years. The two observations may be made different sensors provided they have nearly identical radar system parameters.



Second observation made at some later time.

$$\Delta\phi = \frac{2\pi}{\lambda}(\rho_2 + \rho_2) - \frac{2\pi}{\lambda}(\rho_1 + \rho_1)$$

$$= \frac{4\pi}{\lambda}(\rho_2 - \rho_1)$$

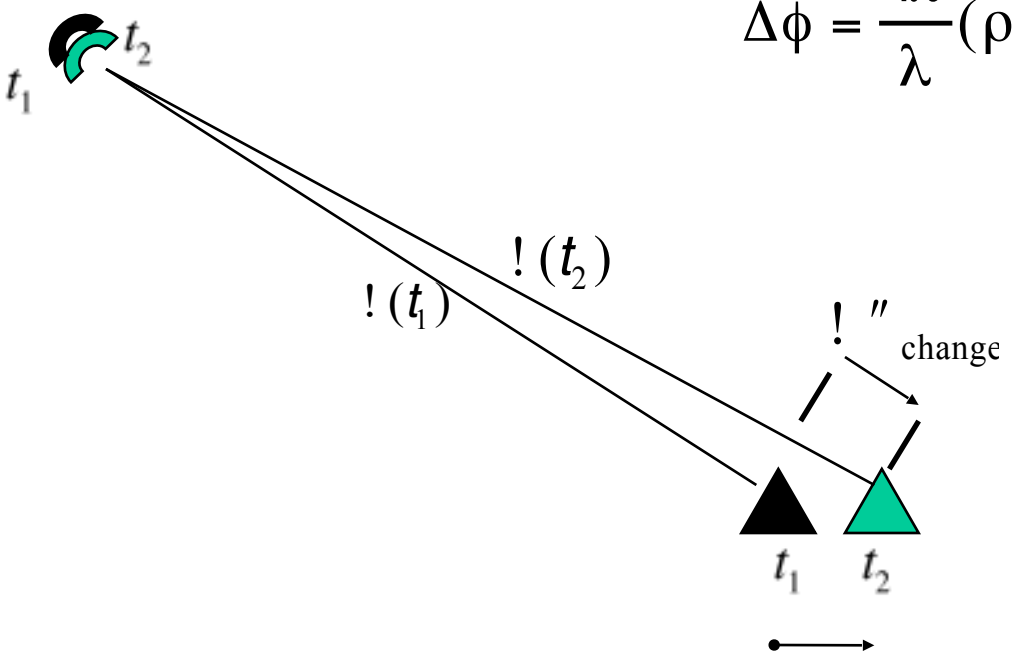
$$\Delta\phi = \frac{2\pi p}{\lambda}\delta\rho,$$

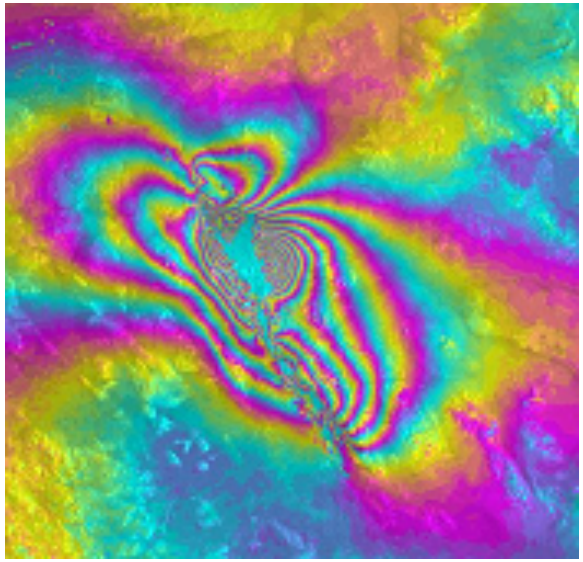
$$p = 2$$

Differential Interferometry

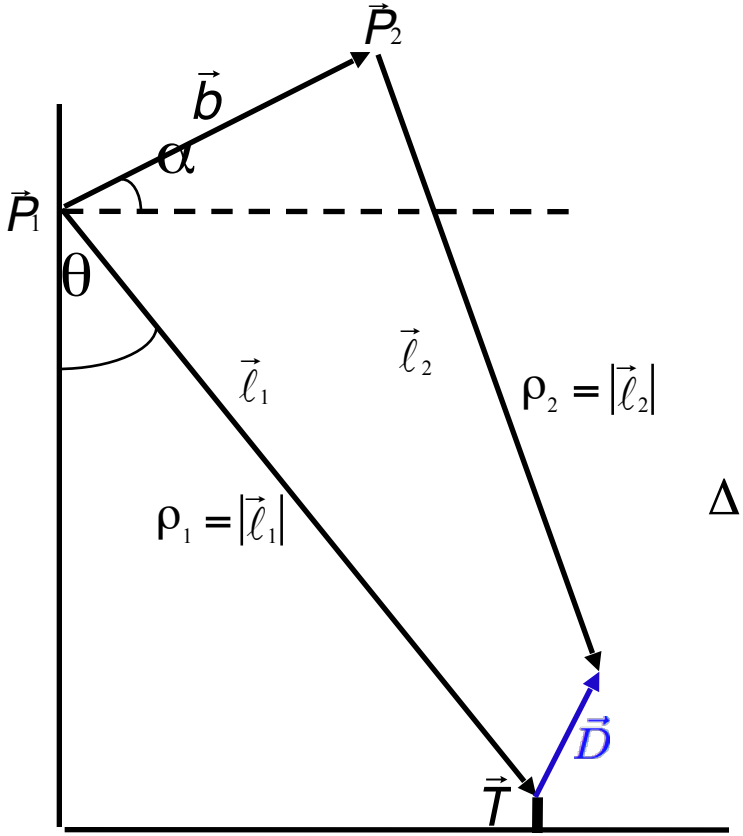
When two observations are made from the same location in space but at different times, the interferometric phase is proportional to any change in the range of a surface feature directly.

$$\Delta\phi = \frac{4\pi}{\lambda}(\rho(t_1) - \rho(t_2)) = \frac{4\pi}{\lambda}\Delta\rho_{\text{change}}$$





Differential Interferometric Phase



$$\vec{b} = \vec{P}_2 - \vec{P}_1 = \vec{\ell}_1 - \vec{\ell}_2$$

$$\vec{\ell}_2 = \vec{T} + \vec{D} - \vec{P}_2 = \vec{\ell}_1 + \vec{D} - \vec{b}$$

$$b = |\vec{b}| = \langle \vec{b}, \vec{b} \rangle^{\frac{1}{2}} \quad , \quad \hat{\ell}_1 = \frac{\vec{\ell}_1}{|\vec{\ell}_1|} = \frac{\vec{\ell}_1}{\rho_1}$$

$$\Delta\phi = \frac{2\pi p}{\lambda}(\rho_2 - \rho_1) = \frac{2\pi p}{\lambda}\left(|\vec{\ell}_2| - |\vec{\ell}_1|\right)$$

$$= \frac{2\pi p}{\lambda}\left(\langle \vec{\ell}_2, \vec{\ell}_2 \rangle^{\frac{1}{2}} - \rho_1\right)$$

$$\Delta\phi = \frac{4\pi}{\lambda}\left(\langle \vec{\ell}_1 + \vec{D} - \vec{b}, \vec{\ell}_1 + \vec{D} - \vec{b} \rangle^{\frac{1}{2}} - \rho_1\right)$$

Assuming that

$$\begin{aligned} |\vec{b}| &\leq \rho \\ |\vec{D}| &\leq \rho \\ |\langle \vec{b}, \vec{D} \rangle| &\leq \rho \end{aligned}$$

yields

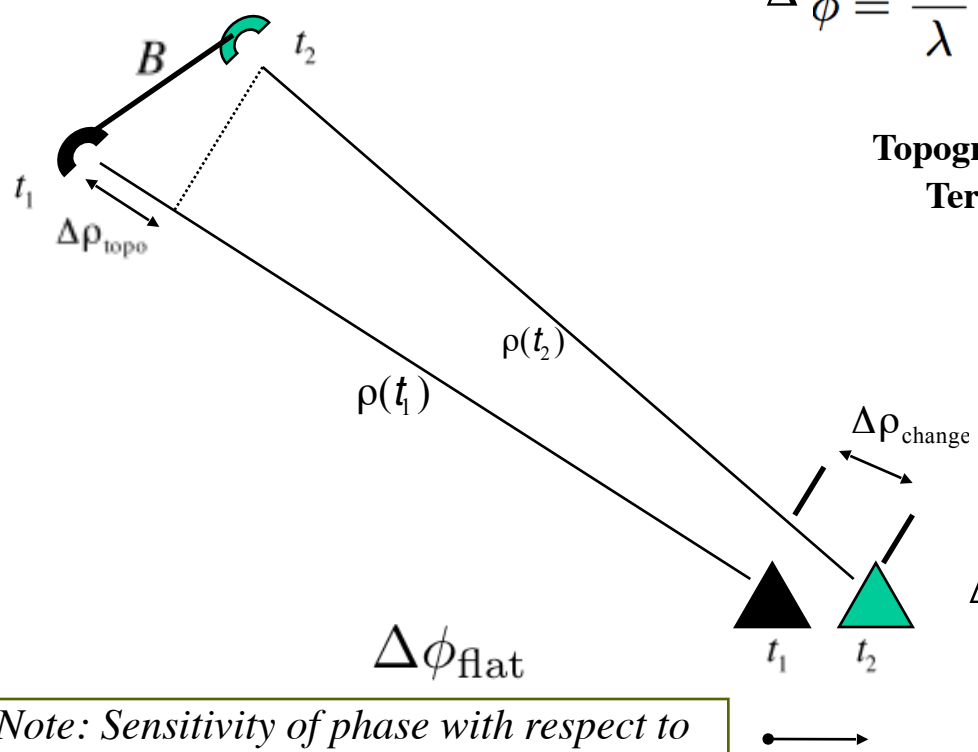
$$\boxed{\Delta\phi = \frac{4\pi}{\lambda}\left(-\langle \hat{\ell}, \vec{b} \rangle + \langle \hat{\ell}, \vec{D} \rangle\right)}$$

Differential Interferometry and Topography

- Generally two observations are made from different locations in space and at different times, so the interferometric phase is proportional to topography and topographic change.

$$\Delta\phi = \frac{4\pi}{\lambda}\left(-\langle\hat{\ell},\vec{b}\rangle + \langle\hat{\ell},\vec{D}\rangle\right)$$

Topography Term **Change Term**



$$\Delta\phi = \frac{4\pi}{\lambda}(\Delta\rho_{change} - \Delta\rho_{topo})$$

$$\Delta\phi = \frac{4\pi}{\lambda}(\Delta\rho_{change} - b\sin(\theta-\alpha))$$

$$\Delta\phi_{flat} = \frac{4\pi}{\lambda}\left(\Delta\rho_{change} - \frac{b_\perp h_T}{\rho\sin\theta}\right)$$

Note: Sensitivity of phase with respect to change is much greater than with respect to topographic relief

If topography is known, then second term can be eliminated to reveal surface change

Extracting the Deformation Term - Pre-Existing DEM

$$\Delta\ \phi_{b_1} = \frac{4\pi}{\lambda}\left(-\langle\hat{\ell},\vec{b}_1\rangle + \langle\hat{\ell},\vec{D}\rangle\right)$$

Phase Simulated from DEM

$$\phi_{topo} = -\frac{4\pi}{\lambda}\langle\hat{\ell},\vec{b}_1\rangle$$

<u>Using Unflattened Phases</u>

$$\begin{aligned}\Delta\rho_{disp} &= \langle\hat{\ell},\vec{D}\rangle \\ &= \frac{\lambda}{4\pi}\left(\Delta\phi_{b_1} - \phi_{topo}\right)\end{aligned}$$

<u>Using Flattened Phases</u>

$$\begin{aligned}\Delta\phi_{flat_1} &= \frac{4\pi}{\lambda}b_{\perp_1}\frac{h_T}{\rho\sin\theta_o} + \frac{4\pi}{\lambda}\Delta\rho_{disp} \\ \Delta\phi_{flat_2} &= \frac{4\pi}{\lambda}b_{\perp_1}\frac{h_T}{\rho\sin\theta_o}\end{aligned}$$

$$\Delta\rho_{disp} = \frac{\lambda}{4\pi}\left(\Delta\phi_{flat_1} - \Delta\phi_{flat_2}\right)$$

Differential Interferometry Sensitivities

- The reason differential interferometry can detect millimeter level surface deformation is that the differential phase is much more sensitive to displacements than to topography.

$$\frac{\partial \phi}{\partial h} = \frac{2\pi p b \cos(\theta - \alpha)}{\lambda \rho \sin\theta} = \frac{2\pi p b_\perp}{\lambda \rho \sin\theta}$$ Topographic Sensitivity

($\phi \Leftrightarrow \Delta\phi$)

$$\frac{\partial \phi}{\partial \Delta\rho} = \frac{4\pi}{\lambda}$$ Displacement Sensitivity

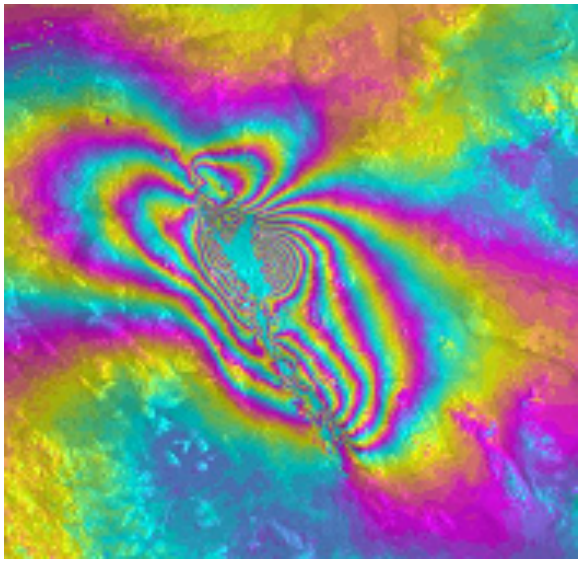
$$\sigma_{\phi_{topo}} = \frac{\partial \phi}{\partial h}\sigma_h = \frac{4\pi}{\lambda}\frac{b_\perp}{\rho \sin\theta}\sigma_h$$ Topographic Sensitivity Term

$$\sigma_{\phi_{disp}} = \frac{\partial \phi}{\partial \Delta\rho}\sigma_{\Delta\rho} = \frac{4\pi}{\lambda}\sigma_{\Delta\rho}$$ Displacement Sensitivity Term

