Treatment of Scale in GAGE and by Other GPS Data Processing Groups

Prepared by Dr. T. Herring, MIT, GAGE Analysis Center Coordinator.

The estimation of scale differences between geodetic networks dates back to triangulation networks where the distances in the networks were all dependent on just a few short “baseline length” measurements. Trilateration, made possible by the use electronic distance measurements (EDM), does not suffer from scale uncertainties to the same extent as triangulation, which involves measurements of angles only except for the baseline length measurements. The practice of estimating scale differences is still commonly used although modern geodetic position measurements are based on light travel times, which would suggest that scale differences between networks should not be an issue (assuming everyone uses the same speed of light and GM for satellite orbits). In practice, the situation is more complex because scale differences can arise from relativistic models; biases in range measurements; atmospheric delay models and, for GPS, phase center models used in the data processing. Once the GPS phase center models are fixed in the analysis, GPS has no intrinsic scale uncertainty and hence scale estimates are not needed although commonly included by various groups in their processing. This note discusses the subtle effects of the treatment of scale changes in the analyses performed by GAGE and other groups. To avoid potential absorption of height signals into the scale estimates, the GAGE analyses do not include estimates of scale changes when aligning solutions to a reference frame. Because the Earth is close to spherical, scale changes act in a radial direction and have little impact on latitude and longitude (horizontal positions) estimates.

On a sphere, a change in the radius of the sphere scales all the features on the surface of the sphere by the ratio the radius change to the radius of the sphere. A change in radius is the same as changing the heights of all the points on the surface of the sphere by the change in radius. For a spherical body there is a direct correspondence between uniform changes in height and the scale of features on the surface. Since the Earth is close to spherical, flattening of ~1/300, uniform small changes in height appear as scale changes for a network of sites and thus scale changes in a network of sites on the Earth’s surface correspond to uniform height changes of all the sites in the network. The ratio the radii between the highest and lowest points on the surface of the Earth is < 0.4% and thus a scale change effectively simply raises or lowers all points in the network by the scale change multiplied by the mean radius of the Earth. The cord distances between the points in the network would change by the cord distance multiplied by the scale change. The scale change to height change conversion is proportional to the mean radius of the Earth (6371 km).

GPS processing of large networks of sites can be done in two different ways: (1) A full network processing using double differences or explicit clock estimation to handle the satellite and station clock errors or (2) a precise point positioning (PPP) method in which the satellite clock errors and accurate satellite orbits are assumed known and single station data can be processed by themselves. For the network processing case, analyses apply loose constraints (typically ±1-10 meters) to the coordinates of all sites in the network and then rotate, translate, and possibly scale the coordinates in the network to align the frame to the coordinates of a set of reference frame defining sites. In the GAGE network processing by NMT, the IGS orbits are fixed and clocks are removed by double differencing. The large GAGE network is divided into a set of subnets that can be processed in parallel and then merged into a single large network. The precise point positioning (PPP) approach uses satellite orbits and clocks determined by global network analysis (JPL analysis in the case of GAGE) and the coordinates of individual sites are computed individually with the satellite orbits and clocks fixed. CWU analyses use the PPP approach. All sites are merged into a single network. In the PPP approach, there are no site-to-site correlations in the merged covariance matrix. The network of site coordinates can then also be rotated, translated, and possibly scaled to align to the coordinates of a set of reference sites. The network rotation and translation are done for each daily solution separately for the NMT and CWU analyses.

There are also multiple ways for the rotation/translation to be estimated and applied. In the GAGE analyses, we add to the covariance matrices supplied by the analysis centers in SINEX files, a covariance matrix, generated by standard propagation of variance-covariance matrix methods, that allows network translation and rotation with standard deviations of 1 meter and 30 milliarcseconds (~1 m at Earth's surface). These values are large compared the intrinsic uncertainty with which these parameters can be estimated. Adding these covariance matrices allows the GAGE networks to translate and rotate and when the reference is realized, the uncertainty in the final determination of the translation and rotation is reflected in the site coordinate standard deviations. For the large GAGE network, which has a large spatial coverage of its reference frame sites, the frame uncertainty contribution is very small. Adding the translational covariance allows the network to translate freely, which is not the case when orbits are fixed. In the GAGE analyses, only translation and rotation are used so as to avoid potentially absorbing height changes into the scale estimates and thus suppressing possible signals in the GAGE time series. The height coordinates are down weighted relative to the horizontal coordinates by a variance factor of 1000 to minimize the height deviations mapping to horizontal coordinates. Users can remove a scale change by computing the mean of the height differences between the daily height estimates and the heights of a set of reference frame sites. This mean of the height differences is then subtracted from the heights of all the sites in the network. We discuss this approach in more detail later.

The most recent global reference frame is ITRF2008, which represents the motions of sites as linear velocities with offsets as needed for equipment changes and earthquakes. The IGS subset of sites from ITRF2008 that are used to define the IGS global reference frame consists of 235 unique sites (492 individual site segments) and the frame is defined by 84 locations with an hierarchical list of sites at each location. The current IGS frame is IGb08 and differs from the original ITRF2008 frame with the addition of some post-2008 discontinuities at the core sites and the modification of the coordinates of some sites due to a change in the phase center model of the antenna used at these sites. For the GAGE analyses, the primary reference frame, NAM08, is derived from IGb08 rotated into a North America fixed frame based on the ITRF2008 North America Euler pole. The GAGE reference frame is densified by including a hierarchical list of ~1500 GAGE sites located at 634 locations (Figure 1). The GAGE reference frame includes linear motions, offsets due to equipment changes and earthquake and, for some earthquakes, logarithmic parameterizations of the postseismic deformation.

In the GAGE analyses, scale changes are not estimated but most other groups processing data from the sites included in GAGE routinely remove scale differences while aligning to their chosen reference frame. For comparison here to see the impact of estimating scale changes we will compare GAGE analyses to the University of Nevada, Reno (UNR) analyses in the UNR North America fixed frame called NA12 and in the IGb08 frame. The IGb08 analyses use a global network of sites while the NA12 analyses use sites only in North America which is similar to the GAGE NAM08 analysis although the reference frame sites used by GAGE span eastern Alaska, Hawaii, the Caribbean, Canada and Greenland. Figure 1 shows examples of the GAGE reference frame sites and the GAGE network for two days in January 2011 and 2015.

To illustrate the impact of estimating scale we examine two sites. One of these sites, P113, is located in Utah and the other, P144, is located in Northern California. The differences in the height estimates between the GAGE and UNR analyses for both of these sites show temporal variations that are common to a large number of sites spread over large regions. The differences between the analyses are largely due to the differences between analyses in the treatment scale changes. The time series of for P113 are shown in Figure 2 for four different analyses. The top two frames show the standard GAGE processing in the NAM08 frame with no scale changes estimated (top) and the UNR NA12 frame processing with scale changes estimated (second from top). There is clearly a much stronger annual signal in the GAGE results compared to UNR results. Visually, the GAGE solution (a), which does not have scale changes estimated, has a downward dip in 2011 and the annual amplitude seems reduced after 2011 for maybe 3 years. In contrast, the UNR NA12 results (b), which estimates scale changes to align to the NA12 reference frame, appear flat with little signal. The difference between the two analyses is due primarily to UNR estimating scale changes in their NA12 realization.