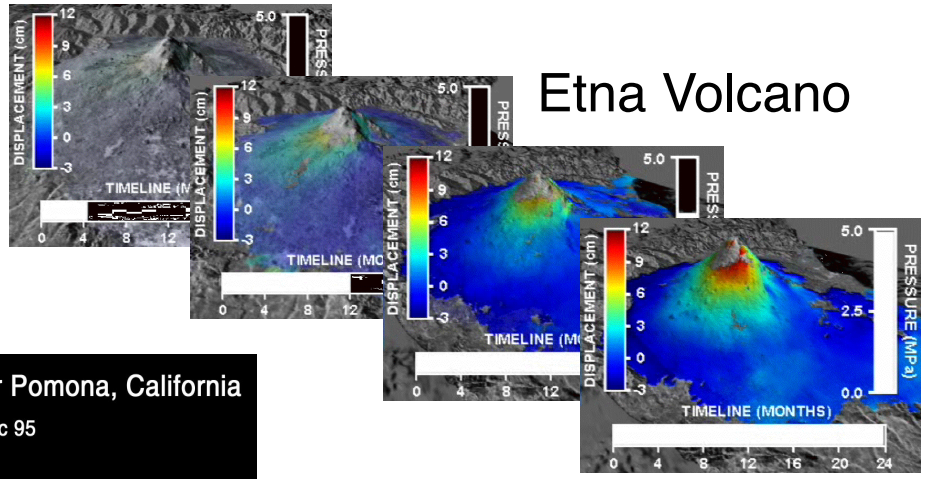
Since $\frac{b}{\rho} << 1$ ==> $\frac{\sigma_{\phi_{disp}}}{\sigma_{\Delta\rho}} >> \frac{\sigma_{\phi_{topo}}}{\sigma_h}$

Meter Scale Topography Measurement - Millimeter Scale Topographic Change

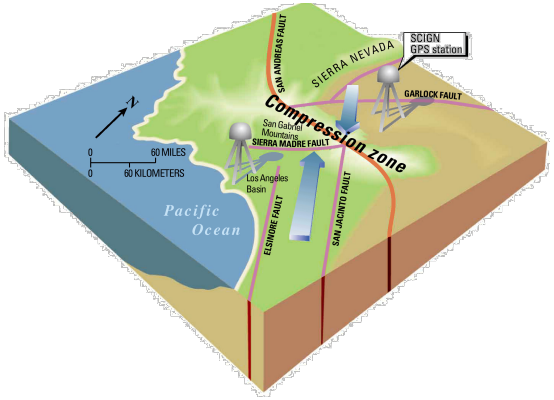
Some Examples of Deformation

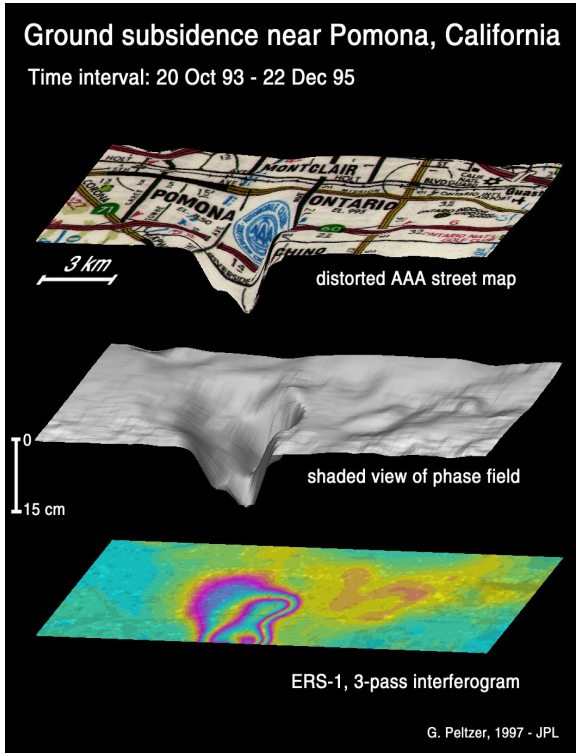


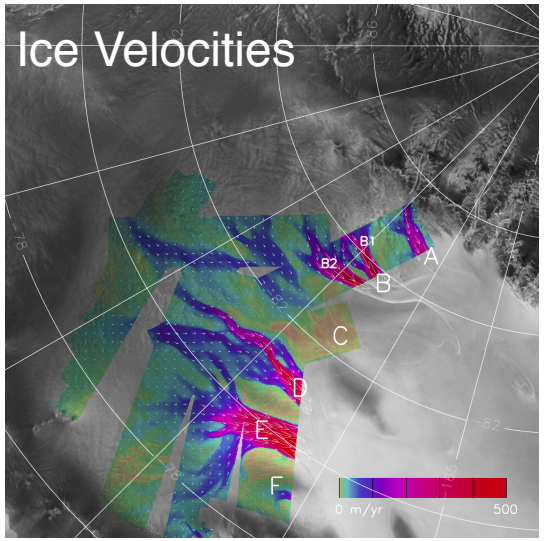
Hector Mine Earthquake



Etna Volcano







Joughin et al , 1999

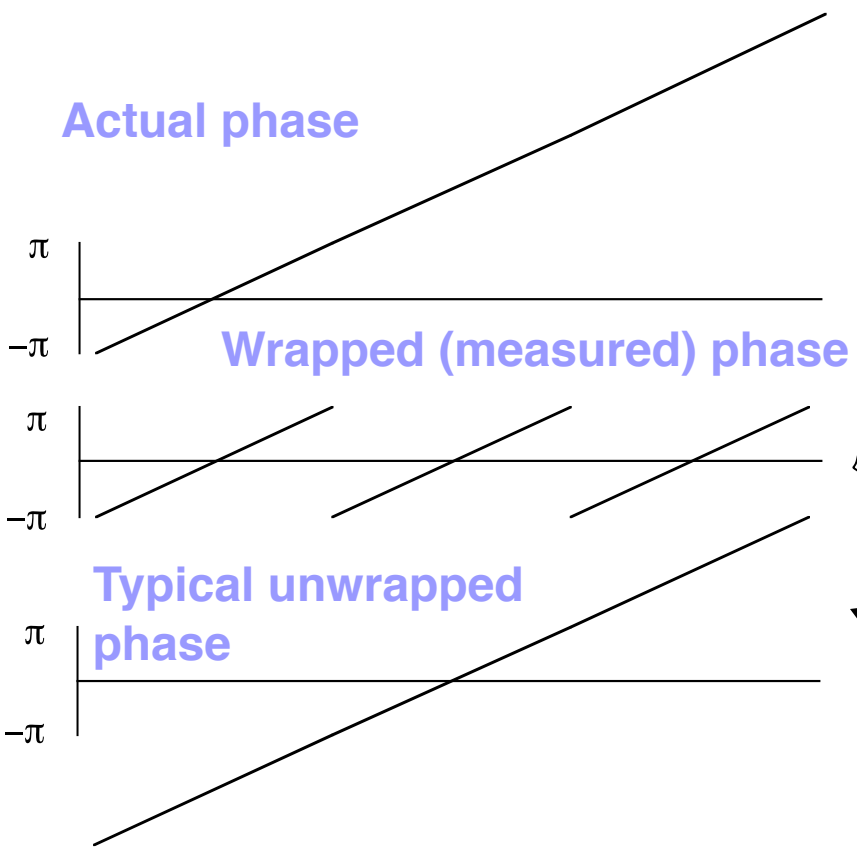
Phase Unwrapping

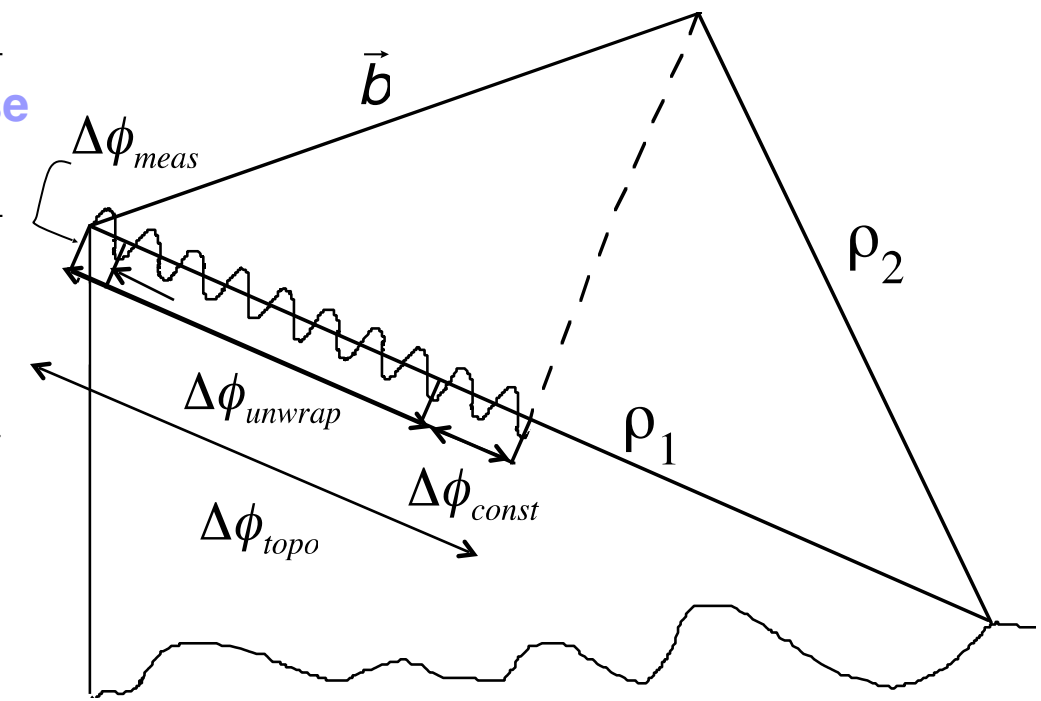
From the measured, wrapped phase, unwrap the phase from some arbitrary starting location, then determine the proper 2π phase "ambiguity".

$$\Delta\phi_{topo} = \frac{2\pi p}{\lambda}\left(\rho_1 - \rho_2\right) = \frac{2\pi p}{\lambda}\vec{b} \bullet \vec{l}$$

$$\Delta\phi_{meas} = \mathrm{mod}\left(\Delta\phi_{topo}, 2\pi\right)$$

$$\Delta\phi_{unwrap}(s,\rho) = \Delta\phi_{topo}(s,\rho) + \Delta\phi_{const}$$





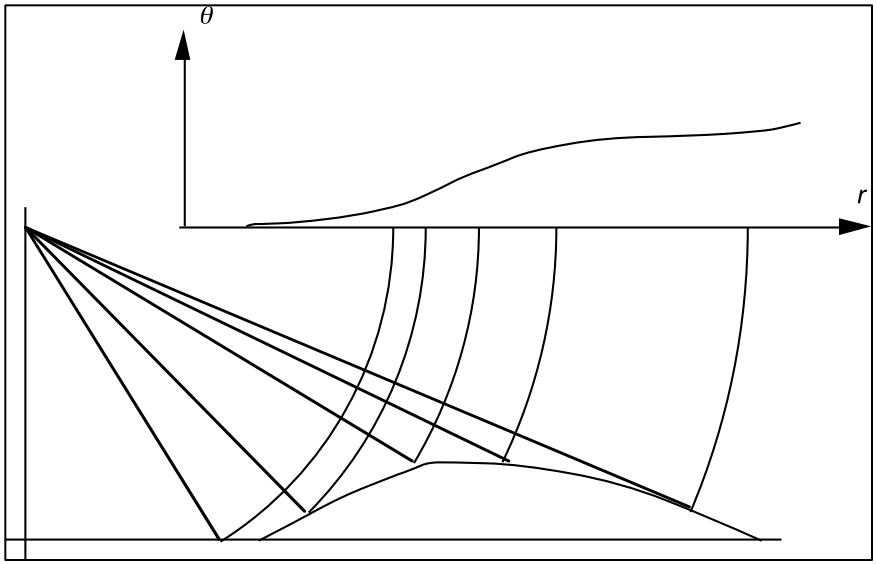
Phase Unwrapping Algorithm Options

- There are a number of approaches to unwrapping:
  - *Residue –or branch cut– methods* that use a simple mechanism to detect where phase consistencies in the phase field exist and generate cuts to avoid them
  - *Least squares methods* that pose the unwrapping problem as an estimation problem and try to efficiently solve the resulting series of equations. (mostly obsolete)
  - *Minimum cost flow algorithms* that pose the unwrapping problem as a network flow problem for which algorithms have been developed over the years

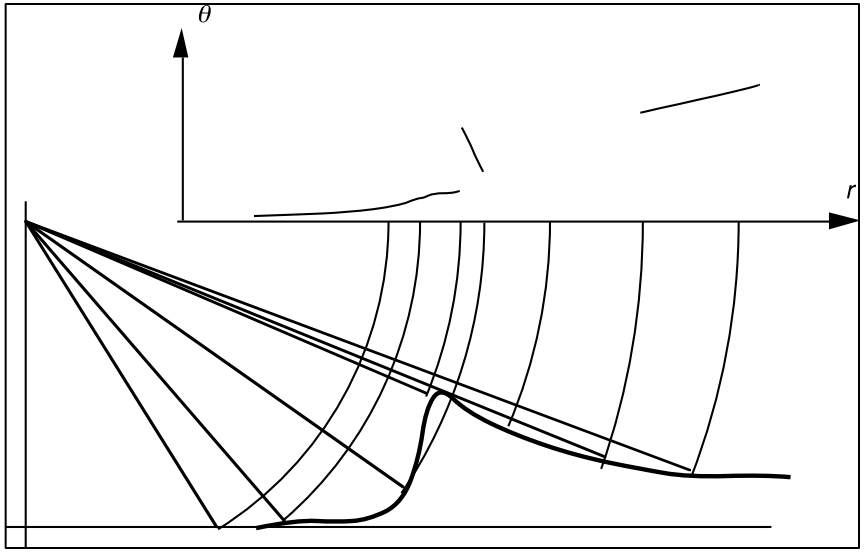
ISCE supports two residue methods and a version of minimum cost flow developed at Stanford (SNAPHU)

Real World Phase Behavior

- Benign topography resulting in a monotonous and well-behaving phase



- Rough topography leading to lay-over and shadows and a generically non-monotonous and non-continuous phase



Two-Dimensional Phase Unwrapping

- Two dimensional phase field values below are in units of cycles
- One-dimensional unwrapping criterion of half-cycle proximity is inconsistent in two dimensions

0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.9	0.6	0.6	0.9	0.0	0.0
0.0	0.0	1.2	0.9	0.9	1.2	0.0	0.0
0.0	0.0	1.2	0.9	0.9	1.2	0.0	0.0
0.0	0.0	0.9	0.6	0.6	0.9	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0

0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	-0.1	-0.4	-0.4	-0.1	0.0	0.0
0.0	0.0	0.2	-0.1	-0.1	0.2	0.0	0.0
0.0	0.0	0.2	-0.1	-0.1	0.2	0.0	0.0
0.0	0.0	-0.1	-0.4	-0.4	-0.1	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0

- **Residues, marked with + and -, define ambiguous boundaries.**

Branch Cuts in Phase Unwrapping

- Branch-cut algorithms (Goldstein, Zebker, and Werner 1986) seek to neutralize these regions of inconsistency by connecting residues of opposite solenoidal sense with cuts, across which integration may not take place.
- Branch cut connections force path independence in the integration of the wrapped phase gradient.
- If done properly, the integrated phase field will be correct. But which is correct?