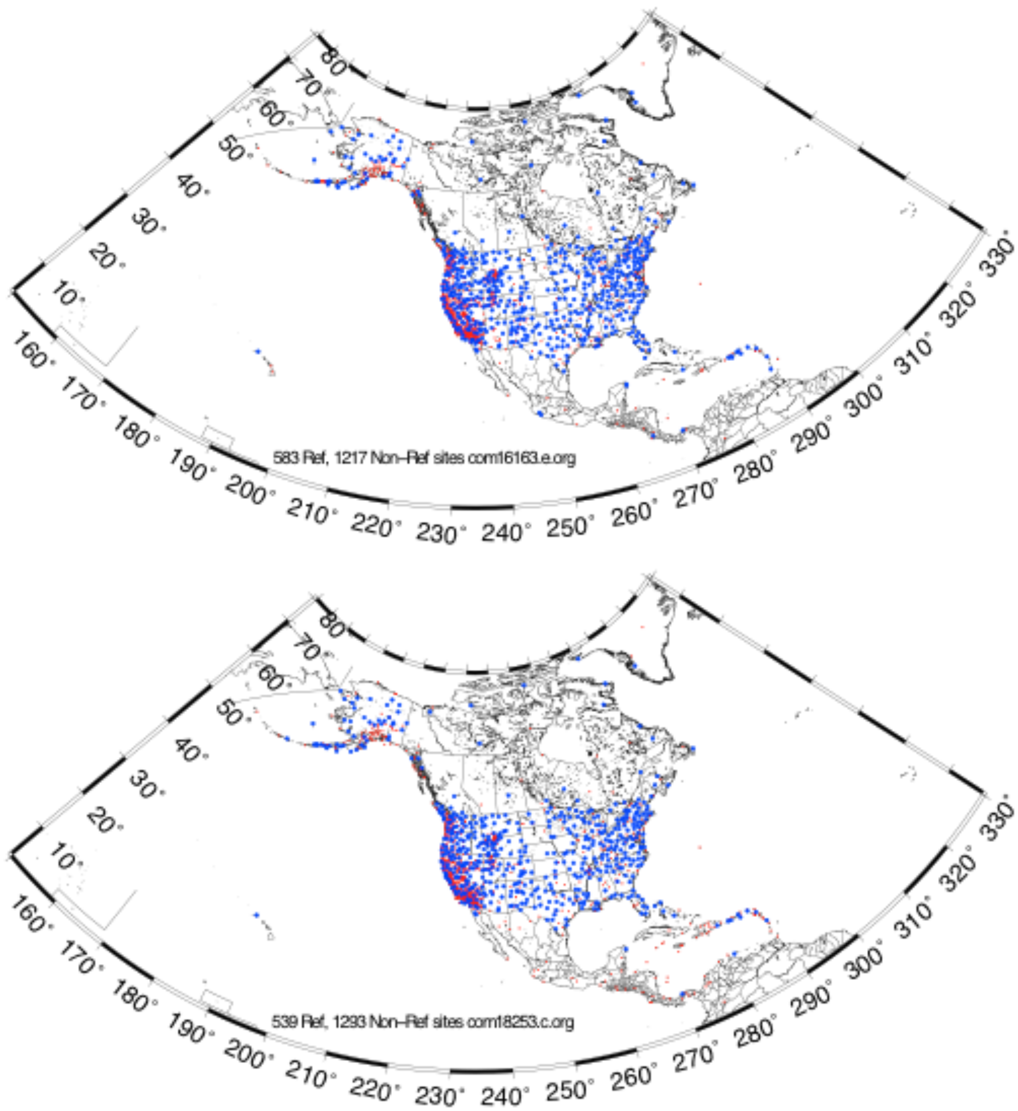

Figure 1: *Typical GAGE reference frame and non-reference sites from analyses in January 2011 (top) and January 2015 (bottom). In these figures, blue dots are reference frame sites and red squares are non-reference frame sites. In the context of the distribution of reference frame sites, the red dots are inconsequential. The reference frame sites are given as hierarchical lists on a 150 km grid spanning the GAGE network so the number of reference frame sites remains relatively constant even as the network grows in size.*

To demonstrate the effects of the UNR estimating scale changes we compare two other representations of the height time series for P113. The lower two frames of Figure 2 show the UNR results in the IGb08 frame where the scale is estimated globally rather than just using North America sites (c) and the GAGE results when scale is estimated using the large northern hemisphere span of the GAGE network (d). Visually, the UNR IGb08 results with scale estimates based on a global sets of sites (and thus less affected by height changes in the NA12 sites) looks similar to the GAGE results with no scale estimated. These results don't necessarily need to match because the UNR IGb08 results still have a scale estimated whereas the GAGE results have no scale removed. They look similar because global scale shows much smaller fluctuations than a North America region scale estimate (discussed in connection with figure 4).