0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.9	0.6	0.6	0.9	0.0	0.0
0.0	0.0	1.2	0.9	0.9	1.2	0.0	0.0
0.0	0.0	1.2	0.9	0.9	1.2	0.0	0.0
0.0	0.0	0.9	0.6	0.6	0.9	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0

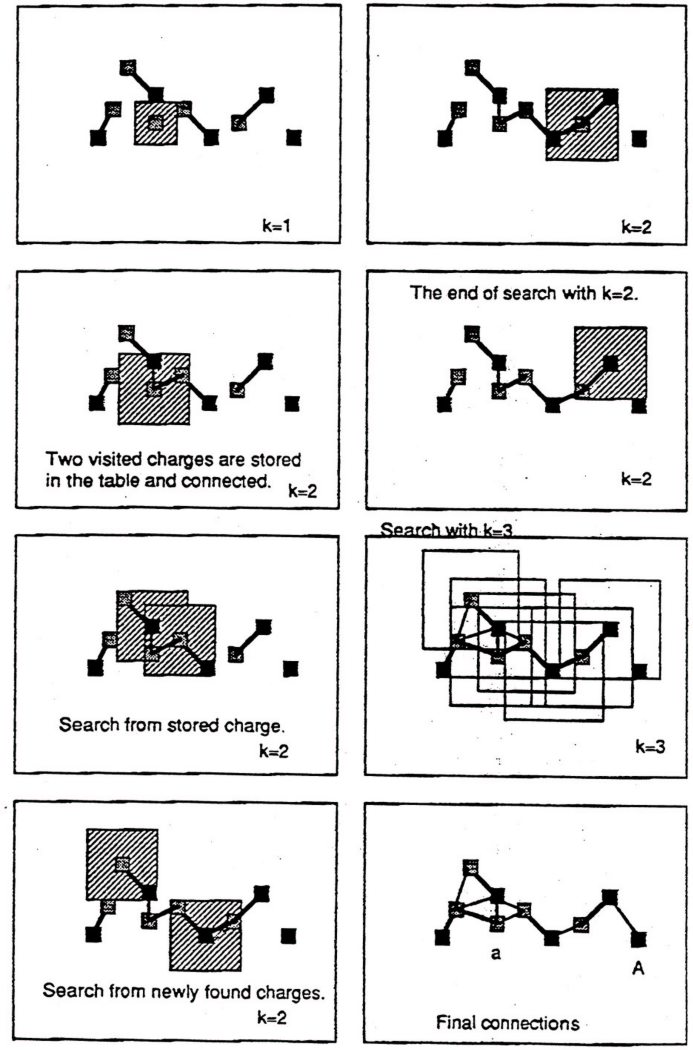
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	-0.1	-0.4	-0.4	-0.1	0.0	0.0
0.0	0.0	0.2	-0.1	-0.1	0.2	0.0	0.0
0.0	0.0	0.2	-0.1	-0.1	0.2	0.0	0.0
0.0	0.0	-0.1	-0.4	-0.4	-0.1	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0

Illustration of Branch Cut Algorithm

Connect all simple pairs

Expand search window and connect where possible

Initiate search from every residue encountered



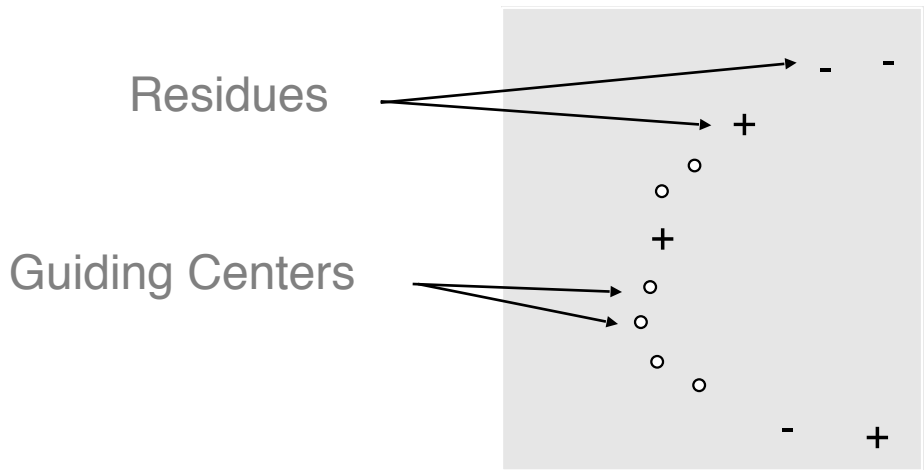
Could not connect last residue with this search window size

Repeat all steps with larger search window size

Branch Cut Strategies

- The standard GZW algorithm is designed to connect residues into a neutral network into the shortest possible connection tree, i.e. to minimize the length of the individual branch cuts comprising a tree
- This will not necessarily create the shortest possible tree, since GZW makes many unnecessary connections in its search for neutrality
- Various criteria have been devised to place guiding centers (unsigned residues) along expected paths to facilitate the right choice in branch cut connection

Phase-field of laid-over mountain

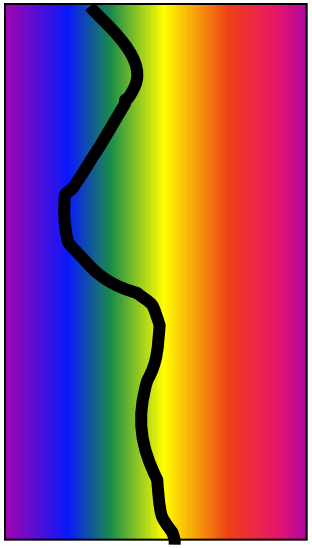


Guiding Center Criteria

- A number of criteria have been devised for selecting guiding centers, each more or less tailored to characteristics of SAR data:
  - when phase slope exceeds threshold (implies layover)
  - when derivative of phase slope exceeds threshold
  - when radar brightness exceeds threshold (implies layover)
  - when decorrelation estimator exceeds threshold (implies noise and/or layover
- Some guiding center selections help in some cases
- Difficult to assess performance in a quantitative way

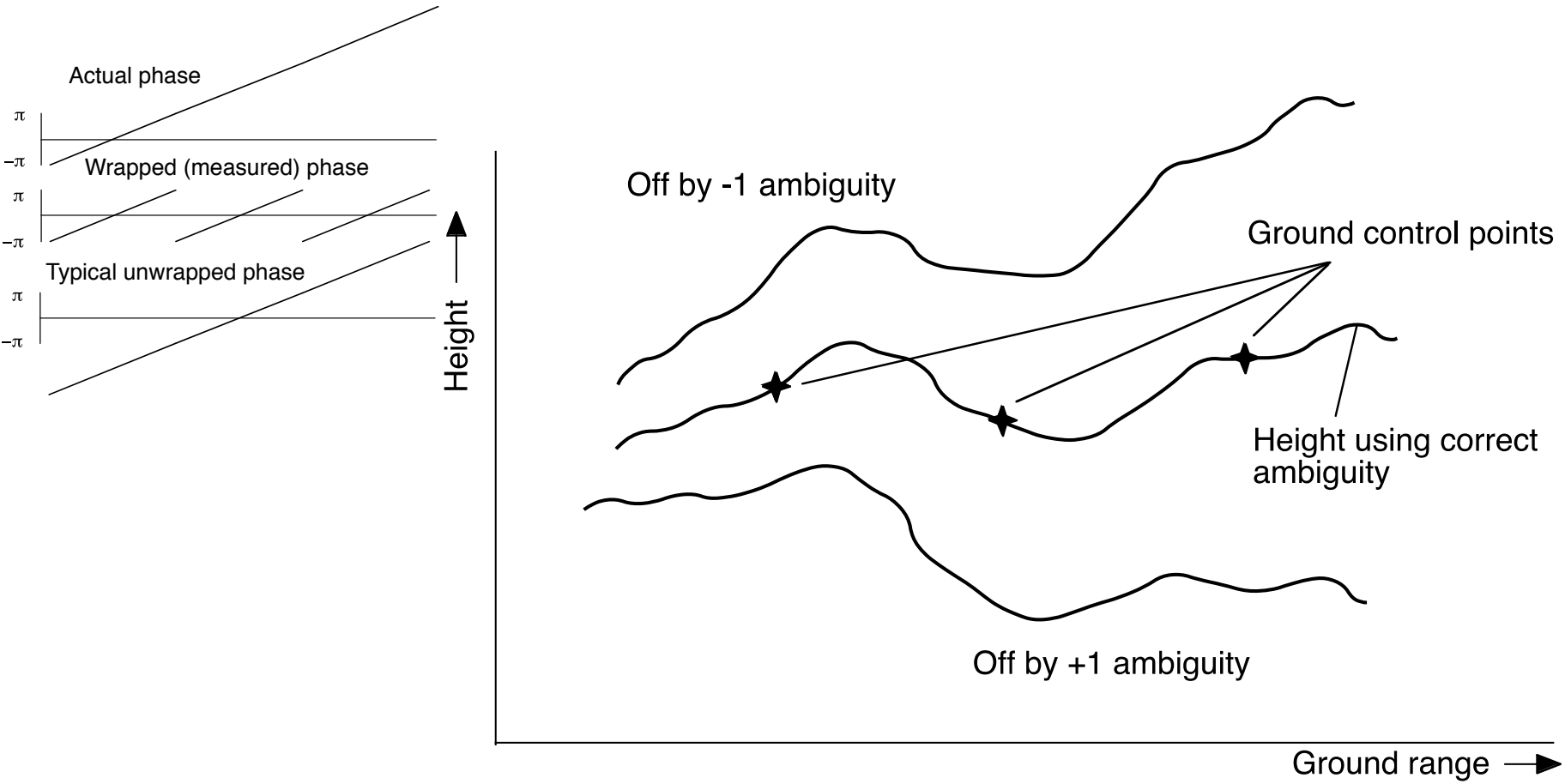
Connected Components

- Connected component* is a term borrowed from topology that denotes a set of points in a patch such that any two points in the set can be connected by a continuous path that does not cross a branch cut or a mask or patch boundary.
- Connected components arise during the unwrapping process when the trees or noise mask separate the patch into multiple regions.



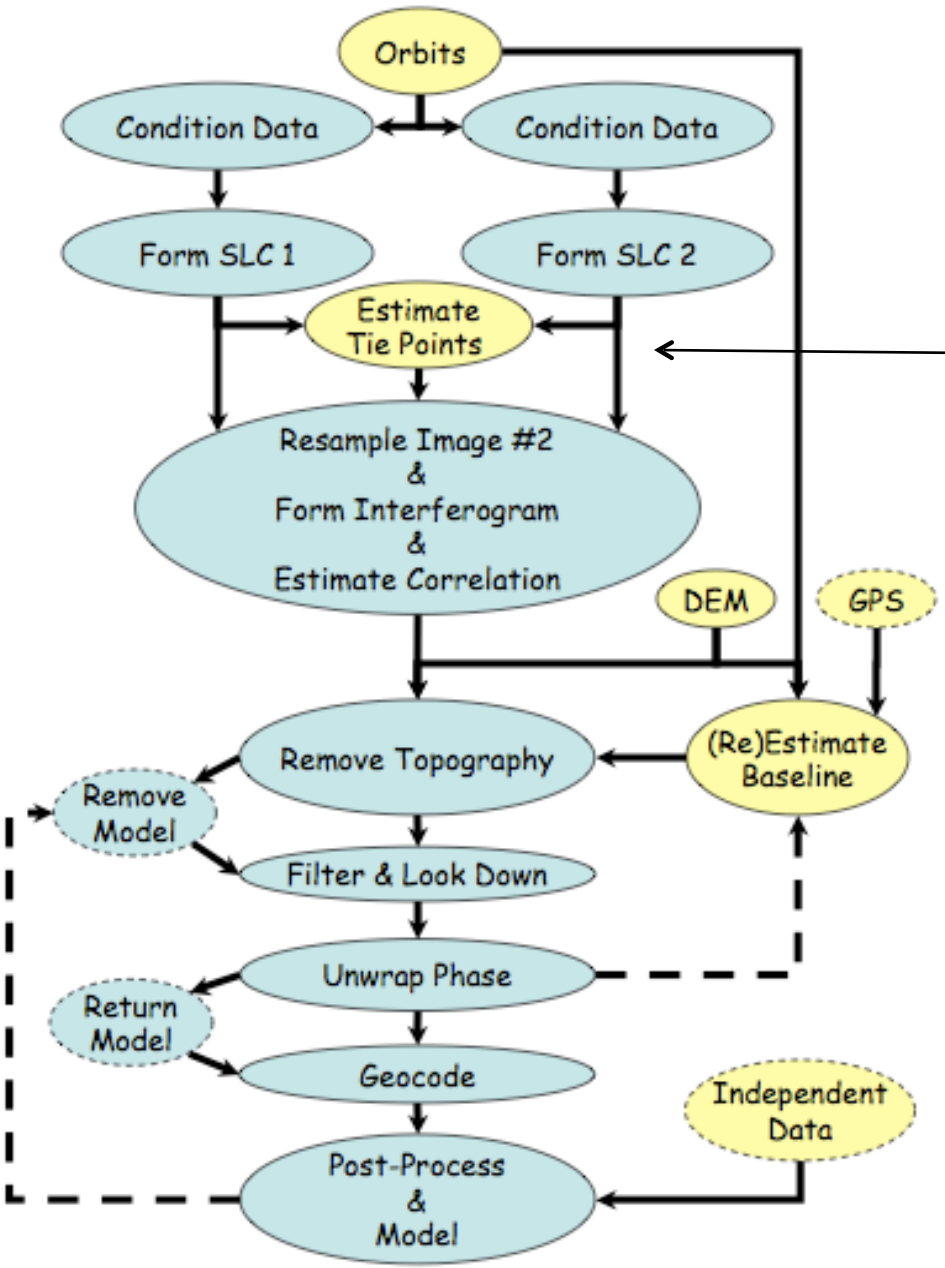
An example of this would be a river running down the middle of the patch with low correlation, hence noisy phase, in the river, effectively bisecting the phase into two distinct regions.

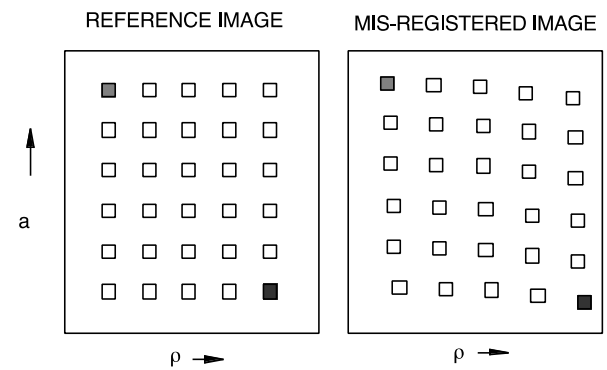
Absolute Phase Determination



Ground control reference points can be used to determine the absolute phase ambiguity

Basic Interferometric Flow





(ISCE does not have baseline re-estimation capability)!

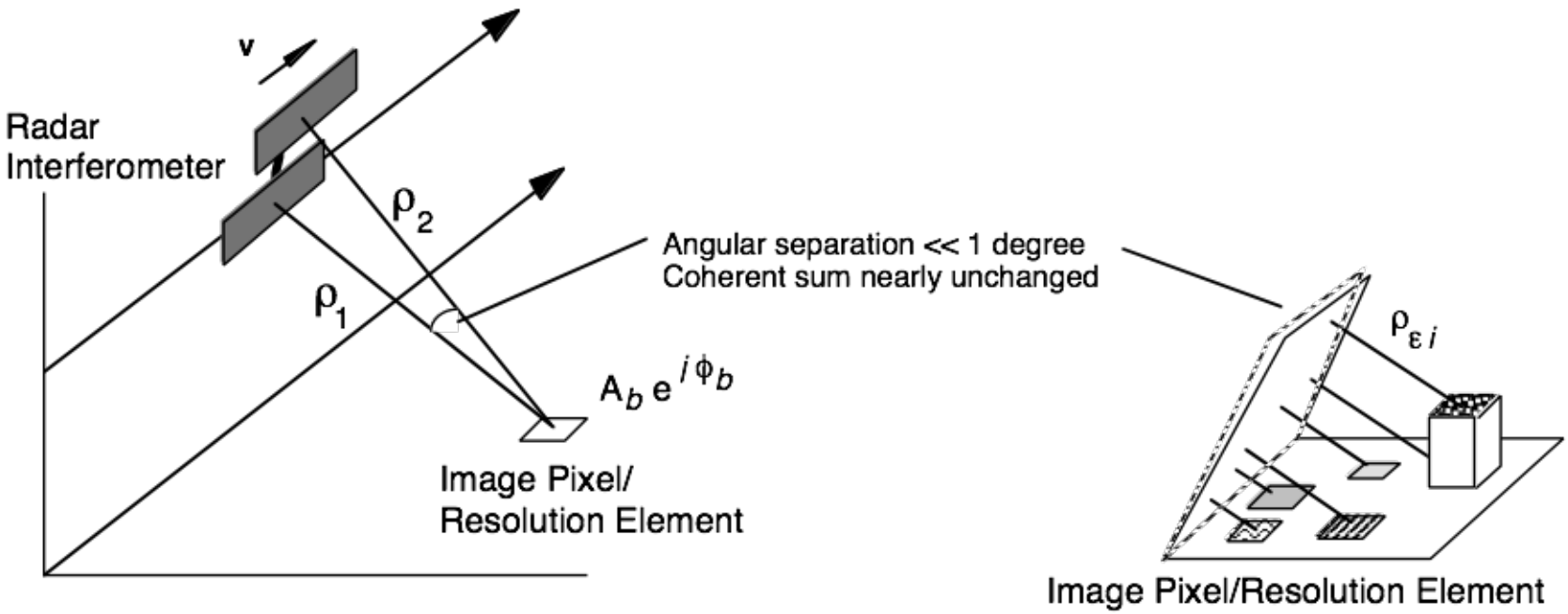
Interferometric Error Sources

- Interferometric Decorrelation
- Propagation delay errors from atmosphere and ionosphere
- Phase unwrapping errors
- Range and azimuth ambiguities due to design constraints

- Layover and shadow in radar imagery from slant range geometry
- Multiple scattering within and among resolution cells
- Range and Azimuth sidelobes due to bandwidth/resolution constraints
- Multipath and channel cross-talk noise as low-level interference
- Calibration errors

Interferometric Phase Characteristics

Pixels in two radar images observed from nearby vantage points have nearly the same complex phasor representation of the coherent backscatter from a resolution element on the ground but a different propagation phase delay



$$s_1 = A_b e^{j\phi_b} e^{-j\frac{4\pi}{\lambda}\rho_1} \qquad s_2 = A_b e^{j\phi_b} e^{-j\frac{4\pi}{\lambda}\rho_2}$$

Correlation* Theory

- Signals decorrelate due to
  - Thermal and Processor Noise
  - Differential Geometric and Volumetric Scattering
  - Rotation of Viewing Geometry
  - Random Motions Over Time
- Decorrelation relates to the phase standard deviation of the interferogram
  - Affects height and displacement accuracy
  - Affects ability to unwrap phase

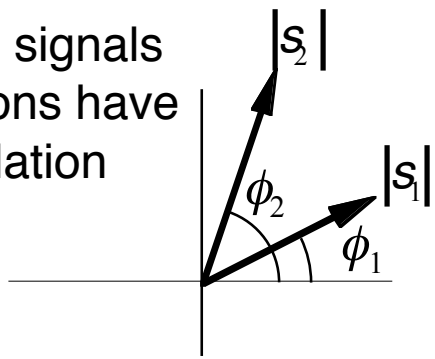
*"Correlation" and "Coherence" are often used synonymously

Correlation Definition

For signals s_1 and s_2 observed at interferometer apertures 1 and 2, the correlation γ is given by

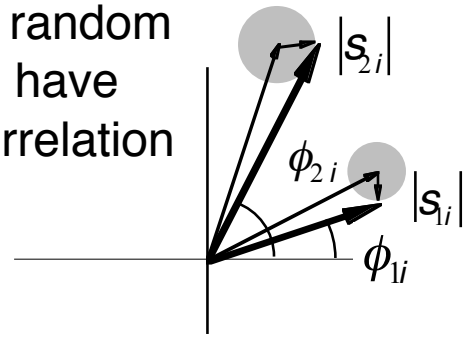
$$\gamma \equiv \frac{\left|\langle s_1 s_2^* \rangle\right|}{\sqrt{\langle s_1 s_1^* \rangle \langle s_2 s_2^* \rangle}}$$

Deterministic signals or combinations have perfect correlation



$$\gamma = \frac{\left|\,|s_1| e^{i\phi_1} |s_2| e^{-i\phi_2}\right|}{\sqrt{|s_1|^2 |s_2|^2}} = 1$$

Signals with random components have imperfect correlation



$$\gamma = \frac{\left|\left\langle |s_{1i}| e^{i\phi_{1i}} |s_{2i}| e^{-i\phi_{2i}} \right\rangle_i\right|}{\sqrt{\left\langle |s_{1i}|^2 \right\rangle_i \left\langle |s_{2i}|^2 \right\rangle_i}} \neq 1$$

Correlation Estimator

The standard estimator of the interferometric correlation between images forming an interferogram that have homogeneous backscatter and constant phase difference is

$$\hat{\gamma} = \frac{\left|\sum_{l,m}^{N,M} s_{1l,m} s_{2l,m}^{*}\right|}{\sqrt{\left|\sum_{l,m}^{N,M} s_{1l,m} s_{1l,m}^{*}\right| \left|\sum_{l,m}^{N,M} s_{2l,m} s_{2l,m}^{*}\right|}}$$

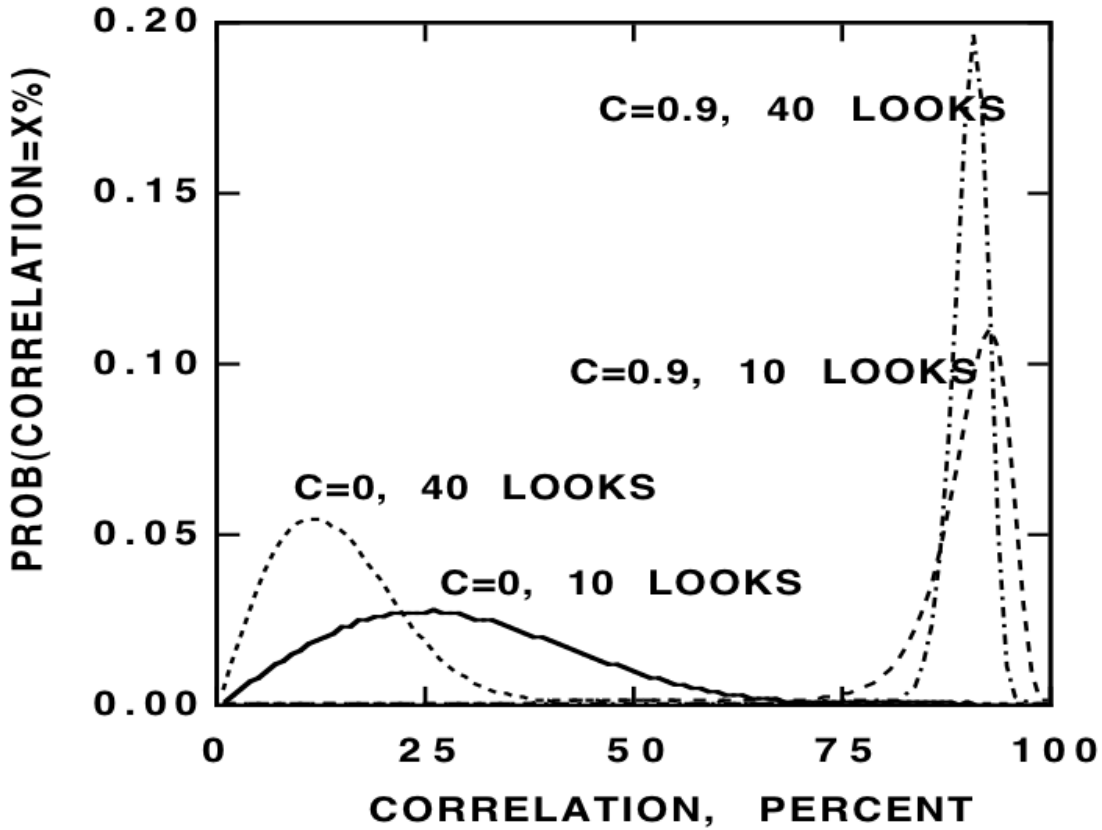
This estimator is biased. As an example, consider $N = 1$, $M = 1$. Then

$$\hat{\gamma} = \frac{\left|s_{1x,y} s_{2x,y}^{*}\right|}{\sqrt{\left|s_{1x,y} s_{1x,y}^{*}\right| \left|s_{2x,y} s_{2x,y}^{*}\right|}} = 1$$

where x and y are the coordinates of a particular image pixel. Other biases arise when backscatter homogeneity and phase constancy are violated.

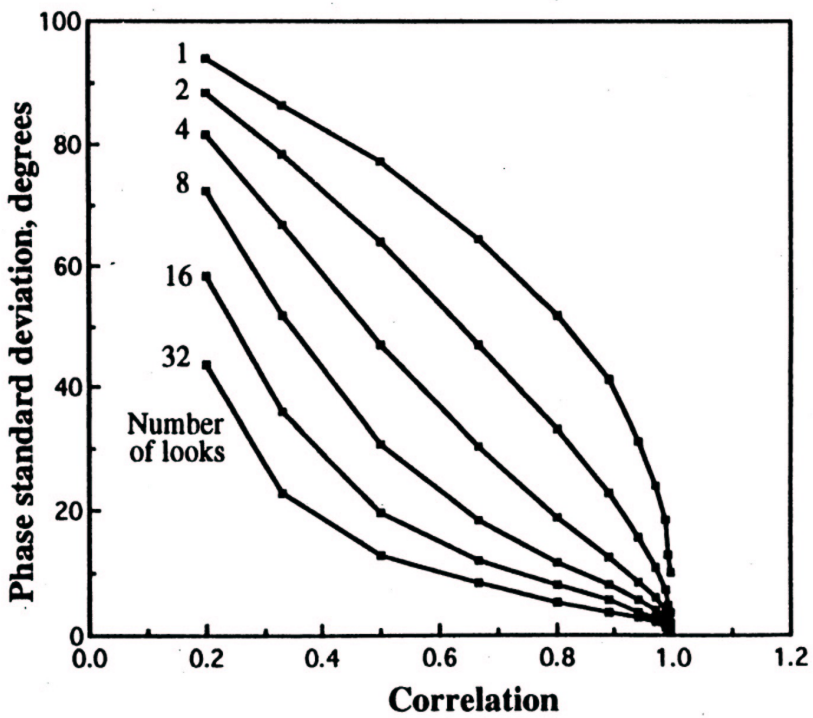
Correlation Estimator Characteristics

A general set of curves of reveals the nature of the estimator bias when backscatter is homogeneous and phase is constant.