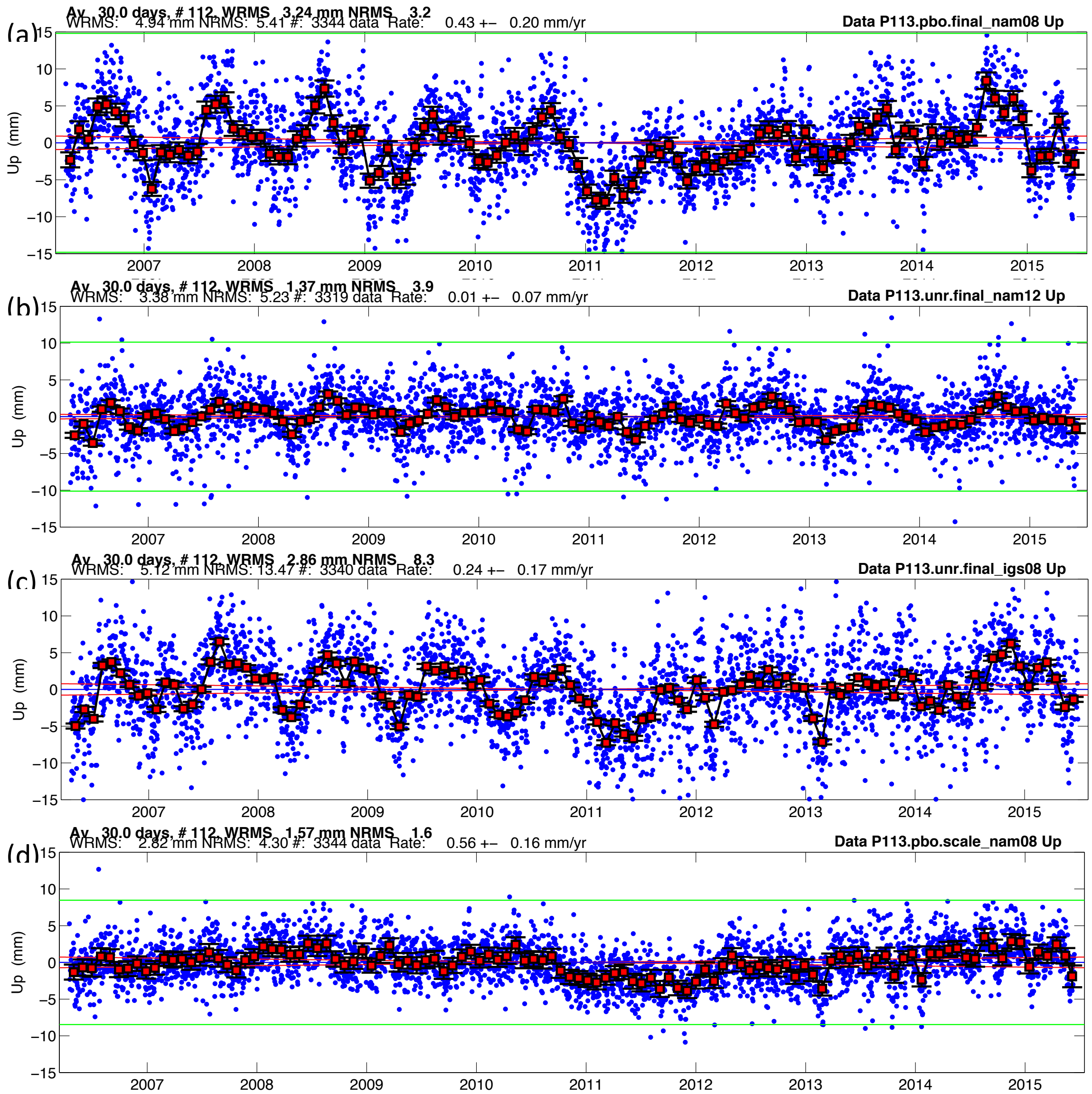

***Figure 2**: Height residuals at P113 after removing a linear trend and with 30-day averages superimposed on the time series. From top to bottom the analyses shown are (a) GAGE NAM08 analysis with no scale changes estimated, (b) UNR NA12 analysis with scale changes estimated based on North American sites only, (c) UNR IGb08 analysis where scale is removed but based on a global network of sites, and (d) GAGE NAM08 analysis but with scale estimated and removed from the time series.*