Relationship of Phase Noise and Decorrelation

- Increased decorrelation is associated with an increase in the interferometric phase noise variance
- If averaging $N > 4$ samples, the phase standard deviation approaches the Cramer-Rao bound on the phase estimator:



$$\sigma_\phi = \frac{1}{\sqrt{2N}} \frac{\sqrt{1-\gamma^2}}{\gamma}$$

Phase noise contributes to height errors in interferometry as demonstrated in the sensitivity equations

Thermal Noise Decorrelation

Radar receiver electronics will add thermally generated noise to the image observations.

$$s_1' = A_b e^{j\phi_b} e^{-j\frac{4\pi}{\lambda}\rho_1} + n_1 \; ; \; s_2' = A_b e^{j\phi_b} e^{-j\frac{4\pi}{\lambda}\rho_2} + n_2$$

The added noise contributes randomly to the interferometric phase from pixel to pixel, causing thermal noise decorrelation. Assuming uncorrelated noise, the correlation between s_1' and s_2' is

$$\gamma = \frac{\left|\left\langle (s_1 + n_1)(s_2 + n_2)^* \right\rangle\right|}{\sqrt{\left\langle (s_1 + n_1)(s_1 + n_1)^* \right\rangle \left\langle (s_2 + n_2)(s_2 + n_2)^* \right\rangle}}$$

$$= \frac{\left|s_1 s_2\right|}{\sqrt{\left\langle (s_1^2 + n_1^2) \right\rangle \left\langle (s_2^2 + n_2^2) \right\rangle}}$$

$$= \frac{\sqrt{P_1}\sqrt{P_2}}{\sqrt{P_1 + N_1}\sqrt{P_{21} + N_2}} = \frac{1}{\sqrt{1 + N_1 / P_1}} \frac{1}{\sqrt{1 + N_2 / P_2}}$$

Thermal Noise Decorrelation

The correlation is related in a simple way to the reciprocal of the Signal-to-Noise Ratio (SNR). For observations with identical backscatter and equal noise power,

$$\gamma = \frac{1}{1 + N/P} = \frac{1}{1 + \mathrm{SNR}^{-1}}$$

Decorrelation is defined as

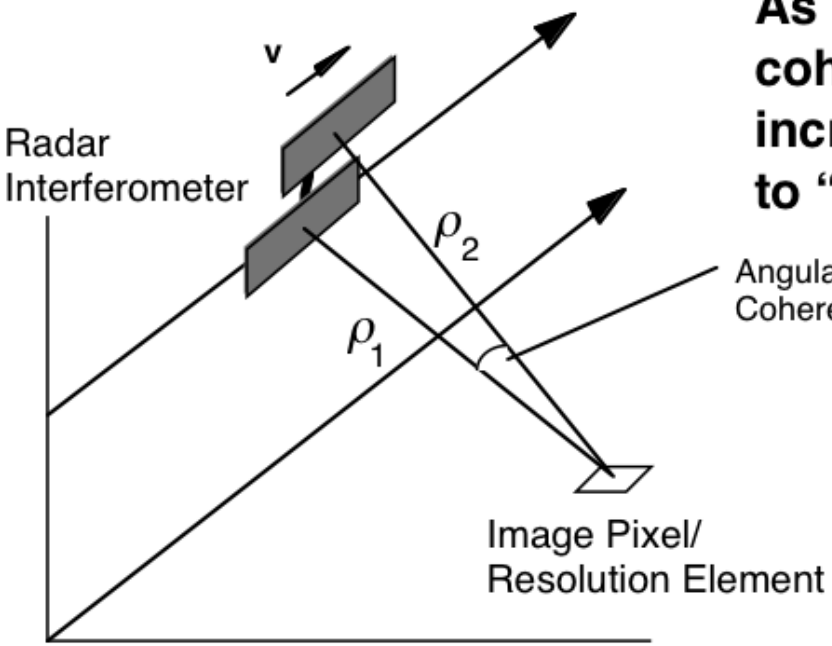
$$\delta = 1 - \gamma$$

The decorrelation due to thermal noise can vary greatly in a scene, not from thermal noise variations, but from variations in backscatter brightness. Extreme cases are:

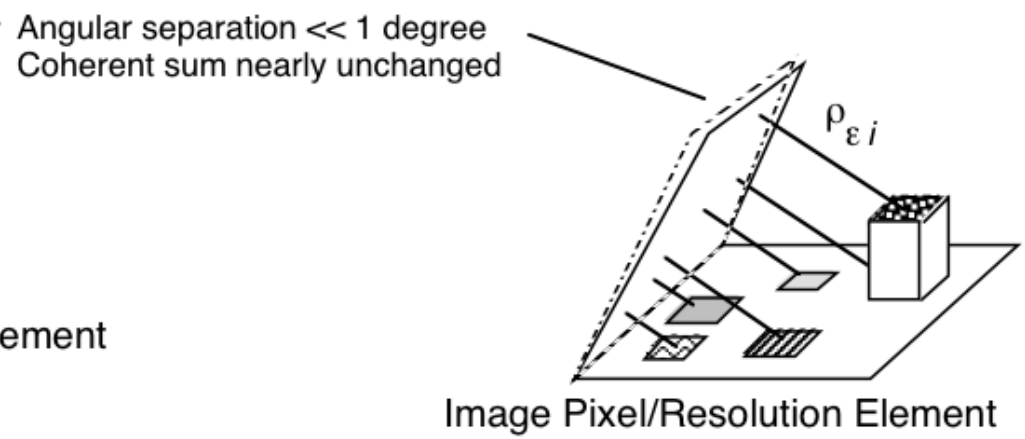
- **Radar shadow, where no signal is returns; correlation is zero.**
- **Bright specular target, where signal dominates return; correlation is 1**

Baseline Decorrelation

Pixels in two radar images observed from nearby vantage points have *nearly* the same complex phasor representation of the coherent backscatter from a resolution element on the ground



As interferometric baseline increases, the coherent backscatter phase becomes increasingly different randomly, leading to "baseline" or "speckle" decorrelation.

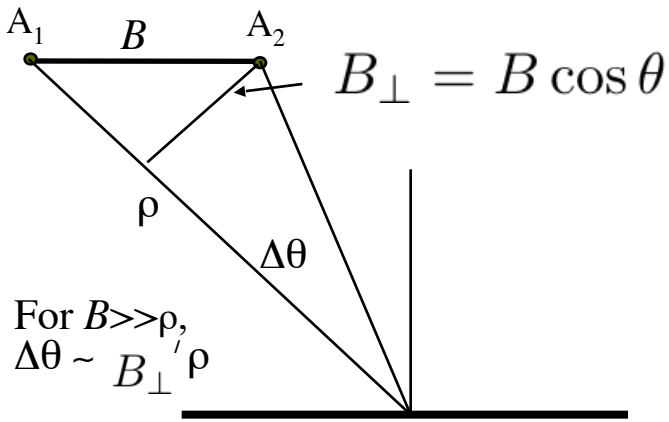


$$s_1 = A_{b1} e^{j\phi_{b1}} e^{-j\frac{4\pi}{\lambda}\rho_1} \qquad s_2 = A_{b2} e^{j\phi_{b2}} e^{-j\frac{4\pi}{\lambda}\rho_2}$$

Form of Baseline Correlation Function

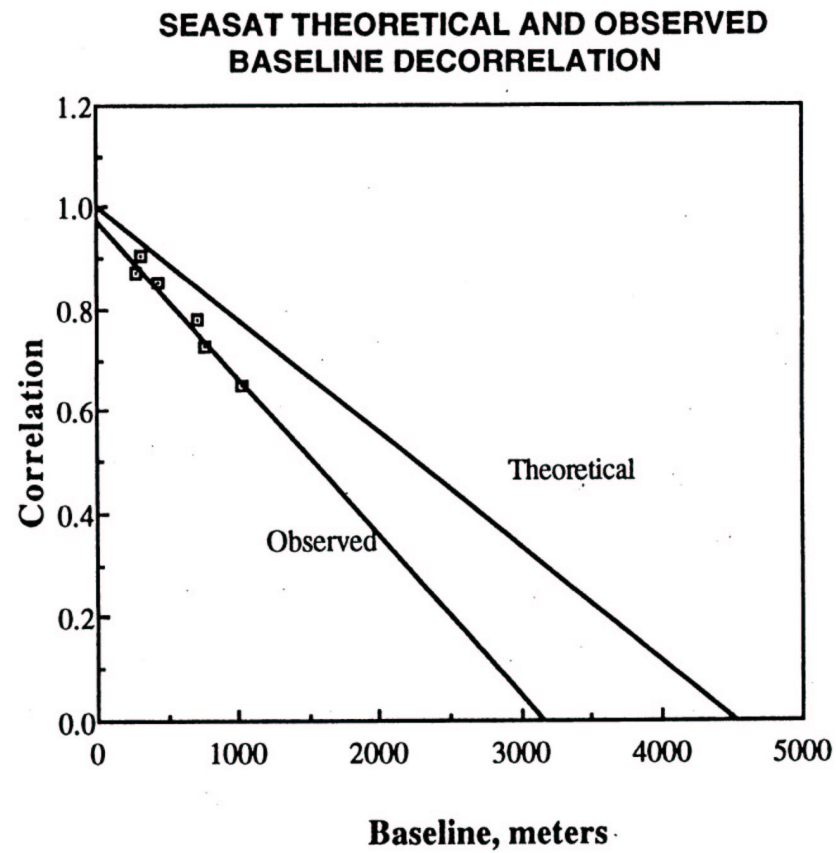
$$\gamma_B = 1 - \frac{2(B\cos\theta)(\Delta\rho_y \cos\theta)}{\lambda\rho}$$

$$\boxed{= 1 - \frac{2B_\perp \Delta\rho_\perp}{\lambda\rho}}$$



$$B_\perp = B\cos\theta$$

For B>>ρ,
$\Delta\theta \sim B_\perp{}'\rho$

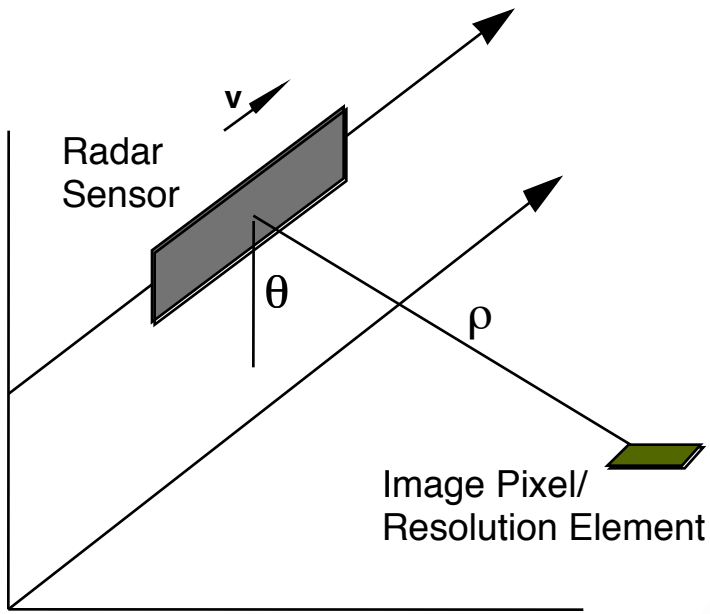


This function goes to zero at the *critical baseline*

$$\boxed{B_{\perp,\mathrm{crit}} = \frac{\lambda\rho}{n\Delta\rho_\perp}} \qquad n = 1, 2$$

Critical baseline depends on wavelength, range and range resolution.

The "Pixel Antenna" View of Baseline Decorrelation

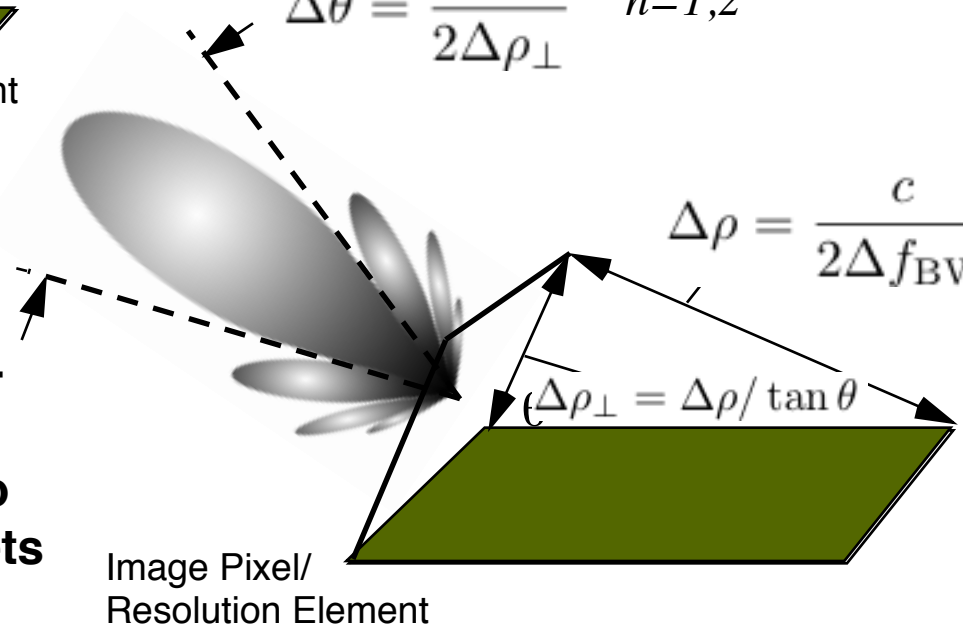


Each resolution element can be considered a radiating antenna with beamwidth of $\Delta\theta_{\Delta\rho_\perp}$, which depends on the range and local angle of incidence.

$$\Delta\theta = \frac{\lambda}{2\Delta\rho_\perp} \qquad n=1,2$$

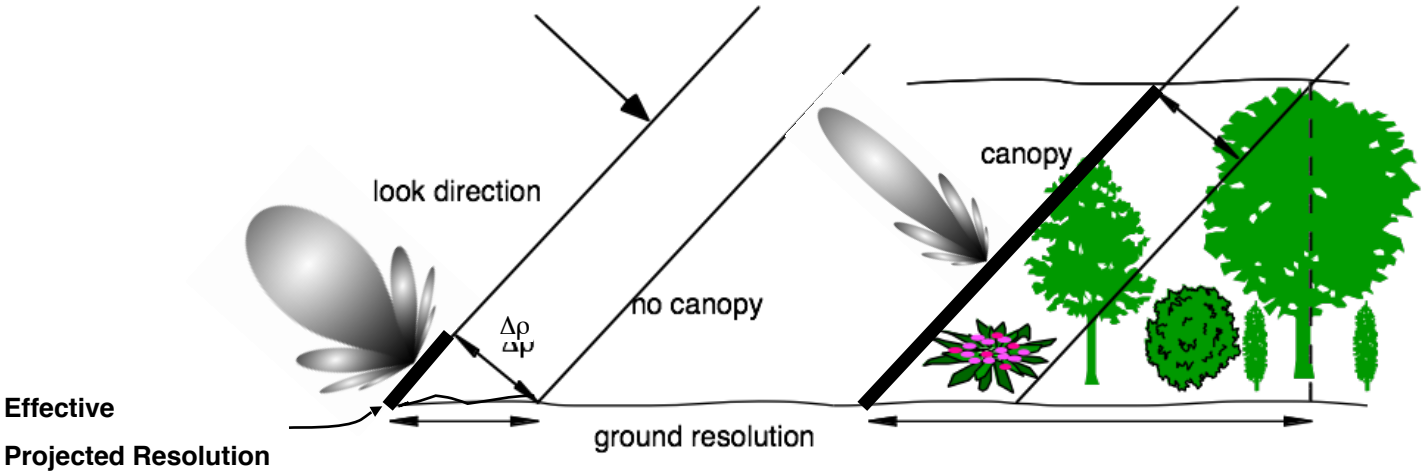
$$\Delta\rho = \frac{c}{2\Delta f_{\mathrm{BW}}}$$

$$\Delta\rho_\perp = \Delta\rho / \tan\theta$$

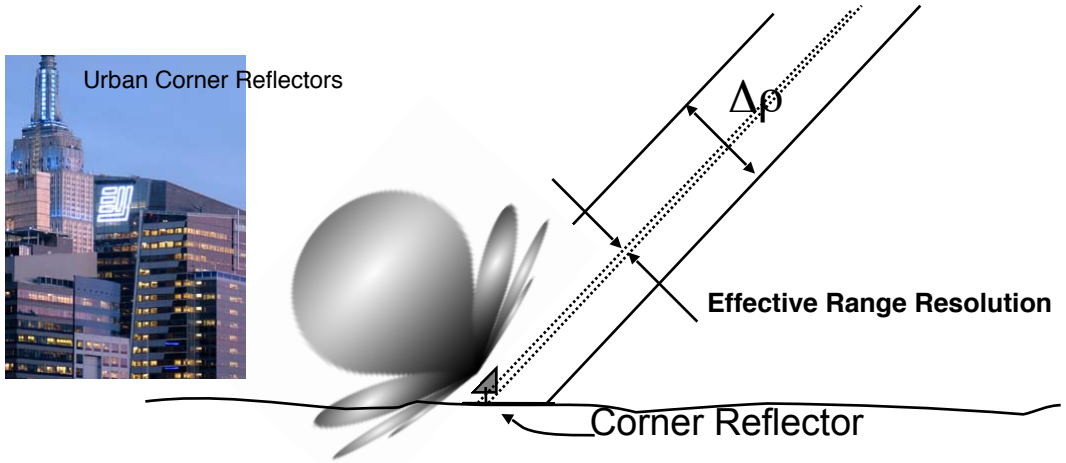


When the two apertures of the interferometer are within this beamwidth, coherence is maintained. Beyond this critical baseline separation, there is no coherence for distributed targets

Overcoming Baseline Decorrelation



Distributed targets correlate over a narrow range of baselines



JPL Corner Reflector

Pixels dominated by single scatterers generally behave though imaged at much finer resolution

Critical Baseline

The critical baseline is the aperture separation perpendicular to the look direction at which the interferometric correlation becomes zero.

$$B_{\text{crit}} = \rho\Delta\theta_{\Delta\rho_{\perp}} = \frac{\lambda\rho\tan\theta}{n\Delta\rho} \qquad n=1,2$$

Interferometers with longer wavelengths and finer resolution are less sensitive to baseline decorrelation. When the critical baseline is reached, the interferometric phase varies as

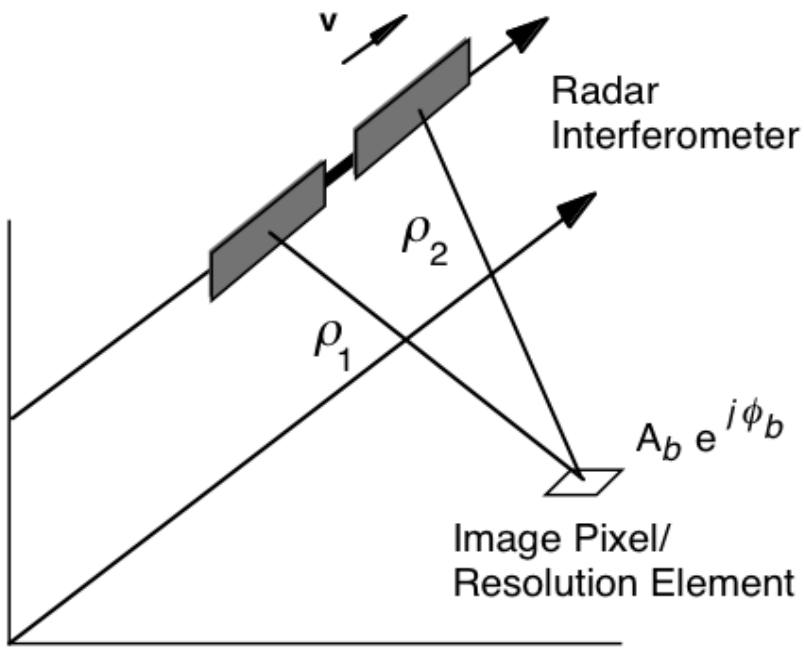
$$\frac{\partial\phi_{\text{crit}}}{\partial\rho} = \frac{\partial\phi_{\text{crit}}}{\partial\theta}\frac{\partial\theta}{\partial\rho}$$

$$= \frac{2n\pi}{\lambda}B_{\text{crit}}\frac{1}{\rho\tan\theta} = \frac{2\pi}{\Delta\rho}$$

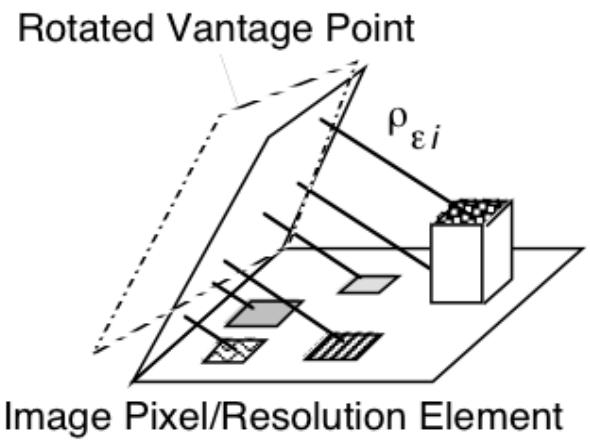
The relative phase of scatterers across a resolution element changes by a full cycle, leading to destructive coherent summation

Rotational Decorrelation

Rotation of scatterers in a resolution element can be thought of as observing from a slightly different azimuthal vantage point. As with baseline decorrelation, the change in differential path delay from individual scatterers to the reference plane produces rotational decorrelation.



The critical rotational baseline is the extent of the synthetic aperture used to achieve the along track resolution.



$$s_1 = A_{b1} e^{j\phi_{b1}} e^{-j\frac{4\pi}{\lambda}\rho_1} \qquad s_2 = A_{b2} e^{j\phi_{b2}} e^{-j\frac{4\pi}{\lambda}\rho_2}$$

Form of Rotational Correlation Function

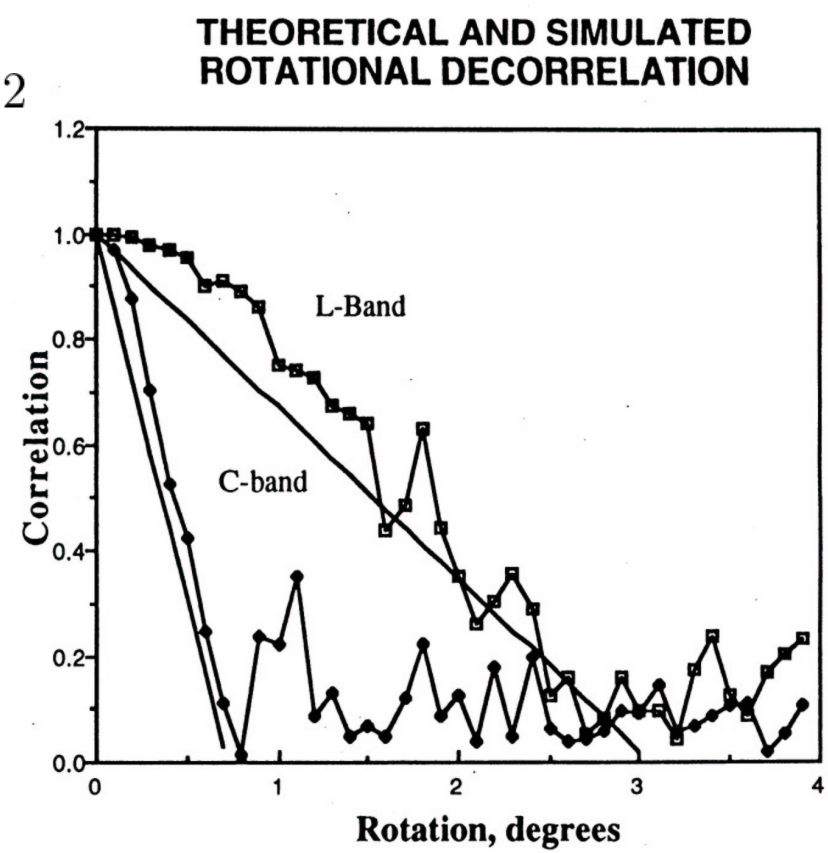
A similar Fourier Transform relation as found in the baseline decorrelation formulation exists

$$\gamma_\phi = 1 - \frac{n \sin\theta B_\phi R_x}{\lambda\rho} \quad n = 1, 2$$

where R_x is the azimuth resolution and B_ϕ is the distance along track corresponding to the rotation angle of the look vector

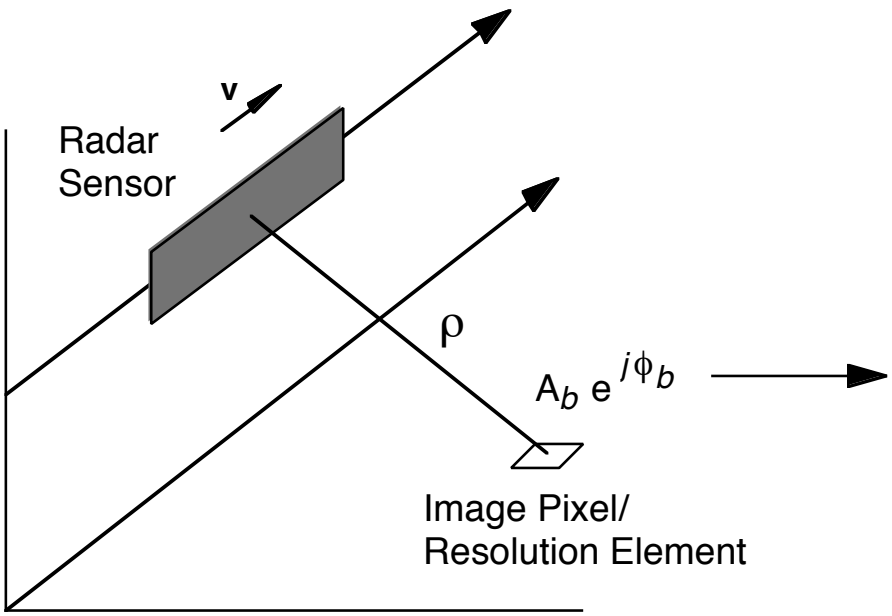
This function goes to zero at the *critical rotational baseline*

$$B_{\phi,\mathrm{crit}} = \frac{\lambda\rho}{n\Delta\rho_\phi}, \Delta\rho_\phi \equiv R_x \sin\theta$$

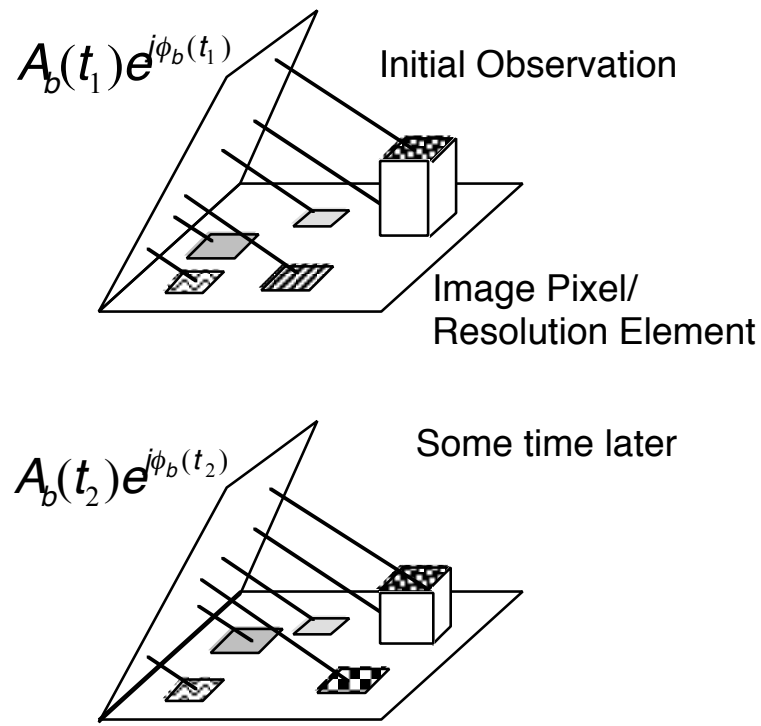


Scatterer Motion

Motion of scatterers within the resolution cell from one observation to the next will lead to randomly different coherent backscatter phase from one image to another, i.e. "temporal" decorrelation.