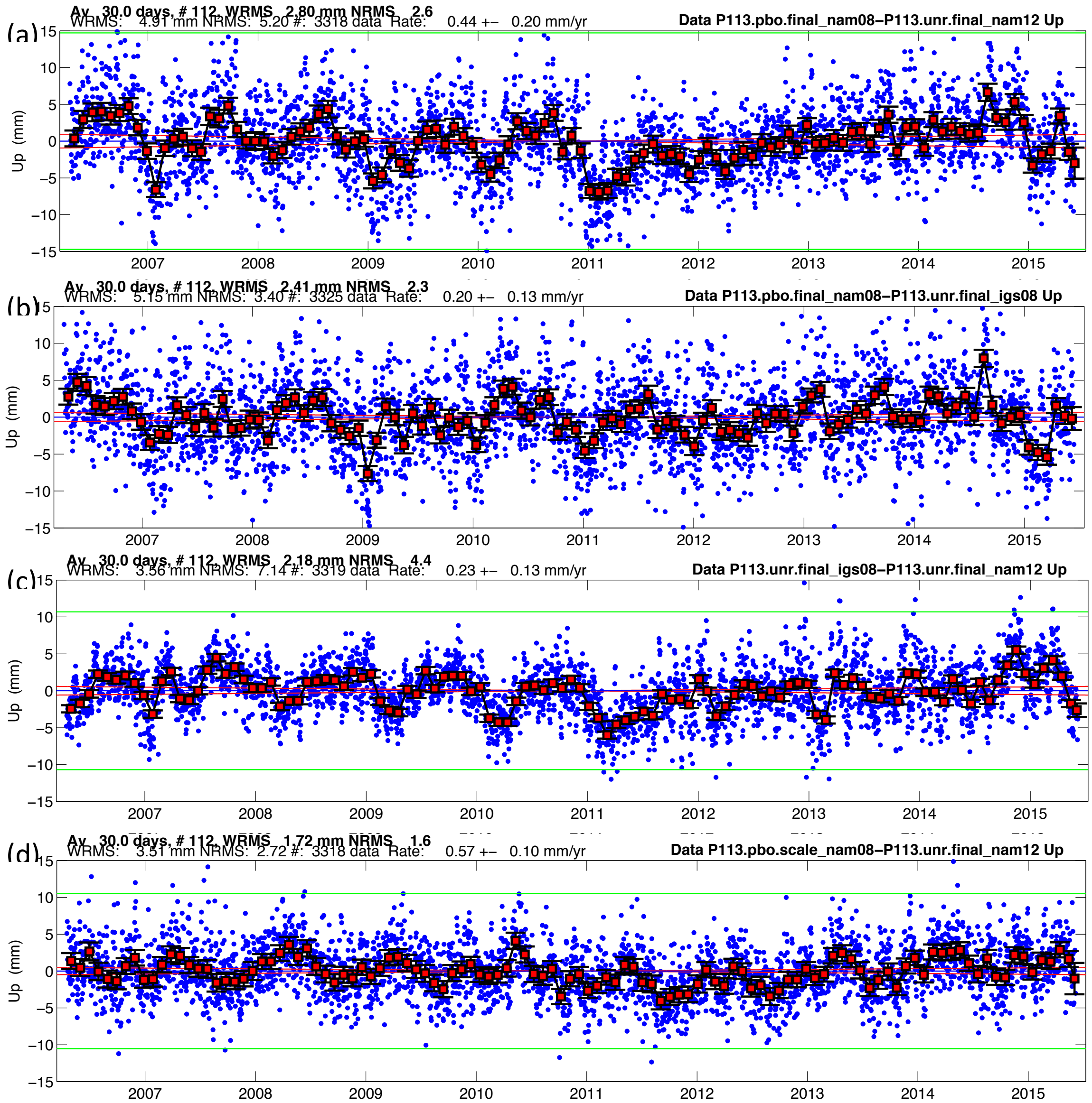

Figure 3: *Differences between the time series shown Figure 2. From top to bottom the plots are (a) GAGE NAM08 no scale changes estimated minus UNR NA12, (b) GAGE NAM08 no scale changes estimated minus UNR IGb08 global scale changes estimated, (c) UNR IGb08 minus UNR NA12, and (d) GAGE NAM08 with scale changes estimated minus UNR NA12 with scale changes estimated.*

Figure 3 shows the differences between the different time series in Figure 2 to highlight that when scale changes are treated similarly the systematic differences between the GAGE and UNR analyses are reduced (as measured by the RMS differences). 30-day averages of the differenced results are shown to highlight the systematic differences between the time series. As expected, the GAGE NAM08 minus UNR NA12 shows the low in 2011 and the trend afterwards. This visual pattern is very similar to the difference between UNR NA12 and UNR IGb08 (c) where these two solutions originate from the same PPP coordinates for the sites. The scope of the reference frame sites, North America only versus global, is the only difference between the analyses. The GAGE NAM08 minus the UNR IGb08 results (b) show less systematic deviations as does the difference between GAGE NAM08 results with scale estimated and the UNR NA12 results (d). There are some small differences in (d) and these might be reduced by using a reference frame network for the GAGE scale estimates that is similar in size to the UNR NA12 frame.

The differences in the nature of the time series are arising from the treatment of scale. In Figure 4 we show estimates of scale changes from the GAGE network and from the ANU/MIT global GPS re-analysis carried out as one of the GPS contributions to ITRF2014. The ANU/MIT is a global analysis, based on network processing, with satellite orbits estimated. Scale change results are shown as the mean of height differences between the daily position estimates and the reference frame sites using 6371 km as the mean radius of the Earth. Scale changes are not explicitly estimated in the global ANU/MIT analyses and the scale changes are inferred from the mean height adjustments to the IGb08 reference frame sites. The ANU/MIT results are shown for scale estimates using the sites distributed globally and using just sites in the northern hemisphere. The two results are similar showing that northern hemisphere sites have larger systematic scale changes and a strong annual period compared to southern hemisphere stations. A large percentage of these scale changes have been attributed snow and water loading during northern hemisphere winters. The southern hemisphere reference frame sites in the IGb08 system are of similar number to the northern hemisphere ones but they tend to be on islands and smaller land masses that do not seem to have large annual loading signals.

Two sets of scale change results are shown for GAGE. One set is obtained from simultaneously estimating translation, rotation and scale in the alignment to the NAM08 reference frame (red line) and the other is obtained simply from the mean of the height differences at the reference frame sites when only translation and rotation are estimated and the heights are down weighted in the estimation of translation and rotation (blue) (standard GAGE processing). The two methods generate effectively the same result. The annual signal in the GAGE and global analyses are correlated with the signal being larger in the GAGE analysis than in the northern hemisphere global but this difference in amplitude might be expected because of the smaller region covered by the GAGE network and reference frame sites. There are also shorter period variations in the GAGE analysis, which tend to have larger variance in the winters. These scale variations may be noise or they may be signal. The GAGE time series still contain these “signals” and users can remove them by removing mean height changes from a selected region if the user deems them to be noise. Time series that have scale estimated remove these signals. If the scale estimates are made available then the full time series can be reconstructed.

GAGE users should be aware that there are also subtle effects in scale estimates when combining PPP results with network generated GPS solutions. Since the PPP results have no inter-site correlations, the GAGE combined solution estimate of scale changes (or mean height differences) are dominated by the PPP results presumably because the small inter-site correlations in the network solutions result in the mean height estimate having a larger uncertainty than the basically square-root-of-N uncertainty generated from the PPP solution. Since GAGE uses 500 to 600 sites in the reference frame realization, the correlation effect seems to have a large impact. The impact of these correlations can be seen in the standard deviations of the scale change estimates from the NMT network solutions and the CWU PPP solutions. The CWU scale change estimates have standard deviations ~5 times smaller than those estimates from the NMT analyses. The standard deviations of the position estimates of individual sites is similar between the two analyses (after applying the GAGE variance scaling factors to the covariance matrices) and this the difference in the scale change estimates is not due to the relative magnitudes of the variances in the two solutions.

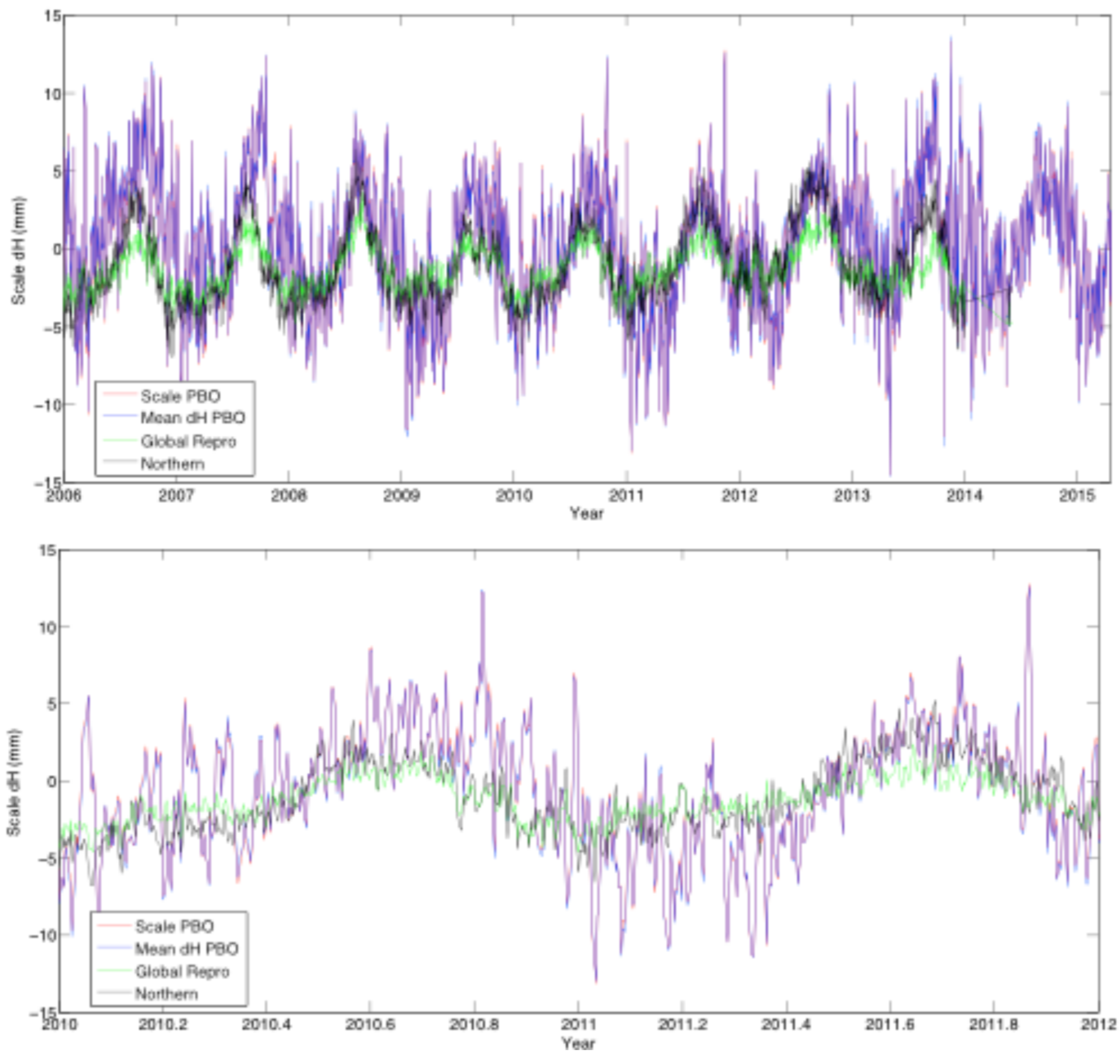

Figure 4: *Scale estimates from GAGE and global GPS analyses. The top figure shows the results from 2006 to date, which is similar to the time range shown in figures 2 and 3; the lower figure is a zoom from 2010-2012 to show some of the details. The red and blue lines, which are almost indistinguishable (and appear purple in the plot), are from estimating scale changes in the GAGE*

network (red) converted to height changes and from the mean height differences at the reference frame sites (blue) when scale changes are not estimated. (If scale changes are estimated the mean height differences are zero). The green and black lines are generated from the ANU/MIT ITRF2014 reprocessing results with the green curve being the mean of the differences of the heights of IGb08 reference frame sites (no scale estimated) globally and the black curve being the mean of the height differences at just the northern hemisphere sites in IGb08. For this interval, there are equals number of northern and southern hemisphere sites but the southern sites tend to be on smaller land masses than the northern hemisphere sites.

The second example site we will discuss is P144. The time series and difference plots for this site are shown in Figures 5 and 6. Of interest here is the apparent drop in late 2014 that is seen in the GAGE NAM08 analysis (5a) but is less obvious in the UNR NA12 analysis (5b). For these plots we also show the error bars for the daily estimates.

The assumptions about noise levels used in the calculation of the error estimates differ between the analysis groups. For the GAGE analysis, the covariance matrices given in the SINEX files submitted by NMT and CWU are multiplied by a separate factor for the two analyses such the average χ^2 per degree of freedom of the fit to the reference frame sites is unity. For NMT, the variance factor is 0.7 while for CWU it is 4.8. For the UNR solution, we plot the errors bars as given in their time series and these values are smaller than the GAGE estimates of the error bars.