Propagation Phase Delay $\phi = -j\frac{4\pi}{\lambda}\rho$



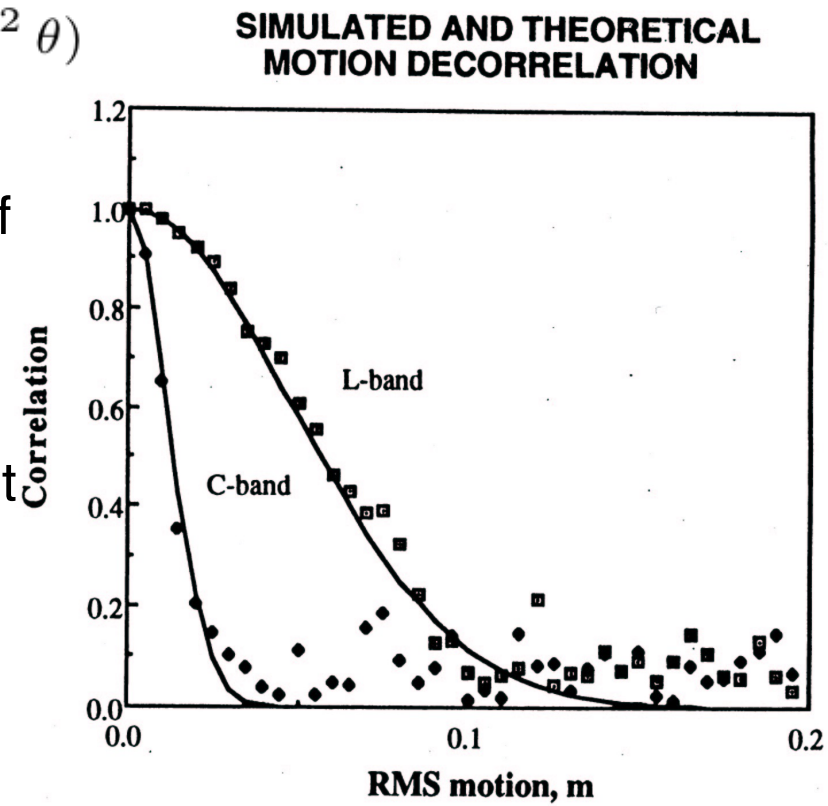
Form of Motion Correlation Function

The Fourier Transform relation can be evaluated if Gaussian probability distributions for the motions are assumed

$$\gamma_t = e^{-\frac{1}{2}\left(\frac{4\pi}{\lambda}\right)^2(\sigma_y^2 \sin^2\theta + \sigma_z^2 \cos^2\theta)}$$

where $\sigma_{y,z}$ is the standard deviation of the scatterer displacements cross-track and vertically

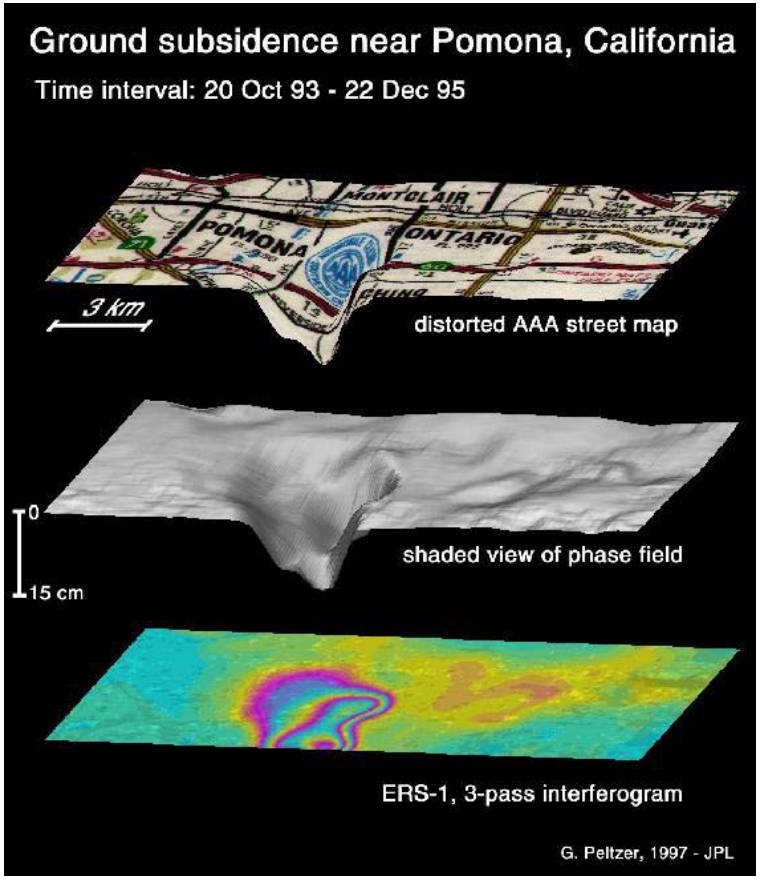
Note correlation goes to 50% at about 1/4 wavelength displacements



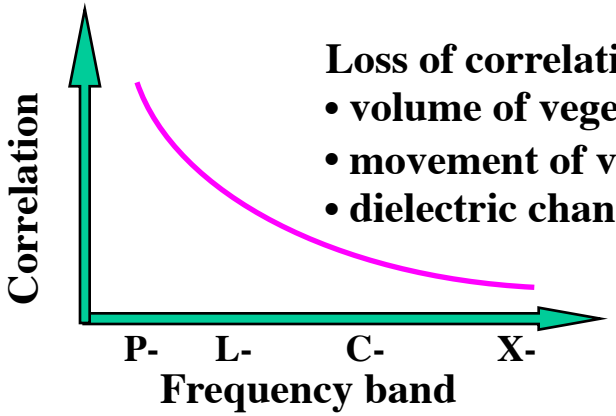
Repeat-pass Interferometry and Coherent Change Detection

L-Band low frequency improves correlation in vegetated areas

Most radars do well in areas of sparse vegetation

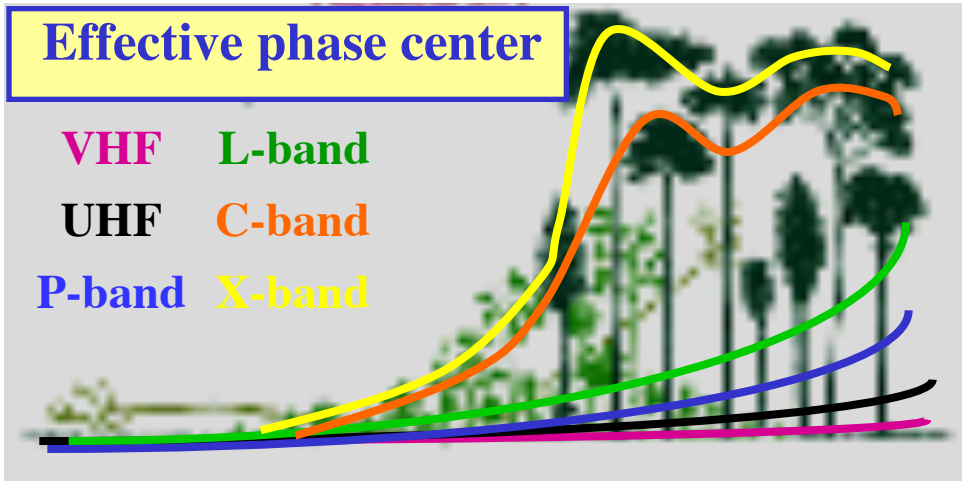


But maintaining correlation in dense vegetation requires longer wavelengths



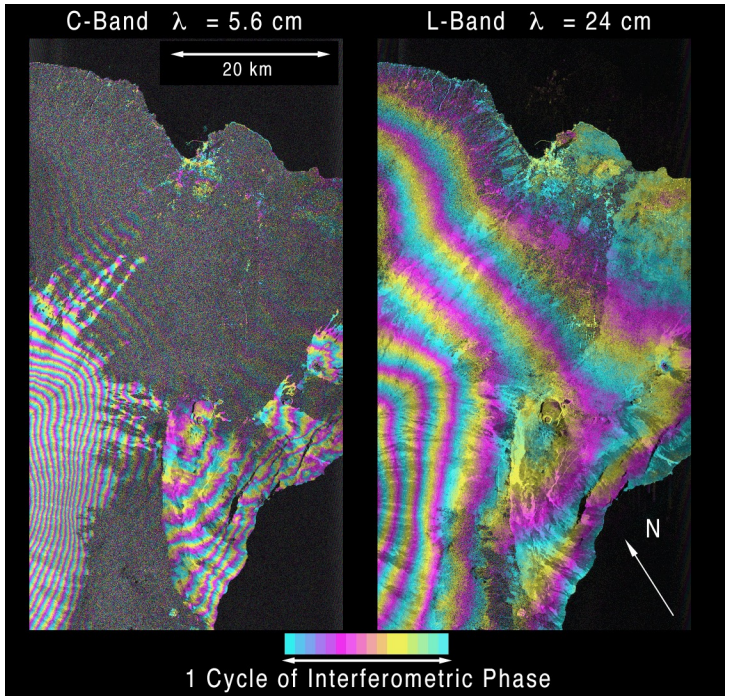
Loss of correlation is due to:

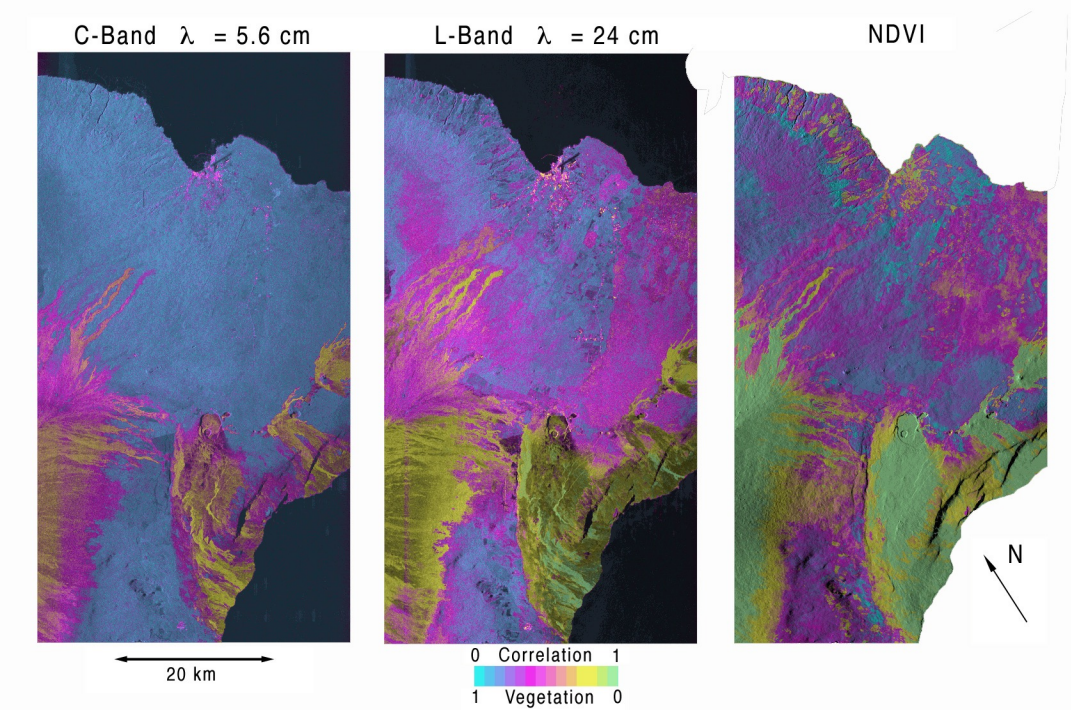
- volume of vegetation
- movement of vegetation
- dielectric change (moisture)



Coherent Change Detection
SIR-C L and C-band Interferometry

6 month time separated observations to form interferograms
Simultaneous C and L band

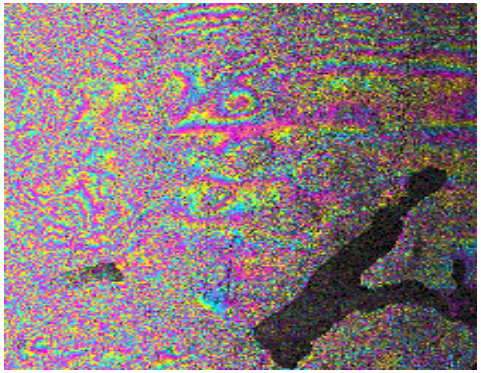




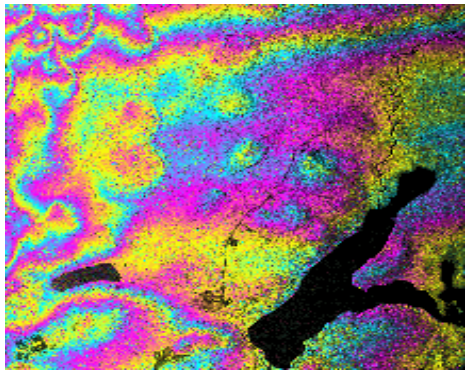
InSAR experiments have shown good correlation at L-band