As in the earlier example, the treatment of scale affects these results. In part (c) of Figure 5, UNR IGb08 analysis, the drop at the end of 2014 can be clearly seen. When scale is estimated for the GAGE NAM08 results, height variations look more similar to the UNR NA12 results than the other results. It is visually interesting that the GAGE results, with scale estimated, seems to show a more precipitous drop than the UNR NA12 solution but when the two time series are differenced as shown in Figure 6(d) the difference is not at all apparent. When the GAGE scale estimated results are overlaid on the UNR NA12 results, with a mean height difference removed, the two time series are consistent within their error bars.

Summary

The aim of this note is to show the impact of removing scale changes when daily GPS results are aligned to a reference frame. The GAGE analyses do not remove scale changes so that users are able to make their own assessments of whether or not to remove scale variations. For users wanting to remove scale changes from the GAGE analysis time series, they simply need to compute the mean of the height differences at a set of reference sites for each day and remove the daily mean from each site's height estimate on that day. GAGE will shortly release a new product that will contain the daily estimates of the mean of the height differences for the reference frame sites used in the GAGE analyses.

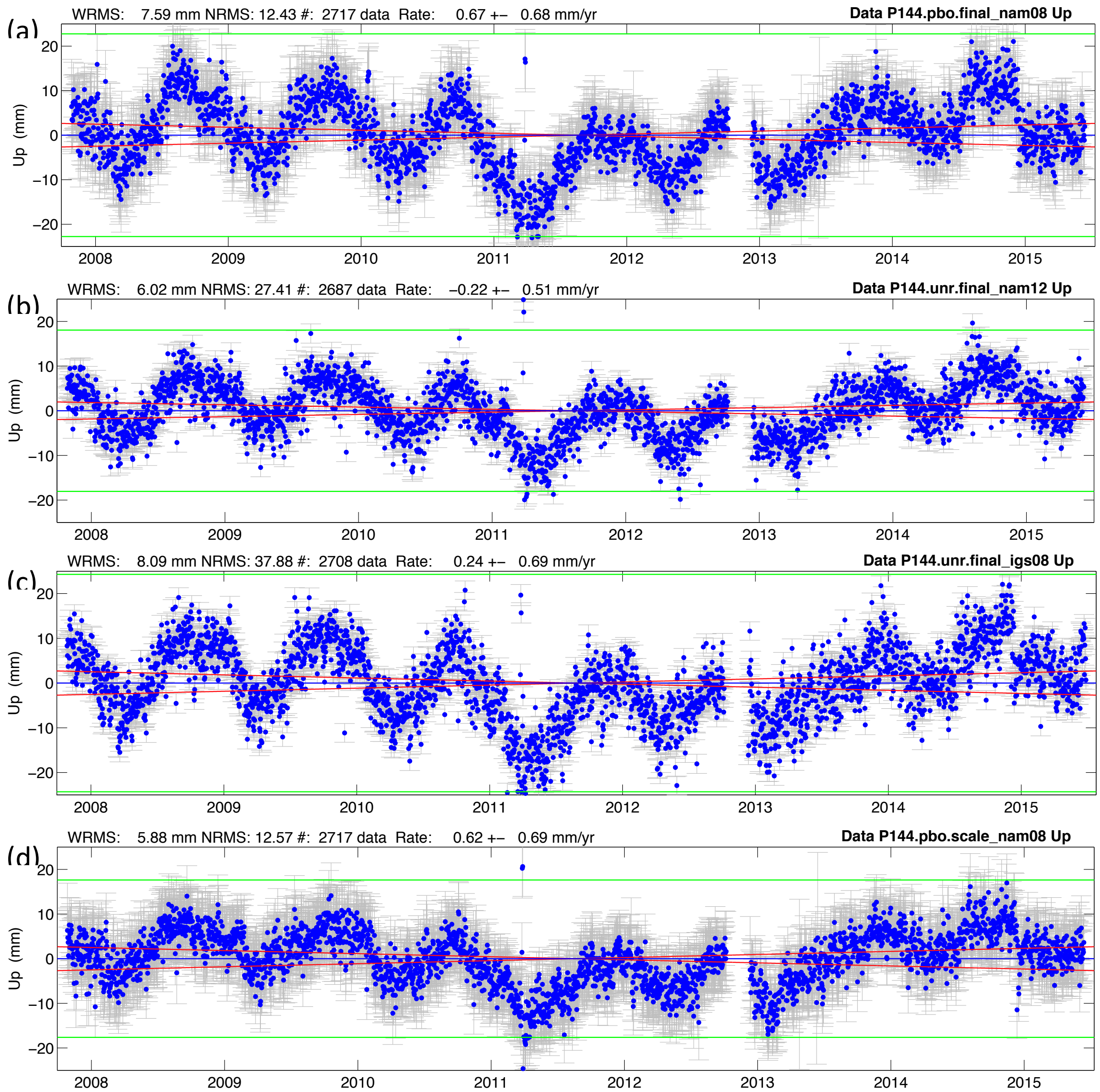

Figure 5: *Height estimates for P144 from different analyses. This figure is similar to Figure 1 but in this figure, the error bars of the daily estimates are shown. From the top to bottom: (a) GAGE NAM08 analysis with no scale estimated, (b) UNR NA12 analysis is scale estimated, (c) UNR IGS08 analysis where scale is removed but based on a global network of sites, and (d) GAGE NAM08 analysis but with scale estimated and removed from the time series.*

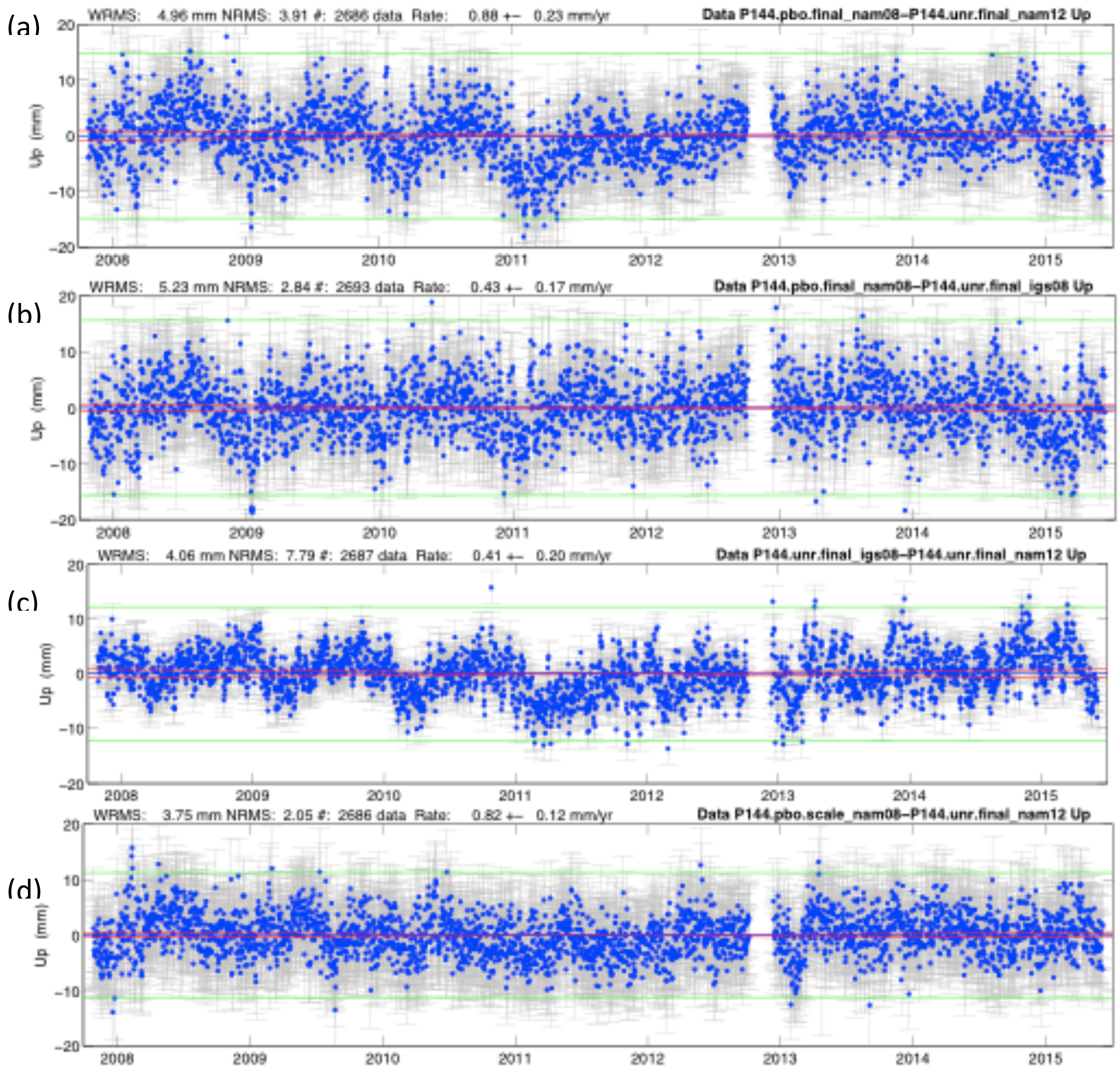

Figure 6: *Differences between the plots shown in figure 4. From top to bottom the plots are (a) GAGE NAM08 no scale minus UNR NA12, (b) GAGE NAM08 no scale minus UNR IGS08 global scale estimated, (c) UNR IGS08 minus UNR NA12, and (d) GAGE NAM08 with scale estimated minus UNR NA12 with scale estimated. One standard deviation errors are shown on the daily position estimates.*