AIRSAR Coherent Change Detection

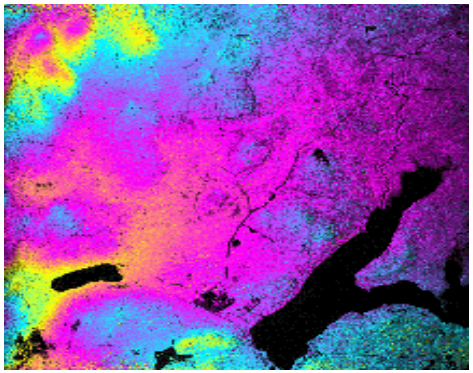
Interferograms



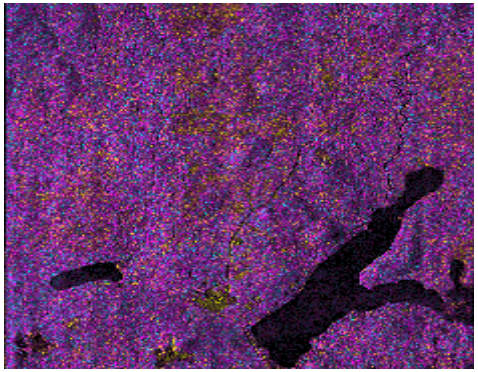
C-band

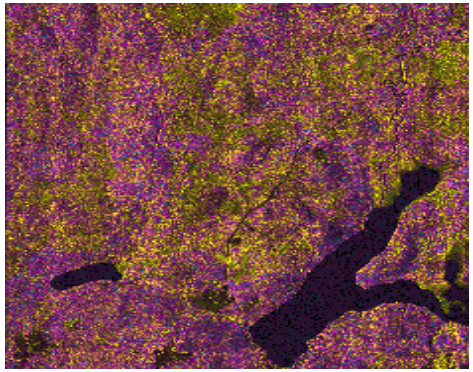


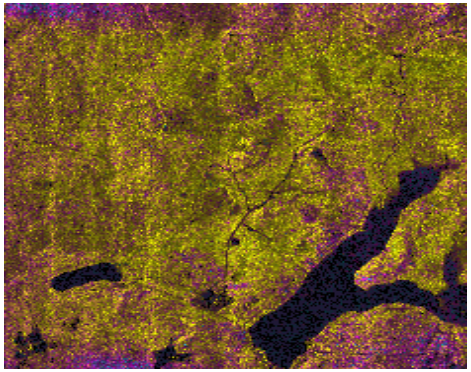
L-band



P-band





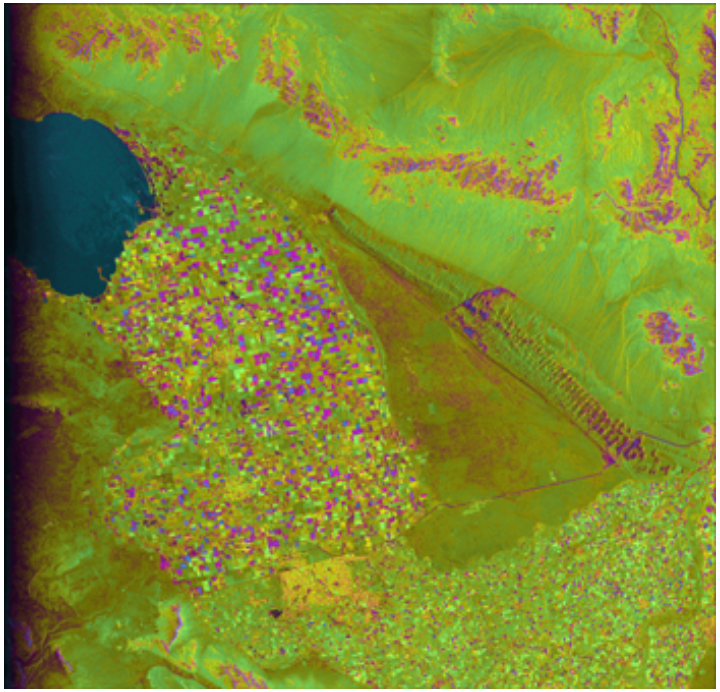


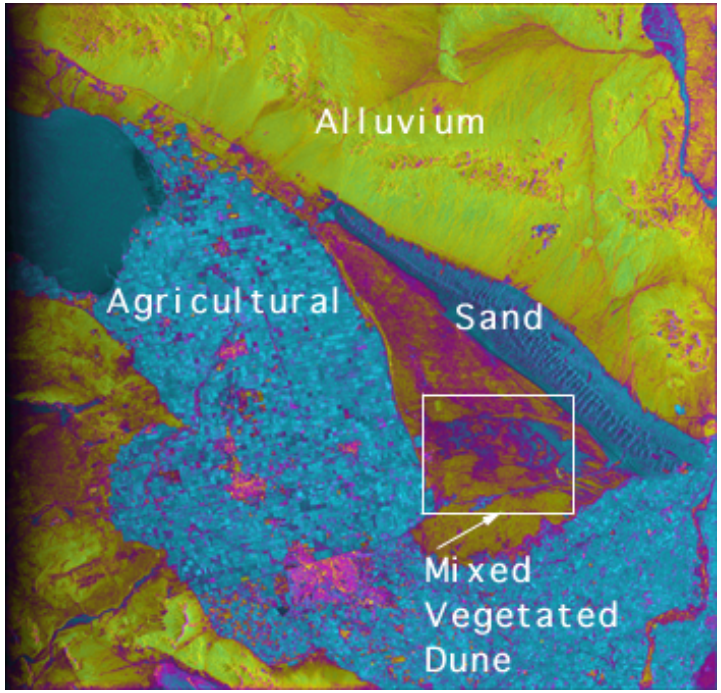
Correlation

Airborne InSAR experiments have shown good correlation at L-band

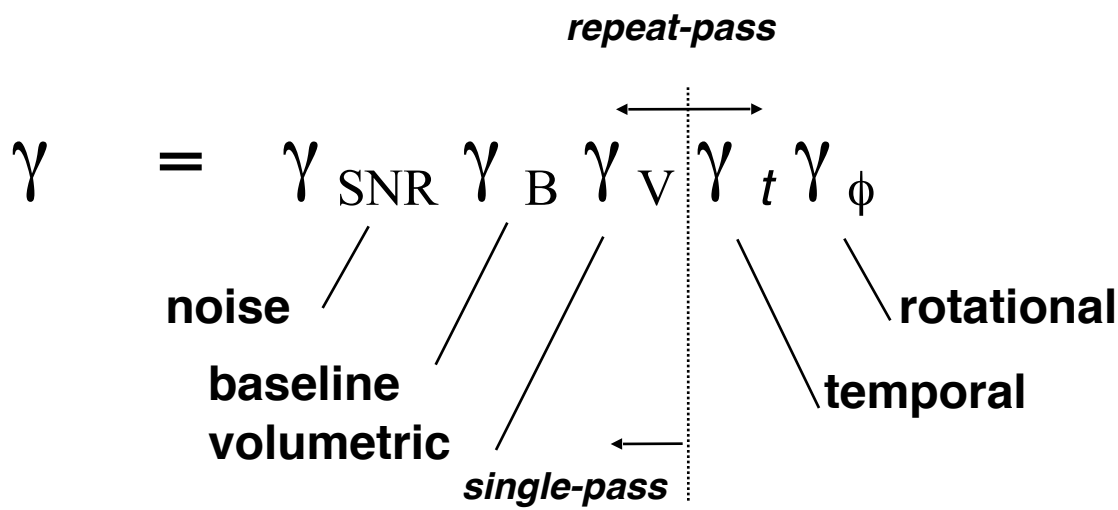
A Correlation Test:

What were the interferometric observation conditions?





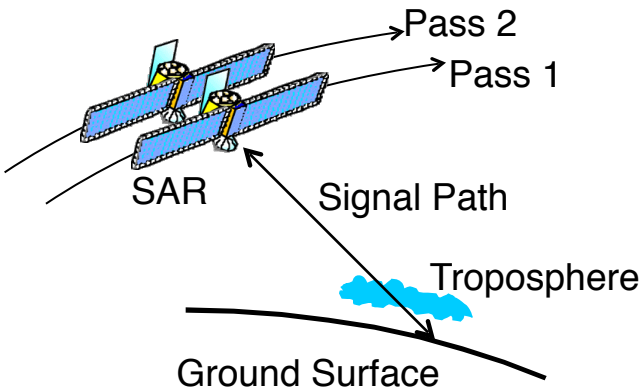
Correlation Discrimination



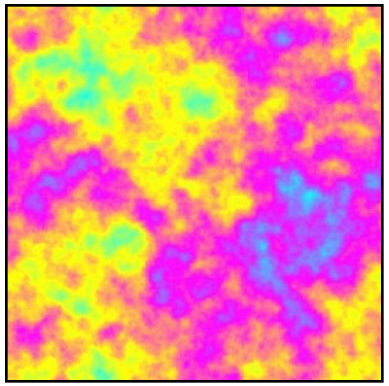
- For a calibrated interferometer, γ_{SNR} can be calculated for each pixel.
- For single-pass systems, $\gamma_t = 1$ and γ_ϕ is usually nearly unity.
  - The baseline decorrelation can be computed from the known geometry, leaving the volumetric correlation, which can used as a diagnostic of surface scatterers.

- For calibrated repeat-pass systems with no rotational decorrelation, the product $\gamma_V \gamma_t$ is separable only if $\gamma_t \neq 0$ and γ_V estimates from several baselines are compared.

Tropospheric Effects on InSAR

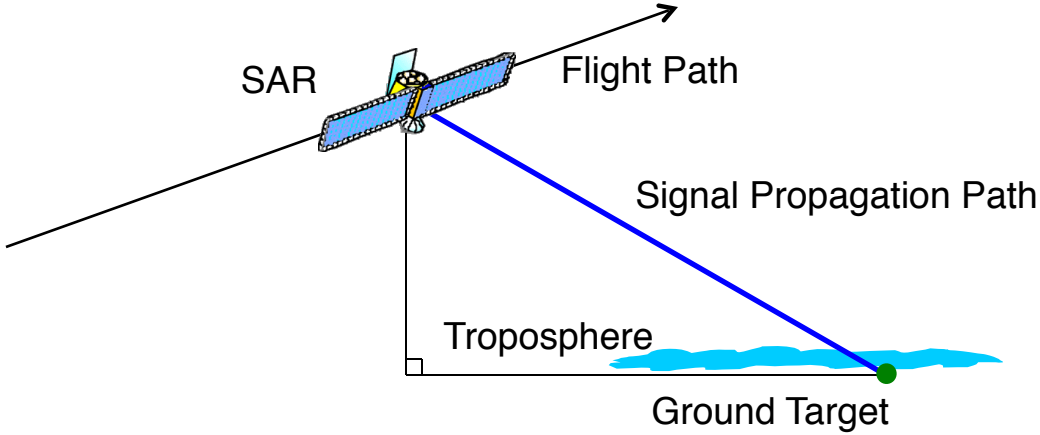


Simulated phase artifacts due to differential tropospheric delay between two passes



- Tropospheric path delays cause artifacts in repeat-pass interferometric synthetic aperture radar (InSAR) measurements of surface displacement
  - Rapidly varying tropospheric delays (both spatially and temporally) are most problematic
  - Such variations are primarily due to changes in water vapor content along signal propagation path

Effects on InSAR Phase

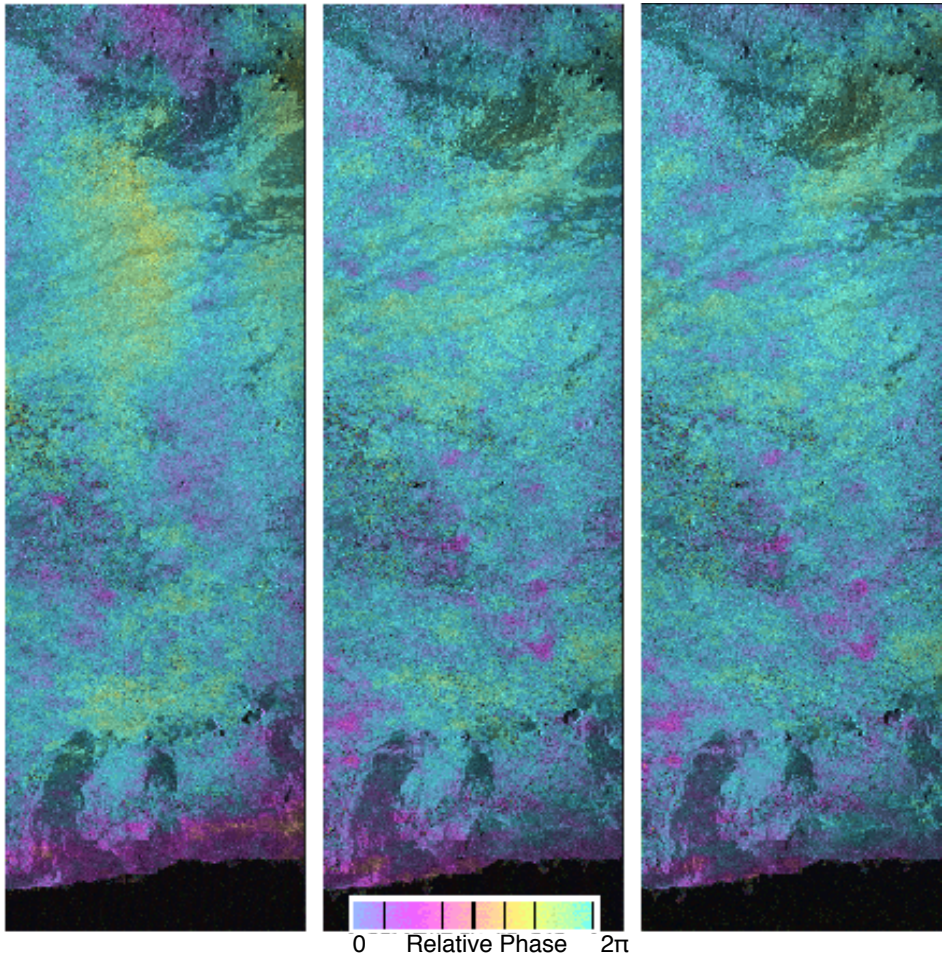


- Observed InSAR phase for broadside acquisition:

$$\phi_{\text{obs}} = -\frac{4\pi}{\lambda}\left(\delta_{LOS} + \delta_{atm}\right) + \phi_{\text{noise}}$$

- Have two unknown parameters (δ_{atm}, δ_{LOS}), but only one equation
- Tropospheric delays δ_{atm} are indistinguishable from line-of-sight (LOS) surface displacements δ_{LOS}

Hawaii SIR-C One Day Repeat Pass Data Showing Tropospheric Distortion



DAYS 7-8 TWO-PASS

DAYS 7-10 TWO-PASS

DAYS 7-8-10 THREE-PASS

Other Resources

M. Simons and P. Rosen,
Interferometric Synthetic Aperture Radar Geodesy
Treatise on Geophysics, Schubert, G. (ed.), Volume 3- Geodesy, Elsevier Press, pp. 391-446, 2007
http://www.gps.caltech.edu/~simons/pdfs/Simons_Treatise.pdf

Rosen, P. A.; Hensley, S.; Joughin, I. R.; Li, F. K.; Madsen, S. N.; Rodriquez, E. & Goldstein, R. M.
Synthetic Aperture Radar Interferometry
Proc. IEEE, 2000***, 88*, 333-382**

Burgmann, R.; Rosen, P. & Fielding, E.
Synthetic Aperture Radar Interferometry to measure Earth's surface topography and its deformation
Ann.~Rev.~Earth Planet.~Sci., 2000***, 28*, 169-209**

Madsen, S. N. & Zebker, H. A.
Synthetic Aperture Radar Interferometry: Principles and Applications
Manual of Remote Sensing, Artech House, **1999***, 3*

Massonnet, D. & Feigl, K. L.
Radar interferometry and its application to changes in the earth's surface
Reviews of Geophysics, **1998***, 36*, 441-500

Bamler, R. & Hartl, P.
Synthetic Aperture Radar Interferometry
Inverse Problems, 1998***, 14*, 1-54**